

Representation-based Nearest Feature Plane for Pattern Recognition

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ABSTRACT. *In this paper, an improved method based on nearest feature plane (NFP), called as representation-based nearest feature plane (RNFP), is proposed for biometric recognition. Borrowing the concept from the nearest neighbor plane (NNP) classifier and center-based nearest neighbor (CNN) classifier, RNFP chooses the valuable representation of the class to reduce the computational complexity of NFP. A large number of experiments on some databases are used to evaluate the proposed algorithm and the result demonstrates that the proposed method take lower computational complexity and achieve similar performance of NFP.*

Keywords: Nearest Feature Plane; Nearest Feature Line; Nearest Neighbor; Pattern Recognition.

1. Introduction. The processing of pattern recognition general needs a two-step process, the first step is feature extraction, for instance PCA [1, 2], LDA [3, 4], ICA [5] and laplacianfaces [6], the second step is the classification. Nearest neighbor (NN) [7] is the one of the important classifier in pattern recognition area. However, the number of prototype samples is usually very small, which makes the recognition rate be improved very difficult. So nearest feature line (NFL) [8] was proposed for face recognition by Stan Z. Li et al in 1999. NFL attempts to enhance the representational capacity of a sample set of limited size by using the line passing through each pair of the samples belonging to the same class. All the lines constituted by samples in the same class are called the feature lines (FL) corresponding to the class. The authors of ref. [8] explain that a feature line provides information about the possible linear variants of two sample points. NFL shows good performance in many applications including face recognition [9-12], audio

retrieval [13], speaker identification [14], image classification [15], object recognition [16] and pattern classification [17]. The authors of NFL explain that the feature line can give information about the possible linear variants of the corresponding two samples very well.

After the NFL being proposed, Chien and Wu proposed the nearest feature plane (NFP) [18] in 2002 year. NFP also attempts to enhance the representational capacity of a sample set of limited size by using the plane passing through each three of the samples belonging to the same class.

NFL and NFP improve the classification ability successfully compared to nearest neighbor (NN). However NFL and NFP also have some drawbacks that limit their further application in practice. For example, NFL and NFP will result in the large computational complexity problem when there are many samples in each class.

Based on the feature line space and feature plane space, Zheng et al proposed the nearest neighbor line (NNL) and nearest neighbor plane (NNP) [17] in 2004, GAO et. al. proposed the center-based nearest neighbor (CNN) [19] in 2007 and Zhou et. al. proposed the nearest feature midpoints (NFM) [20] in 2000. Feng et. al. proposed the nearest feature centre (NFC) [12] classifier in 2012 year.

Motivated by the NNP classifier and CNN classifier, representation-based nearest feature plane (RNFP) for biometric recognition is proposed in this paper. RNFP classifier uses the representation of the class to reduce the computational complexity. At the same time, RNFP tries its best to get the similar performance of NFP classifier. A large number of experiments are designed to evaluate the proposed algorithm and the result demonstrates that the proposed method take lower computational complexity and achieve similar performance of NFP, which is better than that of NN, NFL, NFC, NNP and CNN.

2. Background. In this section, we will introduce nearest feature line, extended nearest feature line and shortest feature line segment. Suppose that $Y = \{y_i^c, c = 1, 2, \dots, M, i = 1, 2, \dots, N_c\} \subset R^D$ denote the prototype set, where y_i^c is the i th prototype belonging to c -class, M is the number of class, and N_c is the number of prototypes belonging to the c -class.

2.1. Background. The core of NFL is the feature line metric. As shown in Figure 1, the NFL classifier doesn't compute the distance of query sample y and y_i^c ; doesn't calculate the distance of y and y_j^c , while NFL classifier calculates the feature line distance between query sample y and the feature line $\overline{y_i^c y_j^c}$

The feature line distance between point y and feature line $\overline{y_i^c y_j^c}$ is defined as:

$$d(y, \overline{y_i^c y_j^c}) = \|y - y_p^{ij,c}\| \quad (1)$$

where $y_p^{ij,c}$ is the projection point of y on the feature line $\overline{y_i^c y_j^c}$, $\|\cdot\|$ means the L2-norm.

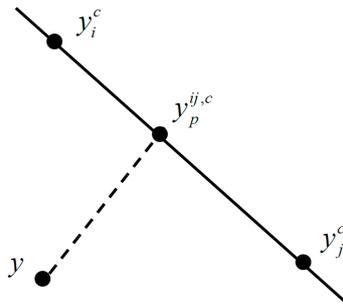


FIGURE 1. the metric of NFL

Given the query sample feature points y , the classification process with nearest feature line algorithm is as follows:

Step 1: according to equation (1), calculate the distance between query sample point y and all the feature lines which are belonged to c -class, where $1 \leq c \leq M$.

Step 2: the distances are sorted in ascending order, each being associated with a class identifier, the corresponding two feature samples, and the corresponding location parameter t .

Step 3: the NFL distance is the first rank distance G

$$d_{\min}^{i_0 j_0, c_0} = \min_{1 \leq c \leq M} \min_{1 \leq i, j \leq N_c, i \neq j} d(y, \overline{y_i^c y_j^c}) \quad (2)$$

The first rank gives the NFL classification composed of the best matched class c_0 .

2.2. Nearest feature Plane. The feature plane metric is defined as the Euclidean distance from the query sample to the feature plane, as shown in Figure 2, the distance between the query sample y to feature plane $\overline{y_i^c y_j^c y_k^c}$ is

$$d(y, \overline{y_i^c y_j^c y_k^c}) = \|y - y_p^{ijk,c}\| \quad (3)$$

where $y_p^{ijk,c}$ is the projection point of query sample y on the feature plane $\overline{y_i^c y_j^c y_k^c}$, $\|\cdot\|$ means L2- norm.

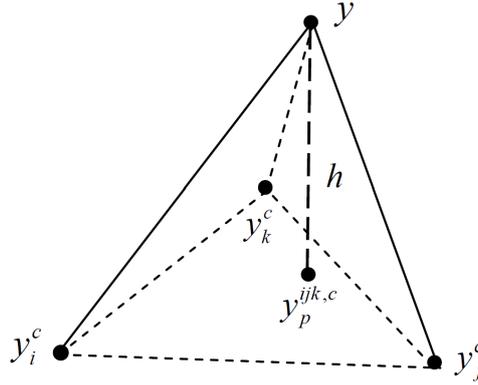


FIGURE 2. the metric of NFP

Given the query sample feature points y , the classification process with nearest feature line algorithm is as follows:

Step 1: according to equation (3), calculate the distance between query sample point y and all the feature lines which are belonged to c -class, where $1 \leq c \leq M$.

Step 2: the distances are sorted in ascending order, each being associated with a class identifier, the corresponding two feature samples, and the corresponding location parameter t .

Step 3: the NFL distance is the first rank distance G

$$d_{\min}^{i_0 j_0 k_0, c_0} = \min_{1 \leq c \leq M} \min_{1 \leq i, j, k \leq N_c, i \neq j \neq k} d(y, \overline{y_i^c y_j^c y_k^c}) \quad (2)$$

The first rank gives the NFL classification composed of the best matched class c_0 .

2.3. NNP Classifier. In the NNP approach, instead of computing all possible feature planes for each class, the feature plane passing through the closest three of points is considered. Hence, during testing, the number of feature planes considered is equal to the number of classes. So the computational complexity of NNP classifier is much less than NFP classifier.

2.4. **CNN Classifier.** Be different with NFL classifier, CNN classifier consider another type line for classification, which is constituted by the mean sample and one sample chose randomly in the class. As shown in the Figure 3, the metric of CNN is defined as $d(y, \overline{y_i^c o^c}) = \|y - y_p^{i,c}\|$

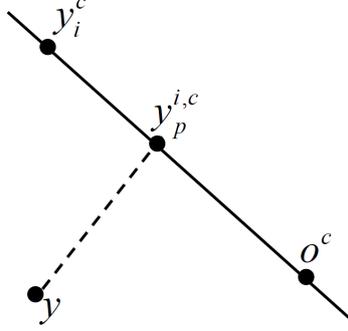


FIGURE 3. the metric of CNN

2.5. **NFC Classifier.** Show in the Figure 4, NFC uses the feature center metric, which is defined as the Euclidean distance between query sample y and the feature center $y_o^{ij,c}$, which is $d_{NFC}(y, \overline{y_i^c y_j^c}) = \|y - y_o^{ij,c}\|$, where $y_o^{ij,c}$ is the center of inscribed circle of the triangle $\Delta y y_i^c y_j^c$.

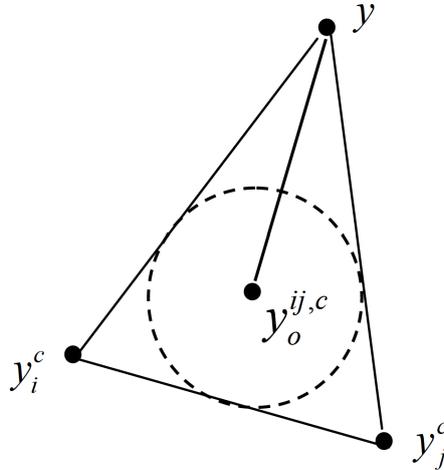
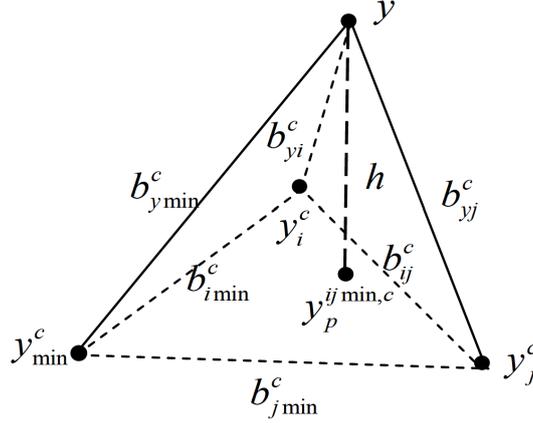


FIGURE 4. the metric of NFC

3. **The proposed methods.** The NFP classifier gain better performance compared to the NFL classifier and NN classifier. However, the computational complexity of NFP is larger than that of NFL and NN. To utilize the advantages brought by the feature planes and counteract the drawbacks of the traditional NFP, an efficient method, called representation-based nearest feature plane classifier, is proposed in this section.

3.1. **The idea of RNFP.** The main difference of RNFP and NFP is as follows. The three points of a feature plane is chose randomly when the NFP classifier is used. However, using the RNFP classifier, the three points of a feature plane is different. The first point is nearest prototype sample of the class from the query sample. Another two points is

FIGURE 5. the *metric* of RNFP

chosen randomly from the rest prototype samples belonged to the same class. It is also shown in Fig. 3.

Let us define $s = (b_{i\min}^c + b_{j\min}^c + b_{ij}^c)/2$. Then, the volume of a tetrahedron is given by:

$$V = \frac{h\sqrt{s(s-b_{ij}^c)(s-b_{j\min}^c)(s-b_{i\min}^c)}}{3} \quad (1)$$

But the volume is also

$$V = \frac{1}{288} \begin{vmatrix} 0 & (b_{ij}^c)^2 & (b_{i\min}^c)^2 & (b_{yi}^c)^2 & 1 \\ (b_{ij}^c)^2 & 0 & (b_{j\min}^c)^2 & (b_{yj}^c)^2 & 1 \\ (b_{i\min}^c)^2 & (b_{j\min}^c)^2 & 0 & (b_{y\min}^c)^2 & 1 \\ (b_{yi}^c)^2 & (b_{yj}^c)^2 & (b_{y\min}^c)^2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix} \quad (2)$$

So we can solve (1) and (2) for h , which is the distance between query sample and the feature plane.

The classification process of RNFP is as follows.

The distance between the query sample y and each feature line $\overline{y_i^c y_j^c y_{\min}^c}$ is calculated, which generates a number of distances. The distances are sorted in ascending order, each being associated with a class identifier and two prototypes. The RNFP distance is the first rank distance.

$$d(y, \overline{y_{i^*}^c y_{j^*}^c y_{\min}^c}) = \min_{1 \leq c \leq M, 1 \leq i < j \leq (N_c - 1)} d(y, \overline{y_i^c y_j^c y_{\min}^c}) \quad (3)$$

The first rank gives the best matched c^* -class and the two best matched prototypes i^* and j^* of the class.

The query sample y will be classified into the c^* -class.

3.2. Computational complexity. In this part, we mainly introduce the computational complexities of NFP, RNFP, NNP, CNN, NFC, NFL, NN. Suppose there are N_c training prototype samples in c -class and each of them is an L -dimensional vector. The number of class is M .

There are $N_c(N_c - 1)(N_c - 2)/3/2$ feature planes in the class c . The cost of computing the distance between query sample and a feature plane is kL , where k is a constant. So with the NFP classifier, the whole cost is $kL \times N_c(N_c - 1)(N_c - 2)/3/2$ in the c -class. The complexity of class c is $O(MLN_c^3)$.

There are $N_c(N_c - 1)/2$ feature lines in the class c . The cost of computing the distance between query sample and a feature line is kL , where k is a constant. So with the NFL classifier, the whole cost is $kL \times N_c \times (N_c - 1)/2$ in the c -class. The complexity of class c is $O(MLN_c^2)$.

In NFC classifier, $N_c(N_c - 1)/2$ feature centers are constituted in the class c . The cost of computing the distance between query sample and a feature center is kL , the cost of computing a feature center is pL , where k and p are the constant. So with the NFC classifier, the whole cost is $(k + p)LN_c(N_c - 1)/2$ in the class c . The complexity of class c is $O(MLN_c^2)$.

In NNP classifier, only one feature plane is constituted in the class c . The cost of computing the distance between query sample and a feature plane is kL , the cost of finding the nearest three sample from query sample in the class c is pLN_c , where k and p are the constant. So with the NNP classifier, the whole cost is $(pN_c + k)L$ in the class c . The complexity of class c is $O(MLN_c)$.

In CNN classifier, N_c feature lines are constituted in the class c . The cost of computing the distance between query sample and a feature line is kL , the cost of computing the center of the class c is pL , where k and p are the constant. So with the CNN classifier, the whole cost is $(k + p)LN_c$ in the class c . The complexity of class c is $O(MLN_c)$.

With the NN classifier, it is easy for us to know that the complexity of class c is $O(MLN_c)$.

With the RNFP classifier, $(N_c - 1)(N_c - 2)/2$ feature planes are constituted in the class c . The cost of computing the distance between query sample and a feature plane is kL , the cost of finding the nearest sample from query sample in the class c is pLN_c , where k and p are the constant. So with the RNFP classifier, the whole cost is $kL(N_c - 1)(N_c - 2)/2 + pLN_c$ in the class c . The complexity of class c is $O(MLN_c^2)$. The complexities of the several classifiers are also shown in Table 1.

TABLE 1. the computational complexity of several classifiers

Classifiers	Computational complexity
NFP	$O(MLN_c^3)$
RNFP	$O(MLN_c^2)$
NFL	$O(MLN_c^2)$
NFC	$O(MLN_c^2)$
NNP	$O(MLN_c)$
CNN	$O(MLN_c)$
NN	$O(MLN_c)$

4. Experimental results. The classification performance of RNFP is compared with NFP, NNP, NFL, CNN, NN and NFC. All experiments are implemented using the MATLAB R2010 under Pentium(R) Dual-Core CPU with a clock speed of 3.06 GHz and 3GB RAM.

“Randomly choose N” scheme is taken for comparison: N images per person are randomly chosen from the tested database. The rest images of the tested database are used for testing. The whole system runs 20 times. To test the robustness of new algorithms, the average recognition rate is used to weigh the performance of new algorithms.

4.1. **Comparison on AR database.** In the first experiment, the “randomly choose N” scheme is adopted on AR face database. The AR [21] database contains over 4000 face images of 126 subjects (70 men and 56 women). To reduce the computational complexity, the subset of AR database includes 1680 face images of 120 individuals with fourteen face images of different expressions and lighting conditions except wearing sun glasses and wearing scarf per subject, and all images in AR database were manually cropped into 50×40 pixels. One subject's face images of AR database are shown in Figure 6.



FIGURE 6. one subject's face images of the subset of AR face database

The result is shown in Figure 7; the lines of NFP and RNFP are almost overlapping. So the RNFP method achieves the similar performance compared with NFP method. However, the complexity of RNFP is much less than that of NFP. Compared with NFL, NNP, CNN and NN, RNFP achieves better performance. When the number of prototype sample of each class is 5, the average recognition rate of RNFP, NFP, NFL, NNP, CNN, NN and NFC are 96.50%, 96.53%, 94.33%, 92.92%, 92.83%, 88.09% and 94.46%, respectively.

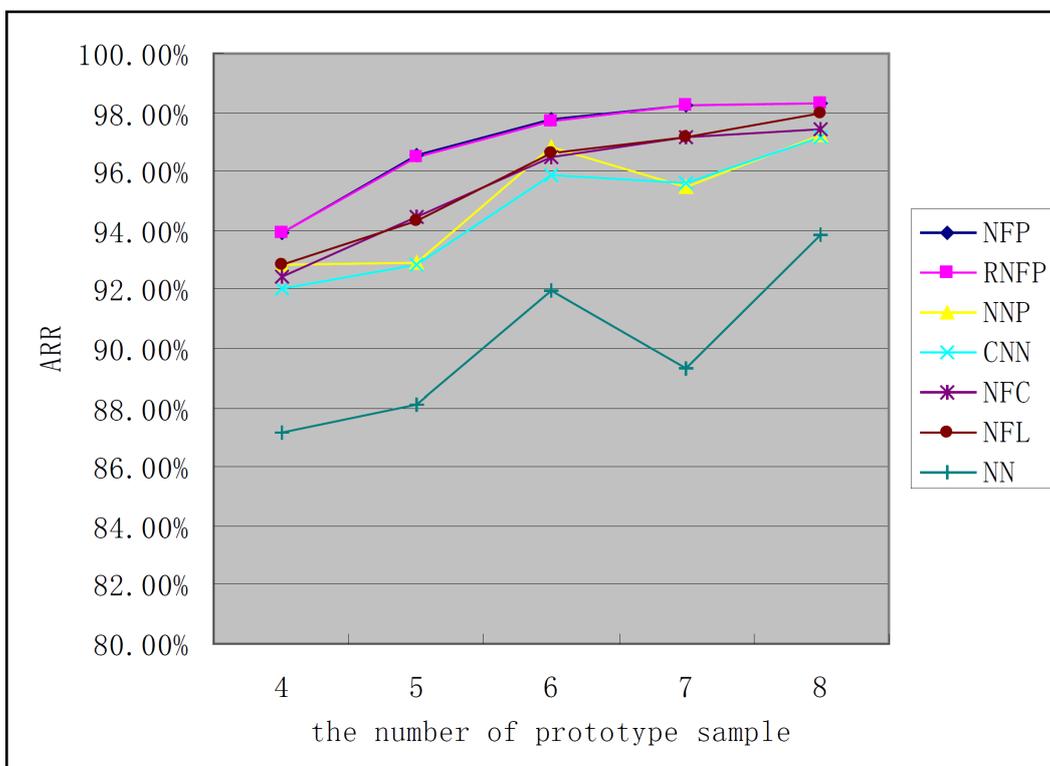


FIGURE 7. the recognition rate on AR face database

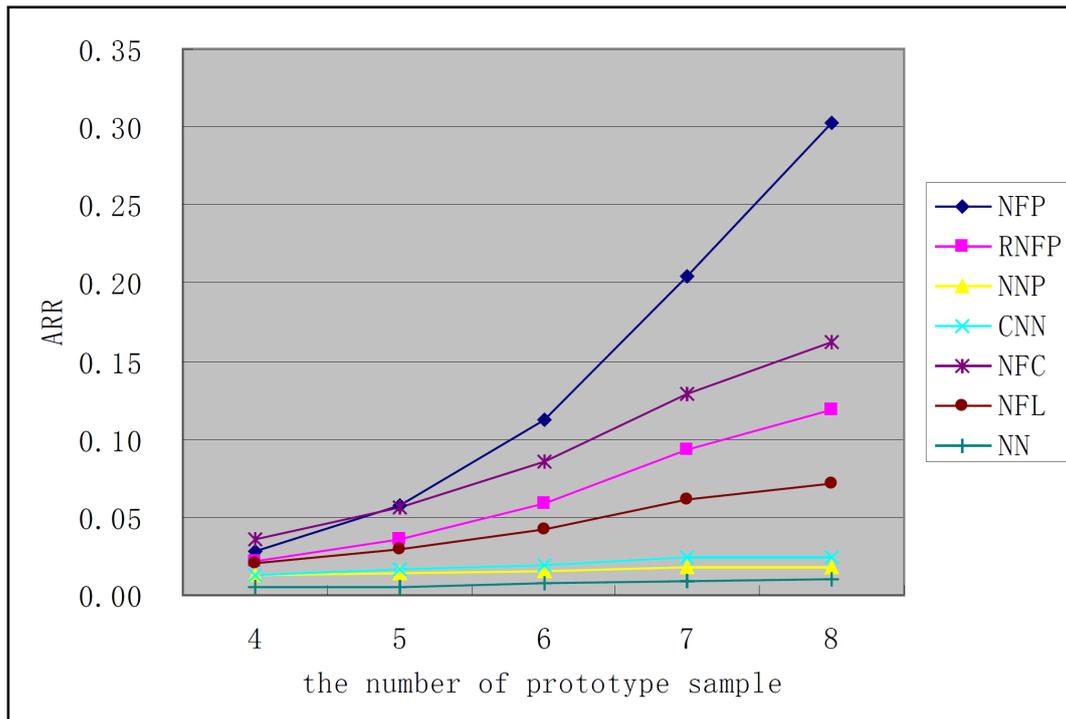


FIGURE 8. the run time on AR face database

5. Comparison on PolyU FKP database. In the second experiment, the “randomly choose N” scheme is adopted on PolyU FKP database [22]. PolyU FKP database is made by the Hong Kong Polytechnic University. (Finger-Knuckle-Print) FKP images were collected from 165 volunteers, including 125 males and 40 females. Among them, 143 subjects were 20–30 years old and the others were 30–50 years old. We collected samples in two separate sessions. In each session, the subject was asked to provide 6 images for each of the left index finger, the left middle finger, the right index finger, and the right middle finger. Therefore, 48 images from 4 fingers were collected from each subject. In total, the database contains 7,920 images from 660 different fingers. The average time interval between the first and the second sessions was about 25 days. The maximum and minimum intervals were 96 days and 14 days, respectively. A subset of PolyU FKP database including 1200 images of 100 left index finger is used in the experiment and all images were manually cropped into 40×80 pixels.

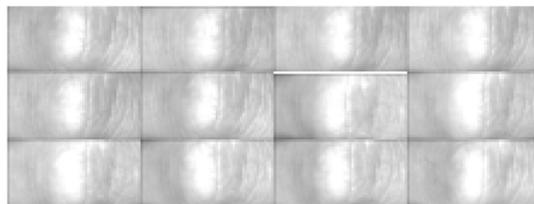


FIGURE 9. one subject's images of the subset of PolyU FKP database

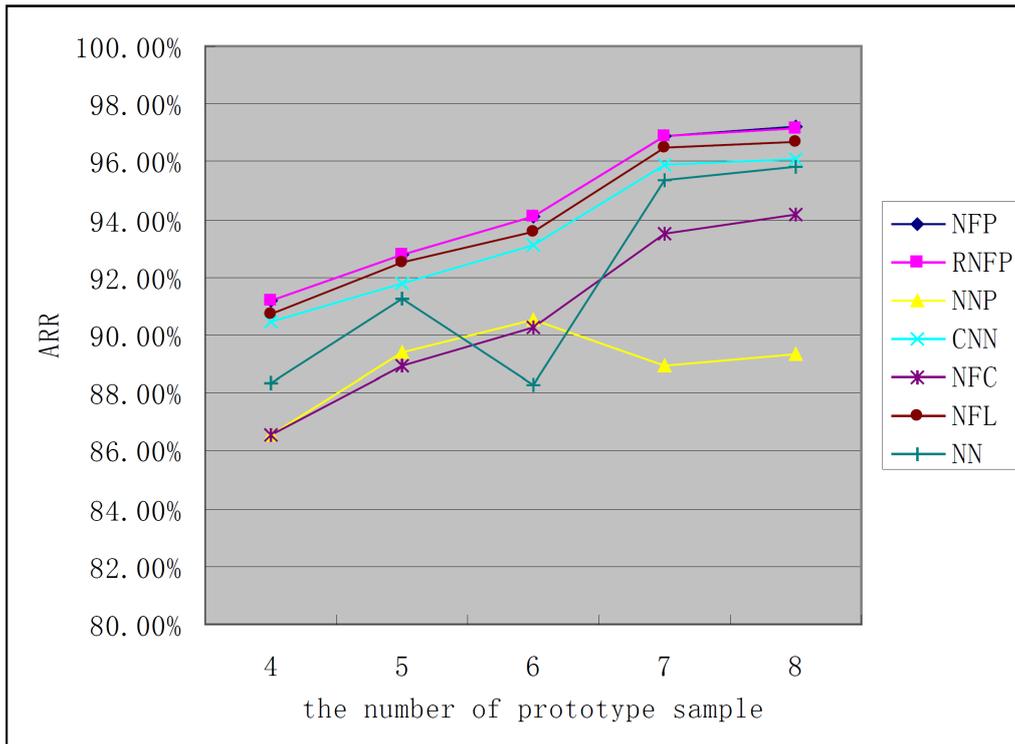


FIGURE 10. the recognition rate on PolyU FKP database

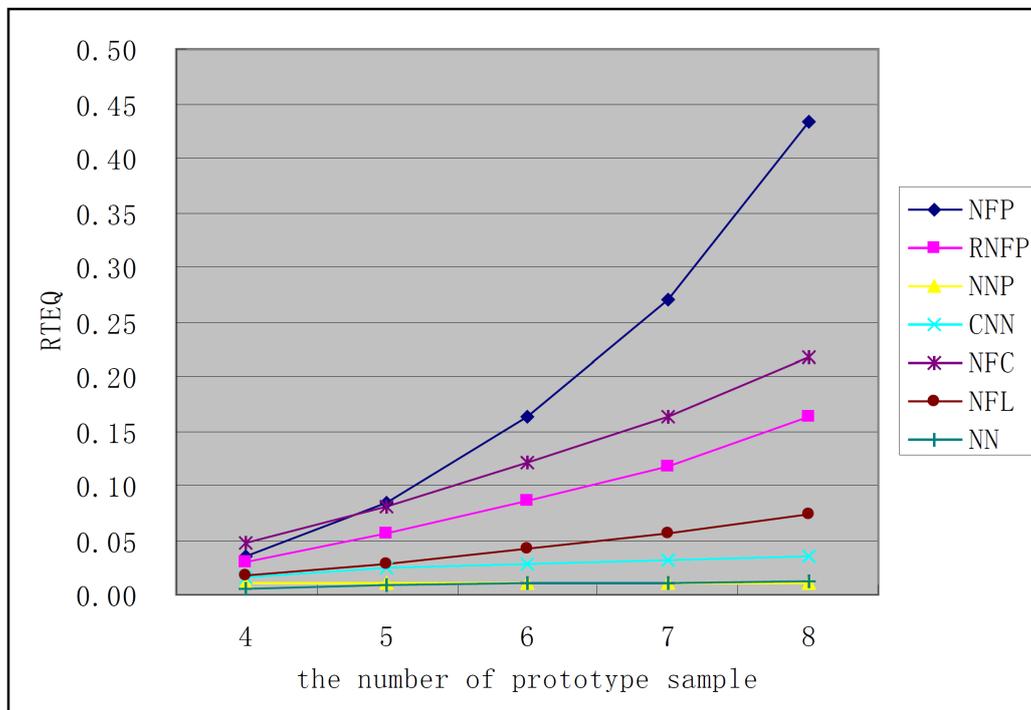


FIGURE 11. the run time on PolyU FKP database

In the second experiment, the “randomly choose N” scheme is adopted on PolyU FKP database. The result is shown in Figure 10; the lines of NFP and RNFP are almost overlapping. So the RNFP method achieves similar performance compared with NFP method. However, the complexity of RNFP is much less than that of NFP, which can be gained in part D. Compared with NFL, NNP, CNN and NN, RNFP achieves better performance. When the number of prototype sample of each class is 5 the average recognition rate of RNFP, NFP, NFL, NNP, CNN, NN and NFC are 92.75%, 92.76%, 92.21%, 90.26%, 91.52%, 90.87% and 89.67%, respectively.

5.1. Comparison on PolyU Multispectral palmprint database. In the third experiment, the “randomly choose N” scheme is adopted on PolyU Multispectral palmprint database [23]. PolyU Multispectral palmprint database is made by the Hong Kong Polytechnic University. Multispectral palmprint images were collected from 250 volunteers, including 195 males and 55 females. The age distribution is from 20 to 60 years old. We collected samples in two separate sessions. In each session, the subject was asked to provide 6 images for each palm. Therefore, 24 images of each illumination from 2 palms were collected from each subject. In total, the database contains 6,000 images from 500 different palms for one illumination. The average time interval between the first and the second sessions was about 9 days. A subset of PolyU Multispectral palmprint database including 1200 images of 100 palms is used in the experiment and all images were manually cropped into 40×50 pixels.

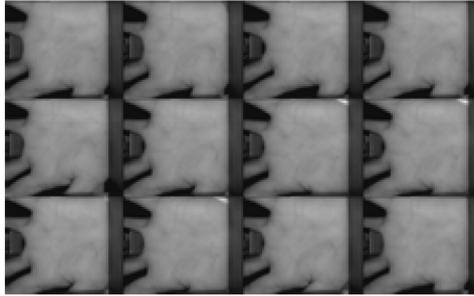


FIGURE 12. one subject’s images of the subset of PolyU Multispectral palmprint database

In the third experiment, the randomly choose N scheme is adopted on PolyU Multispectral palmprint database. The result is shown in Figure 13; the lines of NFP and RNFP are almost overlapping. So the RNFP method achieves almost same performance compared with NFP method. However, the complexity of RNFP is much less than that of NFP. Compared with NFL, NNP, CNN and NN, RNFP achieves better performance. When the number of prototype sample of each class is 5, the average recognition rate of RNFP, NFP, NFL, NNP, CNN, NN and NFC are 94.59%, 94.56%, 93.72%, 93.64%, 93.57%, 92.31% and 92.20%, respectively.

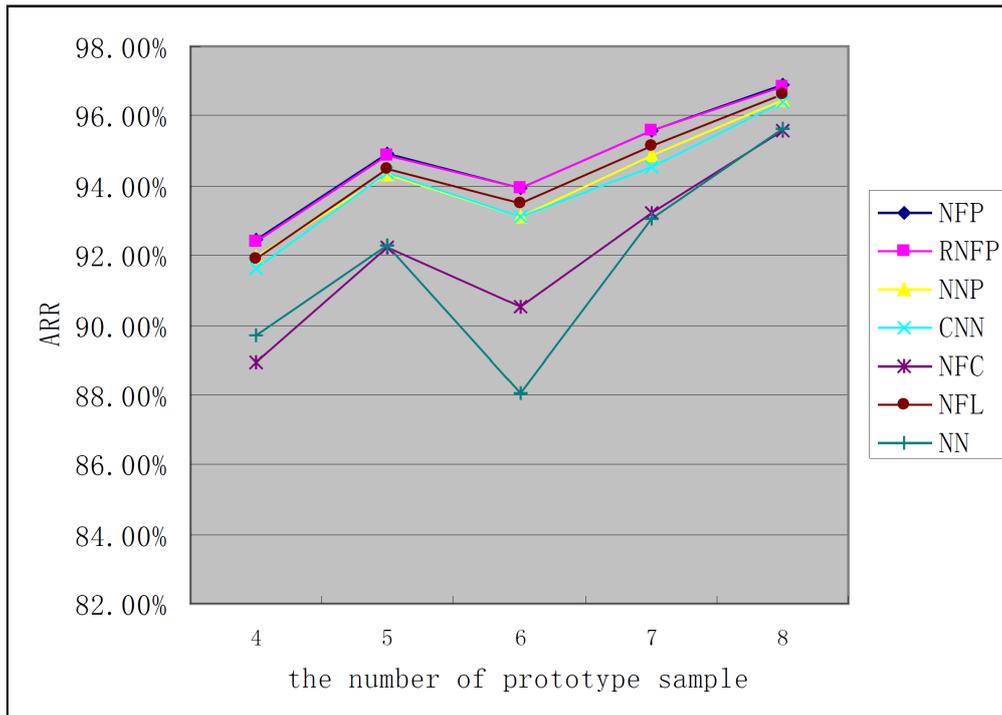


FIGURE 13. the recognition rate on PolyU Multispectral palmprint database

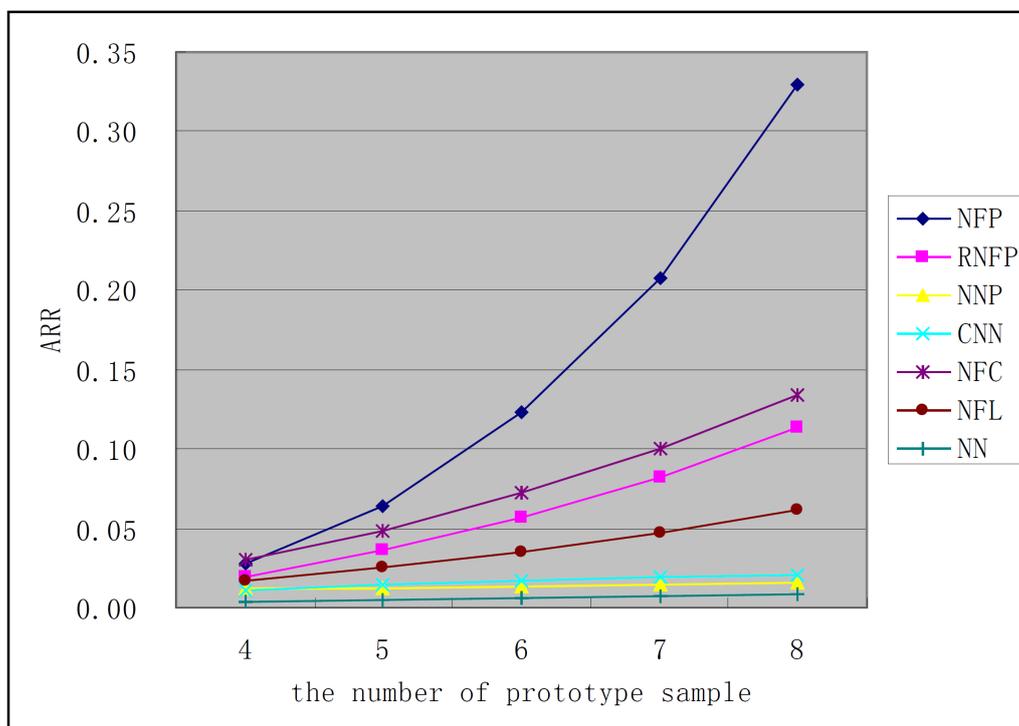


FIGURE 14. the run time on PolyU Multispectral palmprint database

5.2. **Comparison on PolyU Multispectral palmprint database.** In the fourth experiment, the “randomly choose N” scheme is adopted on soil object database. The SOIL data set [24] was recently used as a color object-recognition task and we use it in the

following experiment. In the experimental data set, there are 46 objects and each object has 21 different views (images) that are taken every 9° around an axis passing through the object. Each image is downsampled to a 24×32 color one with R, G, B channels and serves as the features being used. Some subjects images of subset of soil object database are shown in figure 15.



FIGURE 15. some subject's images of the subset of soil object database

In the fourth experiment, the “randomly choose N” scheme is adopted on soil object database. The result is shown in Figure 13; the lines of NFP and RNFP are almost overlapping. So the RNFP method achieves almost same performance compared with NFP method. However, the complexity of RNFP is much less than that of NFP. Compared with NFL, NNP, CNN and NN, RNFP achieves better performance. When the number of prototype sample of each class is 5, the average recognition rate of RNFP, NFP, NFL, NNP, CNN, NN and NFC are 74.40%, 74.43%, 74.02%, 73.61%, 72.28%, 68.99% and 73.17%, respectively.

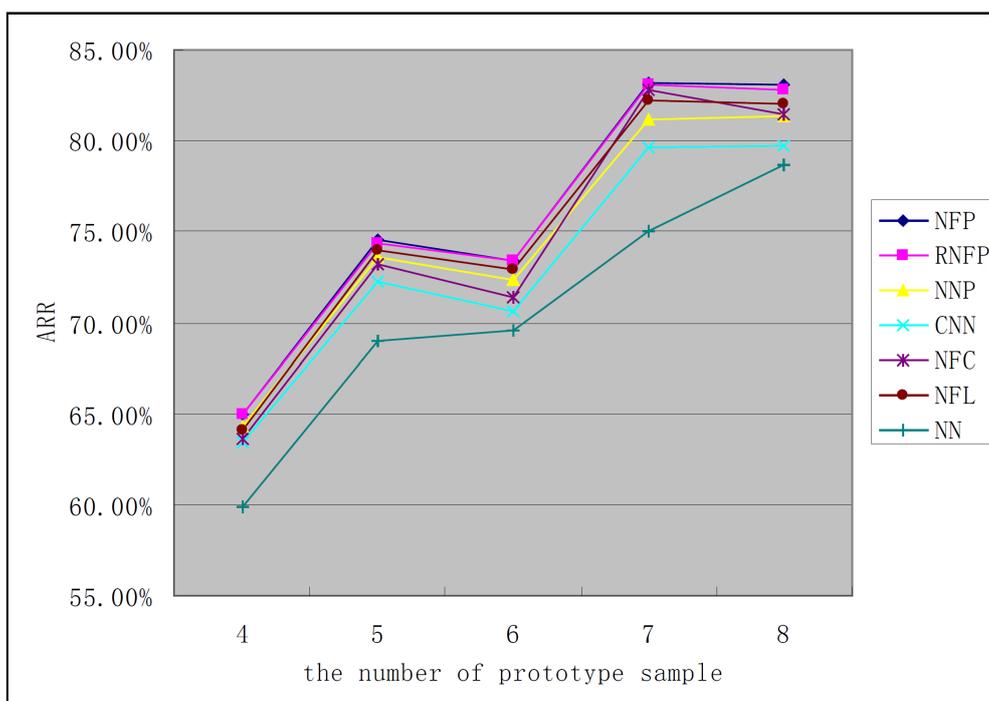


FIGURE 16. the recognition rate on soil object database

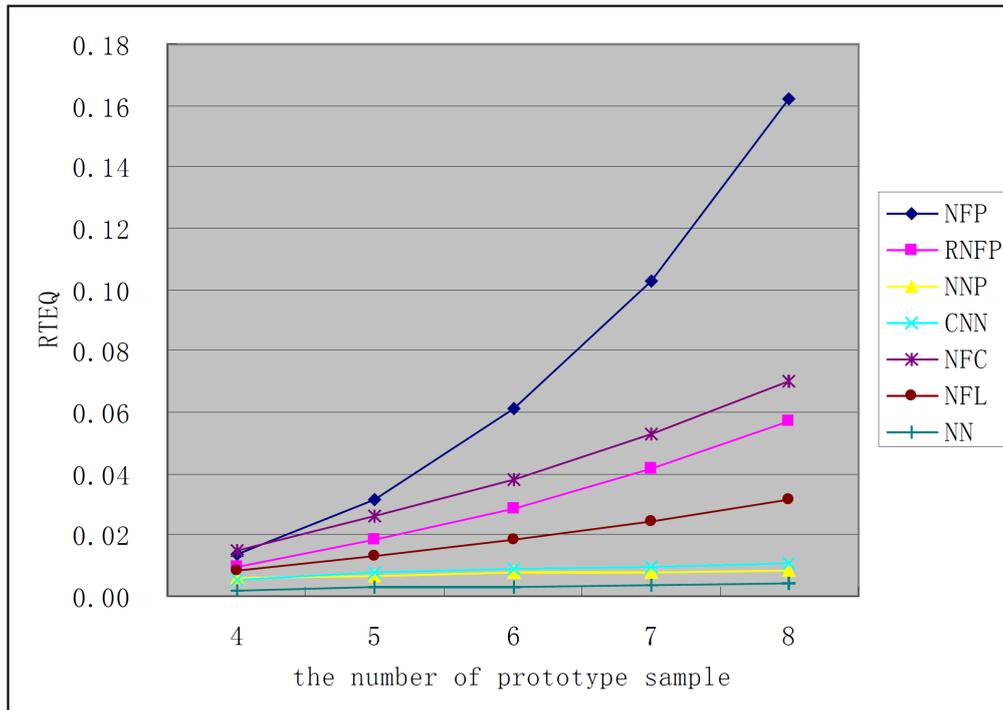


FIGURE 17. the recognition rate on soil object database

6. Conclusions. In this paper, we propose representation-based nearest feature plane classifier for biometric recognition based on NFP classifier. The proposed classifier takes the advantages of NFP and reduces the computational complexity of NFP. It achieves the similar recognition rate of NFP, which is the best recognition rate among variant classifiers. Experimental results confirm the efficiency of the proposed algorithm.

REFERENCES

- [1] M. Turk, and A. Pentland, Eigenfaces for recognition, *Journal of Cognitive Neuroscience*, vol. 3, no. 1, pp. 71-86, 1991.
- [2] H. B. Kekre, and K. Shah, Performance comparison of kekre's transform with PCA and other conventional orthogonal transforms for face recognition, *Proc. of the 2nd International Conference on Emerging Trends in Engineering & Technology*, pp. 873-879, 2009.
- [3] P. Belhumeur, J. Hespanha, and D. Kriegman, Eigenfaces vs. fisherfaces: recognition using class specific linear projection, *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 19, no. 7, pp. 711-720, 1997.
- [4] C. Y. Chang, C. W. Chang, and C. Y. Hsieh, Applications of block linear discriminant analysis for face recognition, *Journal of Information Hiding and Multimedia Signal Processing*, vol. 2, no. 3, pp. 259-269, 2011.
- [5] M. S. Bartlett, J. R. Movellan, and T. J. Sejnowski, Face recognition by independent component analysis, *IEEE Trans. Neural Networks*, vol. 13, no. 6, pp. 1450-1464, 2002.
- [6] X. He, S. Yan, Y. Hu, P. Niyogi, and H. J. Zhang, Face recognition using laplacianfaces, *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 27, no. 3, pp. 1-13, 2005.
- [7] T. M. Cover, and P. E. Hart, Nearest neighbor pattern classification, *IEEE Trans. Information Theory*, vol. 13, no. 1, pp. 21-27, 1967.
- [8] S. Z. Li, and J. Lu, Face recognition using the nearest feature line method, *IEEE Trans. Neural Networks*, vol. 10, no. 2, pp. 439-443, 1999.
- [9] J. Lu, and Y. P. Tan, Uncorrelated discriminant nearest feature line analysis for face recognition, *IEEE Signal Processing Letters*, vol. 17, no. 2, pp. 185-188, 2010.
- [10] Y. L. Zhou, C. S. Zhang, and J. C. Wang, Extended nearest feature line classifier, *Proc. of Pacific Rim international conference on artificial intelligence*, vol. 3157, pp. 183-190, 2004.

- [11] Q. Feng, J. S. Pan, and L. Yan, Restricted nearest feature line with ellipse for face recognition, *Journal of Information Hiding and Multimedia Signal Processing*, vol. 3, no. 3, pp. 297-305, 2012.
- [12] Q. Feng, J.S. Pan, and L. Yan, Nearest feature centre classifier for face recognition, *Journal of Electronics Letters*, vol. 48, no. 18, pp. 1120-1120, 2006.
- [13] S. Z. Li, Content-based audio classification and retrieval using the nearest feature line method, *IEEE Trans. Speech Audio Process*, vol. 8, no. 5, pp. 619-625, 2000.
- [14] K. Chen, T. Y. Wu, and H. J. Zhang, On the use of nearest feature line for speaker identification, *Journal of Pattern Recognition Letters*, vol. 23, no. 14, pp. 1735-1746, 2002.
- [15] S. Z. Li, K. L. Chan, and C. L. Wang, Performance evaluation of the nearest feature line method in image classification and retrieval, *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 22, no. 11, pp. 1335-1349, 2000.
- [16] J. H. Chena, and C. S. Chen, Object recognition based on image sequences by using inter-feature-line consistencies, *Journal of Pattern Recognition*, vol. 37, no. 9, pp. 1913-1923, 2004.
- [17] W. Zheng, L. Zhao, and C. Zou, Locally nearest neighbor classifiers for pattern classification, *Journal of Pattern Recognition*, vol. 37, no. 6, pp. 1307-1309, 2004.
- [18] J. T. Chien, and C. C. Wu, Discriminant waveletfaces and nearest feature classifiers for face recognition, *IEEE Tran. Pattern Analysis and Machine Intelligence*, vol. 24, no. 12, pp. 1644-1649, 2002.
- [19] Q. B. Gao, and Z. Z. Wang, Center-based nearest neighbor classifier, *Journal of Pattern Recognition*, vol. 40, no. 1, pp. 346-349, 2007.
- [20] Z. Zhou, and C.K. Kwoh, The pattern classification based on the nearest feature midpoints, *Proc. of the 17th International Conference on Pattern Recognition*, vol. 3, pp. 446-449, 2000.
- [21] A. M. Martinez, and R. Benavente, The AR face database, *CVC Technical Report*, vol. 24, 1998.
- [22] PolyU FKP Database, 2010, available at <http://www.comp.polyu.edu.hk/biometrics/FKP.htm>.
- [23] PolyU multispectral palmprint Database, 2010, available at <http://www.comp.polyu.edu.hk/biometrics/MultispectralPalmprint/MSP.htm>.
- [24] S. A. Nene, S. K. Nayar, and H. Murase, Columbia object image library (COIL-100), *Technical Report CUCS-006-96*, Columbia University, New York, USA, 1996.