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Resolution Improvement for MUSIC and ROOT MUSIC Algorithms

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Abstract. *Smart antennas are used to transmit/receive the signals according to their data using digital signal processing. One of the most important signals processes in smart antennas is the estimation of the signal direction. Generally, it is known as Direction of Arrival algorithm DOA. MUSIC and ROOT MUSIC are one of the most attractive DOAs regarding their performances. However, if two or more sources are too close to each others in angles, these algorithms are not always giving accurate results. In this paper a reasonable modification for MUSIC and ROOT MUSIC algorithms had been made by adding factors related to: number of snapshot, number of element in the array and the power of input signal respectively. An improved performance thus was obtained for both MUSIC and ROOT MUSIC algorithms. Matlab software was used to simulate these modifications. Moreover, this modification may achieve the desired goal of network efficiency when it is applied to smart antennas.*
Keywords: Smart antennas, DOA algorithms, Music, ROOT music, High resolution

1. **Introduction.** Smart antennas are system's that combine multiple antenna elements with a signal processing capability to optimize its radiation/reception pattern automatically according to the systems signal environment [2]. This is typically done by combining the signals from array elements to form a movable beam proportional to the target signal information. This is called beamforming and it gives the smart antennas their influence. This can provide higher system capacities by concentrating the beams toward the desired users while eliminating the interferer or other users to obtain higher signal to interference ratios. Moreover, power consumption will be decreased and frequency reuse will be allowed more easily within the same cell using what is called space division multiple accesses (SDMA) [3].

The smart antennas are thus acquired their importance from the digital signal processing in it. One of the most important signal processing steps is to find out the direction from which the signal had been transmitted. Many algorithms had been found and studied in this area [1, 5 , 6, 7]. They are called Direction of Arrival algorithms DOA, or Angle of Arrival AOA algorithms. DOAs can be implemented in many applications in wireless networks such as mobile communication, radar application, sonar, rescue services...etc. Those algorithms can be divided into three main categories; conventional beamforming techniques, subspace-based techniques, and maximum likelihood techniques [2]. The most famous algorithms are MUSIC, ROOT MUSIC, ESPRIT, Bartlett, Capon algorithms...etc. Generally, the implementation of such algorithms in smart antennas will be used to eliminate the interference and combine the required signals to improve the performance.

From literature, it had been found that most of the available algorithms have some common problems such as low resolution; i.e.: the algorithm cannot detect the directions for two or more signals when they are adjacent to each others in angles. Multiple Signal Classification (MUSIC) got the highest resolution compared with Bartlett Estimation, Minimum Variance Distortionless Response (MVDR) and Linear Prediction Method algorithms [4].Thus, the performance of the highest resolution algorithm could be improved to achieve the goal and get better estimation for the directions when the sources are so close to each others in angles.

In this paper a method for improving the resolution of MUSIC and ROOT MUSIC algorithms proposed and discussed. Matlab simulation had been done and the results with and without the proposed modifications were provided for comparison purposes.

2. Estimation of Angle of Arrival Algorithms.

Many DOA algorithms are available to estimate the signal angle of arrival. In this section, we will discuss the basic theory for MUSIC and ROOT MUSIC algorithms. Then the proposed modification method will be illustrated.

2.1. MUSIC Algorithm.

MUSIC stands for Multiple Signal Classification. It had been firstly presented by Schmidt [8]. It is an algorithm to estimate the direction of the source. MUSIC is mainly dependent on the correlation matrix of the data and the ability to extract the signal and noise eigenvectors from input correlation matrix. It works only with incoherent sources. The steps for MUSIC algorithm can be simplified as follows. Suppose the input signal is:

$$\mathbf{x}(t) = \mathbf{A}s(t) + \mathbf{n}(t) \tag{1}$$

Where $\mathbf{s}(t)$ is the original signal from the d sources:

$$\mathbf{s}(t) = [s_1(t), s_2(t), ...s_d(t)]^T \tag{2}$$

$\mathbf{n}(t)$ is the channel noise for M element array:

$$\mathbf{n}(t) = [n_1(t), n_2(t), ...n_M(t)]^T \tag{3}$$

$\mathbf{x}(t)$ is the signal received by the array antenna with M elements and can be defined by:

$$\mathbf{x}(t) = [x_1(t), x_2(t), ...x_M(t)]^T \tag{4}$$

$$\mathbf{A} = [\mathrm{a}(\mu_1), \mathrm{a}(\mu_2), ...\mathrm{a}(\mu_d)] \tag{5}$$

Where $\mathbf{A}$ is the array steering vector for d sources and have different structure according to the array configuration and the spatial frequency μ_i. Here the uniform linear array is assumed, thus the spatial frequency and steering vector are given by:

$$\mu_i = \frac{(-2\pi)}{\lambda} \Delta sin\theta_i \tag{6}$$

$$\mathbf{a}(\mu_i) = [1, e^{j\mu_i}, e^{j2\mu_i}, ...e^{j(M-1)\mu_i}]^T; \qquad 1 \le i \le d \tag{7}$$

Where θ is the angle of arrival, and Δ is the separation distance between array elements. Substituting equation (7) in equation (5), the structure of $\mathbf{A}$ matrix corresponding to spatial frequency is given by:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{j\mu_1} & e^{j\mu_2} & \cdots & e^{j\mu_d} \\ \cdots & \cdots & \cdots & \cdots \\ e^{j(M-1)\mu_1} & e^{j(M-1)\mu_2} & \cdots & e^{j(M-1)\mu_d} \end{bmatrix} \tag{8}$$

The input covariance matrix $\mathbf{R}_{xx}$ can be written as:

$$\mathbf{R}_{xx} = \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \delta_N^2\mathbf{I}_M \tag{9}$$

Where $\mathbf{R}_{ss}$ is the signal correlation matrix, δ_N^2 is the noise common variance, and $\mathbf{I}_M$ is the identity matrix of rank M. In practice $\mathbf{R}_{xx}$ is estimated by averaging over the number of snapshot(N).

$$\mathbf{R}_{xx} = \frac{1}{N}\sum_{n=1}^{N} x_n x_n^H \tag{10}$$

The covariance matrix $\mathbf{R}_{xx}$ will have M eigenvalues. The matrix $\mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H$ is positive with rank d. Hence, the smallest (M-d) eigenvalues of $\mathbf{R}_{xx}$ represent the noise and should equal to the noise variance δ_N^2 .Their corresponding eigenvectors $\mathbf{V}_n$ are orthogonal to the steering vector of the input signal $\mathbf{a}(\theta)$. Thus knowing these two values, MUSIC spectrum is constructed to find the orthogonal points (maximum peaks) which represents the desired angles as:

$$P_{MUSIC}(\theta) = \frac{1}{\mathbf{a}(\theta)^H\mathbf{V}_n\mathbf{V}_n^H\mathbf{a}(\theta)} \tag{11}$$

Where $\mathbf{V}_n$ is the noise subspace eigenvectors and $\mathbf{a}(\theta)$ is the steering vectors corresponding to signal components as in equation (7). The peaks of this spectrum represent the required angles of arrival.

2.2. **ROOT MUSIC Algorithm.** This algorithm had been introduced as an improvement of above mentioned MUSIC algorithm by Barabell [9].Using the MUSIC spectrum, the root of the polynomial is used to estimate the angles of arrival. This algorithm is more practical, since the results are given in a numerical format instead of spectrum plotting in MUSIC algorithm. The algorithm can be summarized by the following steps:

- Estimation of the covariance matrix $\mathbf{R}_{xx}$ by weighting over number of snapshots as in equation (10).
- Extraction of the smallest eigenvalues of $\mathbf{R}_{xx}$ and their corresponding eigenvectors to estimate noise subspace.
- Generation of the polynomial from the MUSIC spectrum using the following equation:

  $$P_{MUSIC}(\theta)^{-1} = j(\theta) = \mathbf{a}^H(\theta)\mathbf{V}_n\mathbf{V}_N^H\mathbf{a}(\theta) = 0 \tag{12}$$

  Where the steering vector was defined previously as in equation (7) and it depends on the spatial frequency μ. Thus, calculating the root for the polynomial will give the values for the variable μ. Here the uniform linear array was assumed and μ is defined by equation (6).
- The zeros of the polynomial are calculated in term of (N-1) pairs within the unit circle.
- The closest d roots to unit circuit are then selected. Finally, the values of the angles of arrival are directly calculated using the following formula:

  $$\theta = sin^{-1}[\frac{\lambda}{2\pi\Delta}\mu] \tag{13}$$

2.3. **Proposed Modification.** When the separation angles between sources are very small, MUSIC and ROOT MUSIC algorithms couldn't estimate the angles correctly. Thus an improvement for these algorithms was proposed by adding new factors to achieve this goal and estimate the adjacent angles correctly. This was achieved even with separation of 1 degree only between sources. These factors are related to signal and antenna parameters which are affecting the accuracy of MUSIC and ROOT MUSIC algorithms. Those factors represent the ratio by which each corresponding parameter should be increased in order to get the required super high resolution. Those factors will virtually maximize the values of the corresponding parameters during calculation and enhance the estimation accuracy of the algorithms. Those factors are defined as follows:

A.Number of Snapshots Factor (SF). This factor represents the ratio for incrementing the snapshots. The multiplication of SF with the snapshot number during processing will give higher number of snapshots. Thus, increasing the efficiency of the estimation and obtain sharper peaks in MUSIC and less errors in ROOT MUSIC. By adding more snapshots, the estimation variance for covariance matrix will decrease and thus sharper peaks for MUSIC are obtained. However, very high values for SF will lead to processing delay since the processing time will also increase. Hence, suitable value for it should be selected according to the requirements. The new number of snapshot N' can then be written as:

$$N' = N \times SF \tag{14}$$

B.Number of Elements Factor (EF). EF represents the ratio for incrementing the number of elements in the array M. It had been added to raise the value of number of elements during processing. This was done by multiplying EF with the actual number of elements in the array. Thus, it gives more signal component for the algorithm. Here the importance of this modification appears during manufacturing process. By selecting the suitable EF value, the suitable number of elements required for the antenna design can be selected easily. Then we can substitute the new number of element M' in MUSIC and ROOT MUSIC. M' can be expressed by:

$$M' = M \times EF \tag{15}$$

C.Signal Power Factor (PF). The third factor (PF) is related to the power of the signal. It represents the amplification which should be applied for the signal. Multiplying PF by the input signal will give higher resolution for the DOA algorithms. Thus the amplified signal $\mathbf{x}'(\mathrm{t})$ can be written as follows:

$$\mathbf{x}'(t) = x(t) \times PF \tag{16}$$

Substitute these new values in equation (10), the new covariance for the MUSIC and ROOT MUSIC can be calculated by:

$$\mathbf{R}'_{xx} = \frac{1}{N'} \sum_{n=1}^{N'} x'_n x'^H_n \tag{17}$$

Then eigenvalues and eigenvectors of $\mathbf{R}_{xx}$ are calculated. The noise eigenvectors $\mathbf{V}'_n$ are estimated, which are the smallest (M'-d) eigenvectors of $\mathbf{R}'_{xx}$. Then we substitute $\mathbf{V}'_n$ and the new signal steering vector $\mathbf{a}'(\theta)$ in the MUSIC spectrum formula:

$$P'_{MUSIC}(\theta) = \frac{1}{\mathbf{a}'^H(\theta)\mathbf{V}'_n\mathbf{V}'^H_n\mathbf{a}'(\theta)} \tag{18}$$

For ROOT MUSIC algorithm the calculation of roots will also depend on these new values for $\mathbf{R}'_{xx}$ and $P'_{MUSIC}(\theta)^{-1}$. Thus, using the three factors simultaneously both algorithms MUSIC and ROOT MUSIC will give higher resolution, and adjacent signals will be estimated correctly.

3. Simulation Results.

3. Simulation Results. Using Matlab software, the above mentioned factors were added for the MUSIC and ROOT MUSIC respectively. The assumptions were made in the simulation as follows:

- Uniform Linear array antenna structure with basic 8 elements.
- A channel with a white Gaussian noise.
- The distance between the elements $\Delta = 0.5\lambda$, half wave length.
- All sources are incoherent.
- Number of snapshot=1500.
- Number of snapshot factor (SF)=6.
- Signal power factor (PF)=4.5.
- Number of element factor (EF)=8.

Below are some of the results before and after modification in order to let the reader recognize the improvement which had been acquired in the performance of the algorithms.

3.1. MUSIC Algorithm Results. Normal case: here three signals were emitted from directions 15, 20 and 30°. The ordinary MUSIC algorithm could not estimate the neighboring signal at 15° since the separation was 5 degrees only. As shown in Figure 1.

For the same signals located at 15, 20 and 30° the factors were added and the MUSIC spectrum was calculated as in equation (18) this is shown in Figure 2. The three angles were all detected clearly. Hence, an increased resolution for MUSIC algorithm was obtained.

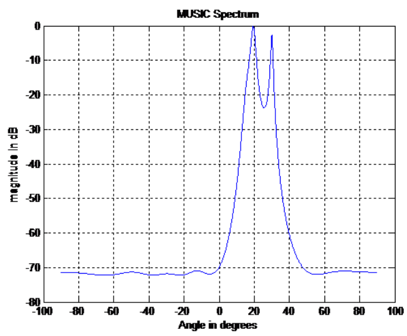

Figure 1. MUSIC without factors.

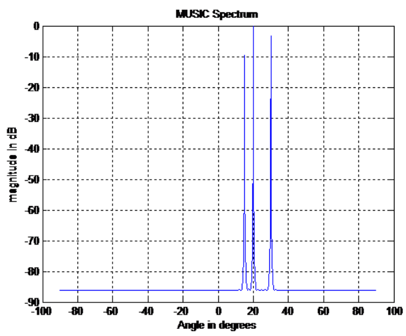

Figure 2. Modified MUSIC algorithm with factors.

Moreover, Figure 3 also shows the results of the modified MUSIC algorithm for closer sources, from angles 15, 18 and 30°.All signals were estimated clearly. In addition, more complex situation was simulated using sources at directions separated by 1 degree only. The signals were assumed at directions 19, 20 and 21° as in Figure 4. The three directions were all estimated accurately.

To improve the results of Figure 4, the values of the factors were increased to; PF= 6, SF =10 and EF = 15. This modification shows the super high resolution of this method as in Figure 5.

3.2. ROOT MUSIC Results. The result of ROOT MUSIC is a numerical representation of the signal direction. To analyse the performance of the improved algorithm, a simulation was done for three neighbouring signals. Table1 Compares the normal ROOT

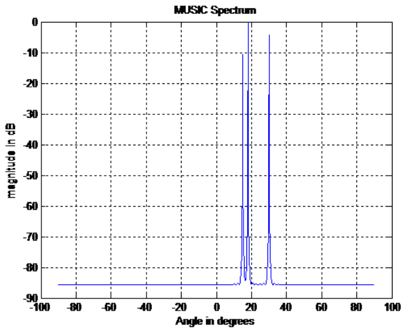

Figure 3. Modified MUSIC for angles 15, 18 and 30°.

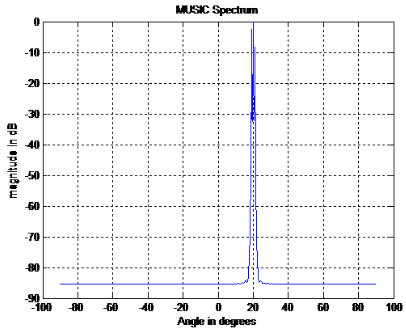

Figure 4. Modified MUSIC for angles 19, 20 and 21°.

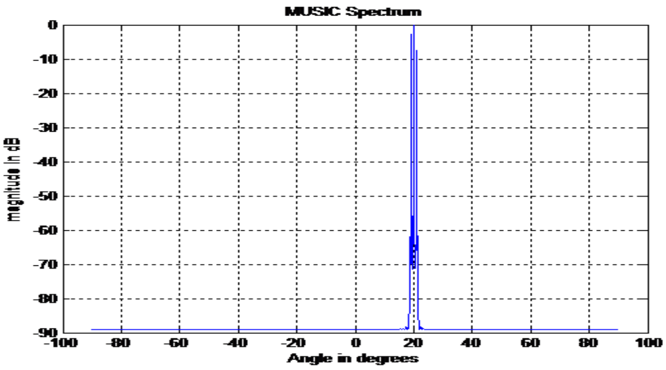

Figure 5. Modified MUSIC for angles 19, 20 and 21° using new factors values.

MUSIC with the modified method using factors concept. Here three signals from directions 10, 25 and 30° were emitted and negligible estimation errors were obtained when factors were used.

For closer signals, normal algorithm could not estimate the neighbouring signals correctly and gave high estimation errors, up to 93.03 % for angle 30°. However, using factors concept the errors were negligible as shown in Table2.

Table 1. ROOT MUSIC algorithm for 10, 25 and 30°.

Concept	**Angles°**	**Estimated Angles°**	**Error%**
Without factors	10	10.1448	1.4485
	25	26.4661	5.8644
	30	29.3159	2.2804
Using factors	10	10.0036	0.0358
	25	25.0013	0.0054
	30	29.9966	0.0113

The results for 3 sources with 1 degree separation are shown in Table 3 to prove the super high resolution of this concept. The three angles were all estimated accurately. By increasing the values of the factors, the error percentages were decreased and the angles estimation was more accurate as shown in table 3.

3.3. **Comparison using different values for the factors.** To clarify the effect of the factors values on the performance of the whole algorithm; the simulation was run for ROOT MUSIC. Since it gives numerical representations for the results. The simulation was run several times using different values for each factor and the corresponding errors

Table 2. ROOT MUSIC algorithm for 10, 28 and 30°.

Concept	Angles°	Estimated Angles°	Error%
Without factors	10 28 30	9.8491 29.0649 57.908	1.5093 3.8031 93.0272
Using factors	10 28 30	9.9967 27.9977 29.9987	0.0330 0.0081 0.0044

Table 3. ROOT MUSIC algorithm for signals from 19, 20 and 21° using factors concept.

Factors Values	Angles°	Estimated Angles°	Error%
same values;PF=4.5, SF=6 and EF=8	19 20 21	18.9935 20.0014 20.9971	0.0343 0.0071 0.0139
Higher values;PF=6, SF=10 andEF=15	19 20 21	19.0014 20.0010 20.9995	0.0073 0.0051 0.0023

in estimation were recorded. Thus, the curves for errors in estimation versus the factor values had been plotted in Figure 6. Each curve represents the effect of changing one factor value while the two other factors values were fixed. Moreover, the simulation was run for estimating two sources at 25 and 30 in each case.

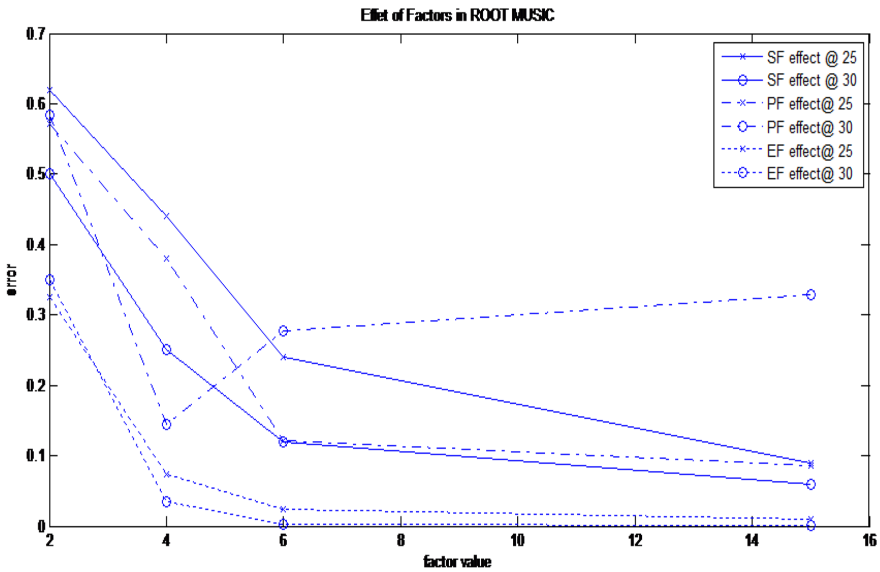

Figure 6. Comparison of ROOT MUSIC using different factors values.

From Figure 6 it is clear that the most efficient factor is the number of element factor (EF). Since the error equals 0 as the value of EF increased. Actually, EF gives more observation for the signal as it is directly related to the steering vector a(θ). Moreover, the covariance matrix will give more points of orthogonality and hence peaks of MUSIC will be sharper and errors will decrease in ROOT MUSIC.

In the second place is the snapshot factor (SF) which is directly proportional to the estimation of covariance matrix. Thus, increasing of SF value will reduce the estimation variance for $\mathbf{R}_{xx}$ and will give sharper peaks in MUSIC and also more accurate values for the roots of the polynomial.

Finally, the power factor (PF) which gives light enhancement to the algorithms since it increases the value of the signal together with the noise.

However, excessive increment for factors values will lead to degradation in performance in form of delay. That is due to the added computational complexity. So that, suitable values for factors should be selected.

4. **Conclusion.** In this paper the theory of smart antennas and the basic concepts of MUSIC and ROOT MUSIC DOA algorithms had been illustrated and discussed. These algorithms when implemented in smart antenna will lead to efficient use of networks. To achieve this goal, the estimation of source direction should be accurate even if sources are too close to each others in angles. Thus new factors had been added in order to increase the resolution of these two algorithms. Those factors are: number of snapshots factor, number of elements factor and signal power factor. They have been entered in the algorithms as amplification ratio to the corresponding parameter in order to increase the parameter values during estimation and thus increase the algorithm accuracy. Matlab software had been used to simulate the method. Some results had been introduced to compare and prove the success of the method in estimating the directions even with separation of 1 degree only between sources. Moreover, some factors can be selected for other DOA algorithms to obtain higher resolutions by selecting the parameters which are proportional to the accuracy of each DOA algorithm.

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