

# Fundamental Frequency Analysis on A Harmonic Power Signal Using Fourier Series and Zero Crossing Algorithms

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**ABSTRACT.** *This paper aims to propose a simple and fast approach to detect the frequency of electrical signals based on zero-crossing point algorithms. First, we will use the Fourier algorithm as a digital filter to fetch the fundamental signal of the acquired signals, and then use a zero crossing algorithm technology which is then applied to the sine or cosine signal of the fundamental signal to calculate the frequency of the fundamental signal. It is intended to apply this theoretical development in personal computers to analyze the operation of the extracted signal with DAQ (Data Acquisition Card) device. This system was developed by using a SCADA (Supervisory Control and Data Acquisition)-based software, LabVIEW, to simulate a signal source with high-order harmonic and noise signals combined together, and the program can be used to estimate the frequency of fundamental sinusoidal signals. Experiment result has shown that the zero-crossing point algorithm is an effective method of measuring the frequency.*

**Keywords:** Frequency analysis, Zero crossing algorithm, Power signals, Fourier algorithm, DAQ, LabVIEW

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1. **Introduction.** Due to the convenience and diversity of the electrical products, people find it extremely difficult to live without using electrical products. Although these products are for people to enhance life's convenience, they also affect the merits of power supply quality. The electrical loading generated by electrical appliances will produce harmonic currents which will result in polluting the power quality [1]. These pollutants are likely to cause damage to the capacitor, communication interference, and transformer and result in cable overloading and overheating accidents [2]. Power quality problems are often noted when associated with industrial electricity issues [3], but these problems are overlooked when associated with the people's regular usage of electricity [4]. When electrical products are damaged, few people think about whether the failures are the result of contaminated electrical signal quality.

Over the past few years, a number of signal frequency detection methods [5-7] have been proposed in the research literature. The bilinear principle is used for frequency deviation

and is also an effective method for on-off nominal frequency estimation [8-9]. Moore et al [10] also proposed a wide range of frequencies measurements using the adaptive algorithm. Discrete Fourier transform and Kalman are also popular technology to be used in the signal-filtering processing. In our research, we tried to develop a simple and fast way to estimate power signal frequency. The measurement scheme is shown in figure 1, in which we take advantage of the NI cDAQ-9174 & NI 9225 signal acquisition card so that not only the development's time is reduced but also the effects are quickly achieved as well. The captured power signals are again applied with the proposed Fourier algorithms to filter out harmonic signals and noise, which then are imposed with a zero crossing algorithm to measure the frequency, phase, amplitude, and other signal parameters. The measurement parameters on a LabVIEW platform could be passed through *WiFi* communication to our intelligent cell phone by using data share technology, so that you can achieve real-time monitoring of power quality measurements.

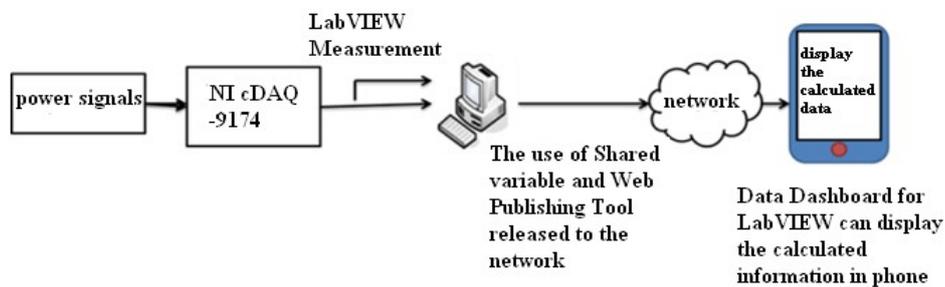


FIGURE 1. Measurement Schematic for fundamental frequency analysis of the power signal.

## 2. Algorithm implementation.

**2.1. Fourier algorithms.** A measured signal of any voltage or current can be expressed as an equation (1).

$$\dot{v}(t) = C \cos(\omega t + \phi) + R(t) \quad (1)$$

Where  $C$  is the amplitude of the fundamental wave,  $\omega = 2\pi f$  is the fundamental angular frequency,  $f$  is the fundamental frequency,  $\phi$  is the phase of the fundamental wave, and  $R(t)$  is the high harmonics and zero-mean noise signal.

If the exact value of the fundamental frequency is unknown, the supposed angular frequency of the fundamental frequency is  $\omega_a$  and  $v_a(i)$  are the estimated amplitude of the fundamental frequency of the signal,  $m$  is the number of samples taken by the supposed fundamental frequency period,  $v_n$  is the sample value of the  $n$  signal.

Each sample value of the fundamental frequency can be presented by a discrete Fourier series (DFS) as follows:

$$\dot{V}_a(i) = \frac{2}{m} \left[ \sum_{n=i}^{m+i-1} V_n \cos\left(\frac{\omega_a T_a}{m} n\right) - j \sum_{n=i}^{m+i-1} V_n \sin\left(\frac{\omega_a T_a}{m} n\right) \right] = A(i) + jB(i) \quad (2)$$

Where  $f_a = 1/T_a$ ,  $T_a = 2\pi/\omega_a$  and  $\psi = (\omega_a T_a)/m = 2\pi/m$ . For the  $i$ -th data window, the sine and cosine components,  $A(i)$  and  $B(i)$  can be calculated as follows:

$$\dot{A}(i) = \frac{2}{m} \sum_{n=i}^{m+i-1} V_n \cos(\psi n) \tag{3}$$

$$\dot{B}(i) = \frac{2}{m} \sum_{n=i}^{m+i-1} V_n \sin(\psi n) \tag{4}$$

Where  $C(i)_a = \sqrt{A(i)^2 + B(i)^2}$  is the estimate amplitude  $\phi_f = B/A$  of the fundamental frequency in the  $i$ th data window, and  $C(i)$  is accurate only when  $\omega_a = \omega$ . Assume the sampling frequency for the measured signal is  $f_s = m/T_a = 1/T_s$ , where  $T_s$  is the sampling period.

If the data window is swept by the signal, the equation (3) and (4) may provide  $A(t)$  and  $B(t)$  with the corresponding value of one period. Sine and cosine components,  $A$  and  $B$ , are the periodic functions of time if the frequency of the fundamental signal is equal to the assumed fundamental frequency in the Fourier series equation ( $T_s \bullet m = T_a = T = \frac{1}{f}$ ). Additionally,  $A(t)$  and  $B(t)$  are mutually orthogonal in the frequency  $f$ , where  $A(t)$  is a pure cosine wave, and  $B(t)$  sine wave. When  $T_a \neq T$ ,  $A(t)$  and  $B(t)$  are not pure sine wave and cosine wave, respectively. However, their fundamental frequency is still  $f$ .

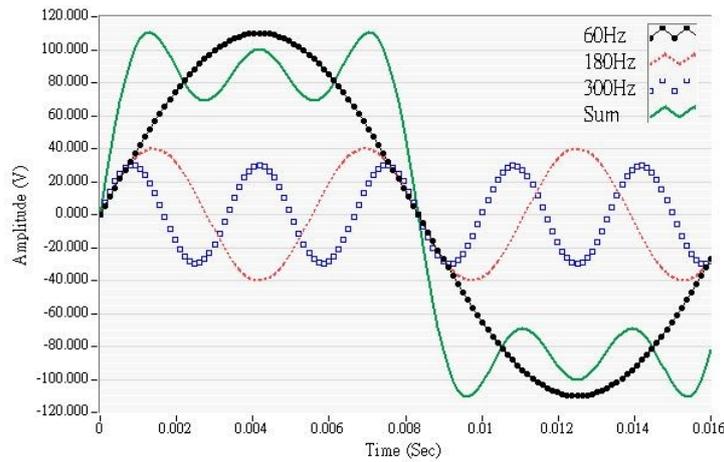


FIGURE 2. The fundamental signal with its 3rd and 5th harmonics

Figure 2 shows a 60 Hz fundamental frequency and its higher harmonic signal, and the electrical voltages of the harmonics are assigned to 100% of the 1st, 40% of the 3rd and 30% of the 5th harmonic waves. In practice, the cosine and sine signals of the equation (3) and (4) can be represented as equation (5) and (6), a vector with length  $m$ .

$$\text{c}\ddot{\text{o}}\text{s} = \frac{2}{m} [\cos(\psi), \cos(2\psi), \dots, \cos((m - 1)\psi), 1]^T \tag{5}$$

$$\text{s}\ddot{\text{i}}\text{n} = \frac{2}{m} [\sin(\psi), \sin(2\psi), \dots, \sin((m - 1)\psi), 0]^T \tag{6}$$

The vector of signal samples is expressed as an equation (7).

$$\dot{S}AM_s = [v_1 v_2 v_3 v_4 v_5 \dots v_n]^T \tag{7}$$

By using auxiliary vectors of cosine and and sin, equation (3) and equation (4) are calculated only by multiplications and additions without any trigonometric calculation, so the calculation is very simple.

After each sampling, the sampling points should be re-arranged as an equation (8).

$$\dot{v}_1 = v_2, v_2 = v_3, \dots \dots v_n = v_{new} \tag{8}$$

Thus, moving the data window and the signal sample values are considered as scalar signal processing. For each data in the data window, according to the relationship of equation (3) and equation (4), it can calculate their corresponding cosine ( $A(t)$ ) and sine ( $B(t)$ ) components.

For example, suppose a signal is sampled with five periods, and one period has  $m=128$  points. Then there will be  $128 * (5-1) +1= 513$  data windows in the calculations. The cosine function ( $A(t)$ ) can be computed as follows:

$$\dot{A}(i) = \frac{2}{m} \sum_{n=1}^{m \cdot (5-1)+1} v_n \cos(\psi n) \tag{9}$$

Where  $m=128$  and  $i = 1,2,3,\dots,128(5-1)+1$

**2.2. Zero-Crossing Algorithm.** The zero-crossing algorithm is an intuitional frequency detection method, simple calculation method, and fast execution without the complex mathematical formulas. For a periodic signal, the period between two zero-cross points is half the signal period. Through calculating the numerical time units of two adjacent zero-crossing points on the time axis, and finding its countdown, it can obtain the actual frequency of the signal [11].

Consider an electrical signal, it can be expressed as a sinusoidal function in an equation (10) and its zero crossing algorithm could be deduced by figure 3.

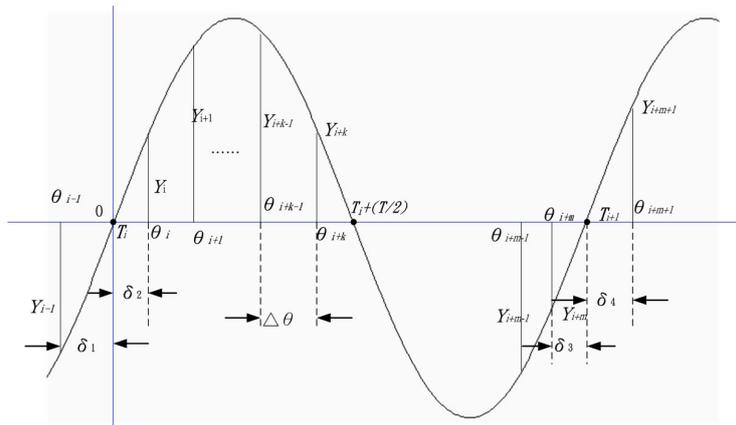


FIGURE 3. Schematic for zero crossing algorithm

Where,  $A$  is the amplitude,  $\omega = 2\pi f$  is the angular frequency, and  $f$  is frequency of the signal. When the signal is represented in a discrete manner, and even if  $t = kT_s$  is substituted, equation (10) can be expressed by the following equation

$$\dot{y}(k) = A \sin(\omega k T_s) \tag{10}$$

In the signal sampling process, the X axis is represented in the scale of the degree of diameter, the angular difference between successive sampling points [12, 13] is

$$\dot{\Delta}\theta = \omega T_s = 2\pi f T_s \quad (11)$$

Where  $T_s$  is the sampling time interval. Assuming a full cycle of sampling points is  $m$ , signal periods are equivalent to sampling interval multiplied by the number of points. Therefore, the signal frequency can be calculated by the following formula

$$\dot{f} = \frac{1}{m T_s} \quad (12)$$

In practical applications, since the sampling frequency is not an integer multiple of the actual signal frequency, the signal period is not an integer multiple of the sampling period. Therefore, Most of the exact  $m$  values are not integer values. As we investigate the  $T_i$  period of the signal in figure 3, there are two zero crossing points  $(T_i, 0)$  and  $(T_{i+1}, 0)$  with voltage magnitude transition from negative to positive in this period, where  $T_i$  is between the points  $(\theta_{i-1}, Y_{i-1})$  and  $(\theta_i, Y_i)$ , and  $T_{i+1}$  between  $(\theta_{i+m}, Y_{i+m})$  and  $(\theta_{i+m+1}, Y_{i+m+1})$ . The exact periodic of this sinusoidal wave can be expressed as

$$\dot{T} = m T_s + \delta_2 + \delta_3 = m_A T_s = m_A \Delta\theta \quad (13)$$

Where  $T_s = \Delta\theta$  is the sampling interval. Because the signal period is not an integer multiple of the sampling interval,  $m_A$  value is not set to an integer; according to Figure 3,  $m_A$  can be obtained by equation (14).

$$\dot{m}_A = m + \frac{\delta_2}{\Delta\theta} + \frac{\delta_3}{\Delta\theta} = m + \frac{\delta_2}{\delta_1 + \delta_2} + \frac{\delta_3}{\delta_3 + \delta_4} \quad (14)$$

Where,  $\delta_2 = \theta_i - T_i$  is the time difference between the  $i$  sampling point  $(\theta_i, Y_i)$  and zero point  $(T_i, 0)$ , and  $\delta_3 = T_{i+1} - \theta_{i+m}$  the time difference between the last sampling point  $(\theta_{i+m}, Y_{i+m})$  and the zero point  $(T_{i+1}, 0)$ .  $\delta_3 = T_{i+1} - \theta_{i+m}$ . Since we define the time difference of two adjacent sampling points,  $\Delta\theta = T_s$ , the total sampling points corresponding to one period ( $T$ ) from  $(T_i, 0)$  to  $(T_{i+1}, 0)$ , are equal to  $m_A$ . Therefore, the entire period of sine wave can be a fraction of the number of samples, which makes the signal period calculation more accurate. Applying the definition of equilateral Triangle in figure 3, the following formula can be obtained

$$\frac{\delta_2}{\delta_1 + \delta_2} = \frac{|y_i|}{|y_i| + |y_{i-1}|} \quad (15)$$

$$\frac{\delta_3}{\delta_3 + \delta_4} = \frac{|y_m|}{|y_m| + |y_{m-1}|} \quad (16)$$

Equation (14) can be expressed as

$$\dot{m}_A = m + \frac{|y_i|}{|y_i| + |y_{i-1}|} + \frac{|y_m|}{|y_m| + |y_{m-1}|} \quad (17)$$

The accurate frequency of the evaluated signal can be estimated as

$$\dot{f} = \frac{1}{m_A T_s} = f_a \frac{m}{m_A} \quad (18)$$

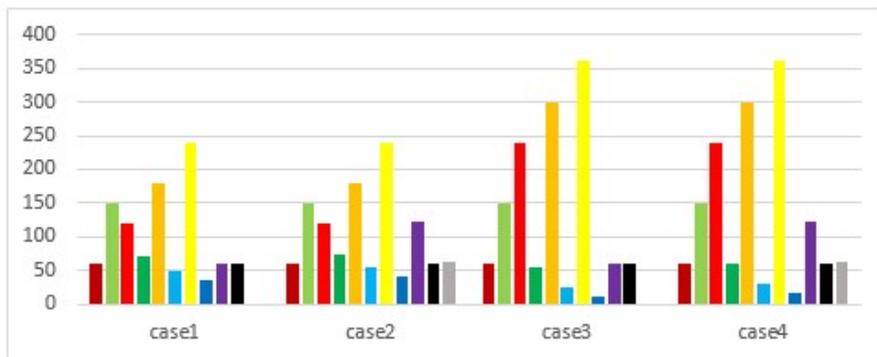
Even  $(\theta_i, Y_i)$  and  $(T_i, 0)$ ,  $(\theta_{i+m}, Y_{i+m})$  and  $(T_{i+1}, 0)$  are exactly overlapping each other, the equation (17) and (18) can still be used for frequency detection.

The electric power measurement algorithms based on the zero crossing algorithms are simple and fast, but there are many restrictions, such as the harmonic of larger amplitude, non-integer harmonic wave, and noise, which are very easy to disturb the determination of the zero-crossing points. Therefore, Fourier Series algorithms are proposed to filter out the higher order harmonic and noise of the signal, then the zero crossing algorithms could be implemented to calculate the fundamental frequency of the signal accurately. This article will identify several items which will undermine the zero-crossing points determination and verify whether adding a Fourier series-based filtering method can improve the accuracy more.

### 3. Simulation and discussion.

3.1. **Large Harmonics.** We set up two experiments to show how the larger harmonics distort the zero-crossing point and result in a wrong estimation of the fundamental frequency. In table 1, comparing case 1 with case 2, where the signal is composed of the

TABLE 1. Zero-crossing influenced by large harmonics



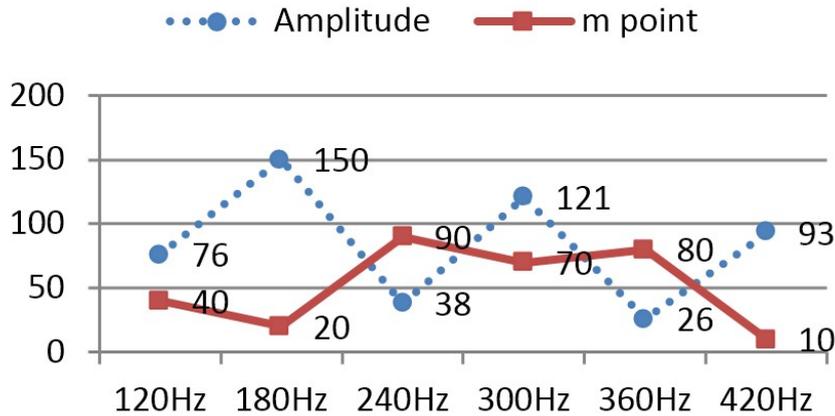
	case1	case2	case3	case4
■ Frequency1	60	60	60	60
■ Amplitude1	150	150	150	150
■ Frequency2	120	120	240	240
■ Amplitude2	70	75	55	60
■ Frequency3	180	180	300	300
■ Amplitude3	50	55	25	30
■ Frequency4	240	240	360	360
■ Amplitude4	36	41	11	16
■ zero-crossing algorithm	60	122.99	60	122.996
■ Zero-crossing with the DFS	60	60	60	60
■ Variance	0	62.99	0	62.996

fundamental frequency (60 Hz) and its 2nd, 3th and 4th harmonics, It is discovered, as shown in table 1, that adding a greater harmonic could undermine zero crossing, which will result in incorrect information calculation by only using zero-crossing algorithms.

Due to the larger harmonic components presence in the Power signal, as shown in case 2, the overall signal generates an excessive zero-crossing point on each 60 Hz cycle that makes the zero-crossing algorithm fail to determine the fundamental frequency. The same result is shown in case 3 and case 4, if only the zero-crossing algorithm implemented for this case, this will cause the error analysis of the results. Through our proposed Fourier series-based filtering approach, the fundamental frequency component can be accurately captured so to reduce the calculation errors caused by the excessive zero-crossing point.

If just a single higher harmonic is encountered with the fundamental frequency, it also exhibits the excessive zero-crossing point. As investigating the applying voltage magnitude of the harmonic signal in the simulations (shown in table 2), the excessive zero-crossing points are more easily e generated by even multiples harmonics (120hz, 240hz, 360hz) than odd multiples harmonics (180hz, 300hz, 420hz). As the sampling points are larger than the maximum sampling points, the excessive zero-crossing points will still need to be determined. Even though the lower sampling rate sometime prevents the disturbance of the excessive zero-crossing points, it discards the accuracy of the frequency measurement.

TABLE 2. The minimum magnitude of the harmonic disturbing zero-crossing point



According to the sampling theorem, high sampling frequency can get more sampling points on each cycle of the signal and result in a more precise calculation of the recovered fundamental frequency. It can be found as shown in figure 4 and figure 5 that even both harmonic signals (120hz and 240hz ) superimpose on the 60Hz fundamental signal, where the harmonic signals have an intersection at the zero- crossing point which is the same as the fundamental signal, so the transition slope at the intersection is not so obvious and fails to be observed. Further, the zero-crossing is calculated based on equivalent triangles, so it will flatten the slope of zero-crossing point and lead the missed determination of the zero-crossing points.

The disturbances of zero-crossing points for the odd harmonic signals (300Hz and 420Hz) are not quite the same way, as shown in figure 6 and figure 7, where the disturbances require the larger amplitude, but the amplitude is too small to interfere with the crossing point. However, if a larger than the critical magnitude of the amplitude appears as shown in table 2, it will nterference the determination of the zero-crossing point. The disturbance will happen since the zero crossing algorithm finds excessive zero crossing points in the every cycle of the 60 Hz fundamental signal.

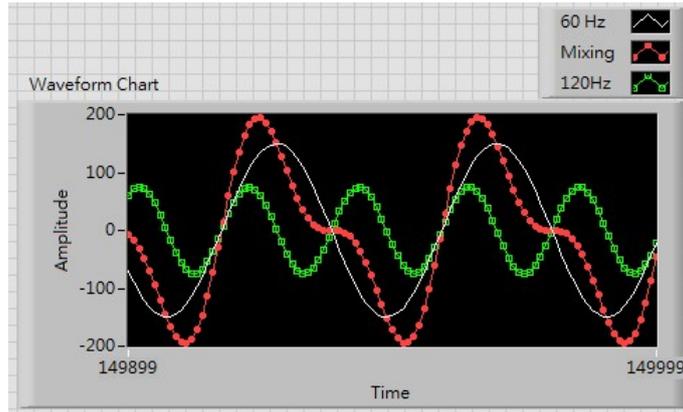


FIGURE 4. The mixed signal of the fundamental signal and its second harmonic

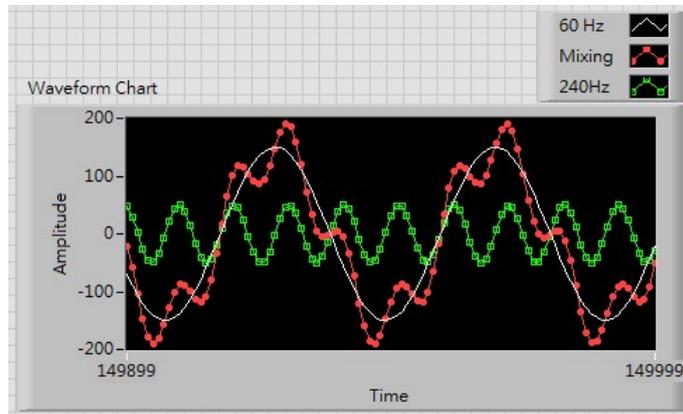


FIGURE 5. The Mixed signal of the fundamental signal and its fourth harmonic

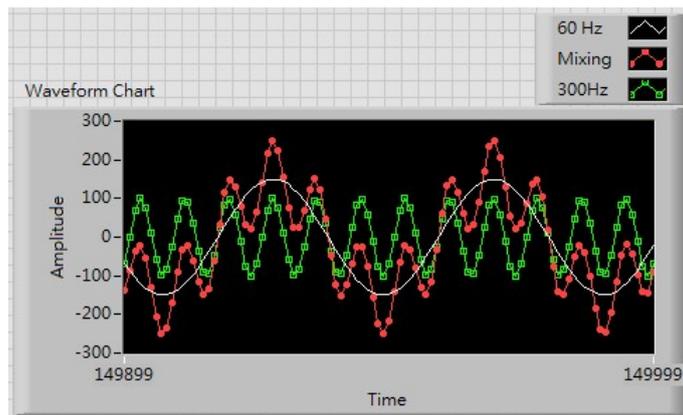


FIGURE 6. The Mixed signal of fundamental signal and its fifth harmonic

In figure 8, it can be found that the fundamental signal superimposing its high harmonics could disturb the zero-crossing point of the fundamental signal, so the digital Fourier series (DFS) can be applied to filter out the fundamental signal and facilitate the further accurate zero-crossing algorithm calculation.



FIGURE 7. The Mixing signal of fundamental and its senenth harmonic

From figure 8, it could be found that the sampling data is about 4.5 cycles of the waveform, but the acquired fundamental signal will miss one cycle of the waveform after the processing of the digital Fourier series (DFS).

Therefore, in the DFS implementation, the cycles of the sampling data must be brought over more than three cycles in wavelength, because in the calculation of the zero-crossing algorithm, it always makes a little modification on the length of the wave data.

Since it could be done by a single waveform calculation for the zero-crossing implementation, the original sampled data is suggested to be more than three cycles of the waveform to make sure enough wave data could be provided for the zero-crossing calculation after it was processed by the DFS algorithm. In our applications, we always provide about 10 cycles of the sampled data to assure we have enough wave data for the zero-crossing calculation and we take the average of the summation of the zero-crossing points of each cycle to improve the calculation accuracy.

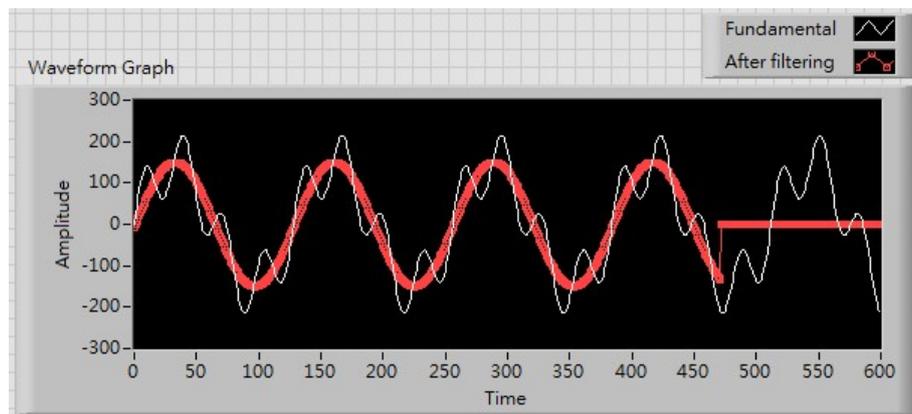


FIGURE 8. The signal with DFS implementation

**3.2. Adding Noise.** Since the zero-crossing algorithm is a kind of time-domain analysis technology, analysis accuracy is very susceptible to any interference on the zero-crossing point of the signal. Therefore, in this study, before the zero crossing algorithm is applied,

the digital Fourier series (DFS) will be applied to filter out unwanted information to get an accurate fundamental frequency component without interference. In figure 9, we can add a larger white noise on the fundamental signal and apply the DFS on the signal to show the anti-noise performance of the DFS implementation in figure 10.

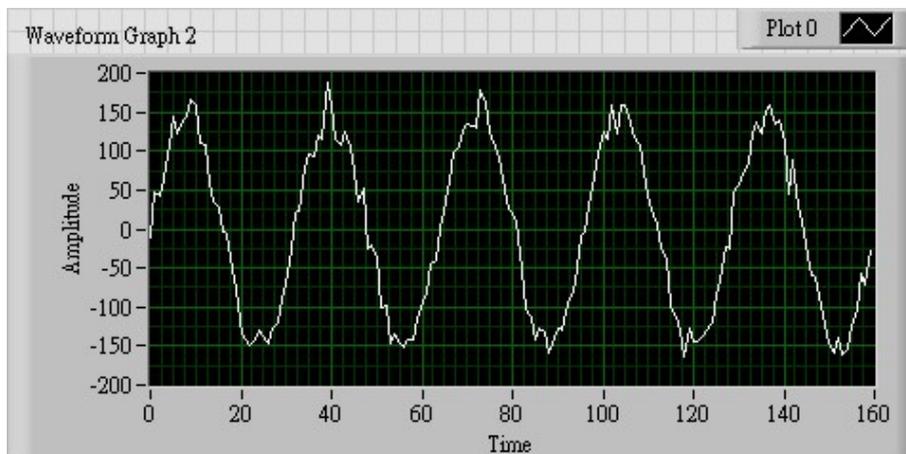


FIGURE 9. The fundamental signal with white noise

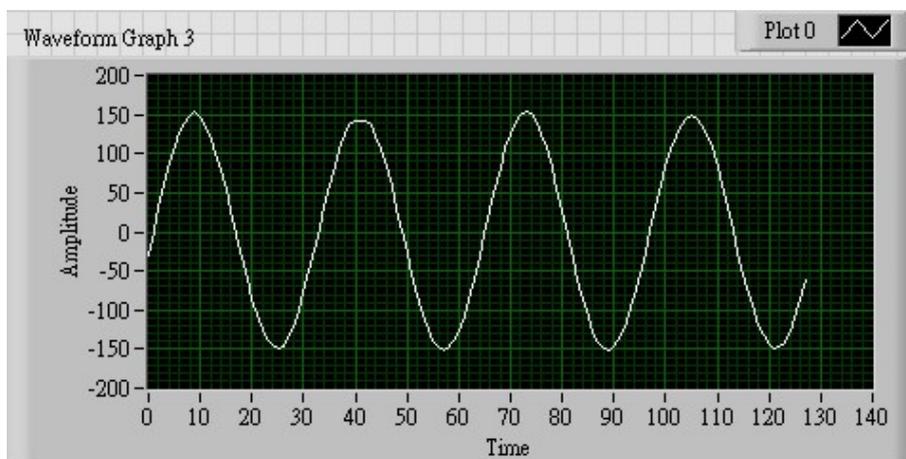


FIGURE 10. Signal processing using the DFS

We also set up a simulation, as shown in table 3, the magnitude of white noise is varied from 1 volt to 13 volts, to investigate the influence of the white noise on the zero-crossing calculation. When the magnitude of white noise increases up to 4% (6 volts) of the fundamental signal, without the DFS implementation involved, the only zero-crossing algorithm might fail to measure the fundamental frequency and show a measured error much more than 18.4%. Even though the measurement errors might increase as well as the magnitudes of the while noise increase, they are not in the proportional relationship, and it depends how the noise disturbs the zero-crossing points of the 60hz fundamental signal. With the implementation of DFS, the measurements are found to have an accuracy of 99.8%, no matter how large the magnitude of the noise being added to the fundamental signal.

TABLE 3. Noise elimination with DFS enhancing the zero-crossing accuracy

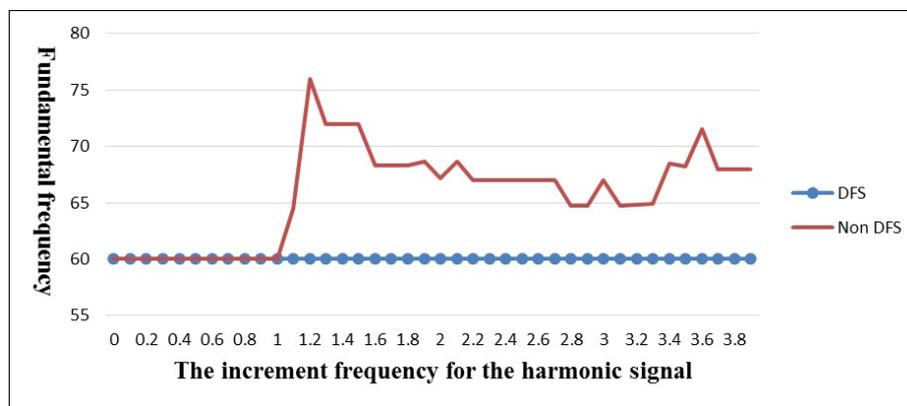
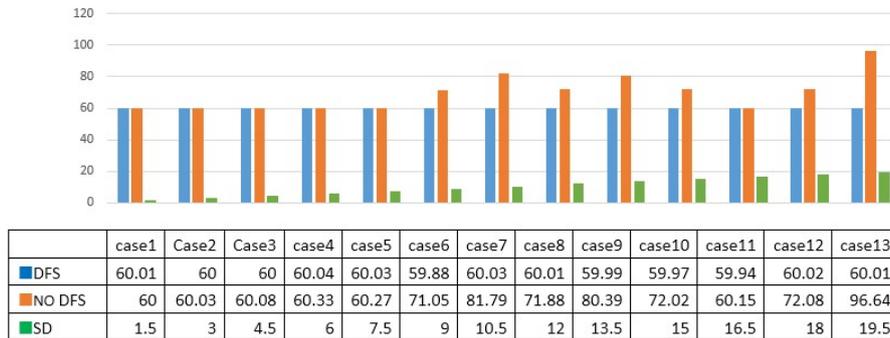


FIGURE 11. The influence of inter-harmonics on the zero-crossing algorithm

**3.3. Interference of inter harmonics.** Non-integer multiples of the fundamental harmonic is commonly known as inter-harmonics. When the power signal contains a harmonic component, it may interfere with the position of the zero-crossing point, which results in the zero-crossing method of failure. The paper proposed the Digital Fourier series algorithm to filter out the inter-harmonics, and set up a few simulation conditions to prove its effectiveness, where the fundamental frequency is 60Hz with amplitude of 150 volts. The frequencies of the 3<sup>rd</sup>, 5th and 7th harmonics signal are varied synchronous from 180 Hz, 300 Hz and 420 Hz to 184 Hz, 304 Hz and 424 Hz with a increment of 0.1 Hz for each step. The amplitude of 3rd, 5th and 7th harmonics signal was set at 45, 15 and 1 volts individually.

It is found from figure 11 that the influences of inter harmonics on the zero-crossing calculation is extremely severe.

From the information obtained from figure 11, it can be found that when the traditional zero-crossing algorithm is encountered with this problem, the accuracy of the calculated information is worse. However, added with Fourier algorithm in front, the inter-harmonic is even too large, we can also accurately calculate the fundamental frequency of the power signal.

**4. Experimental results.** In general, the simulated signal development for the engineering experimental ways takes too much time. However, based on the principle of virtual instruments, LabVIEW can simulate quickly and accurately to obtain the required information in simulating a variety of situations.

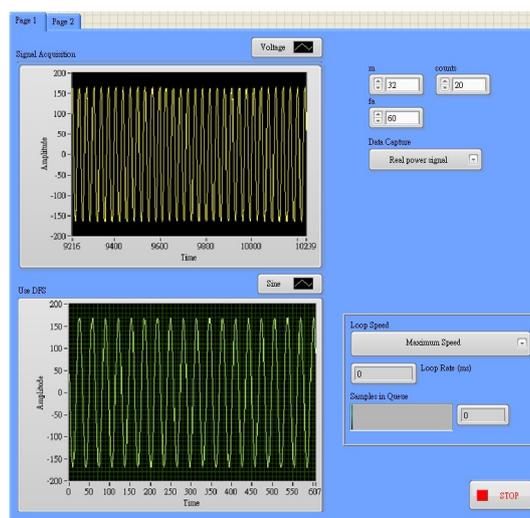


FIGURE 12. User interface I of Labview

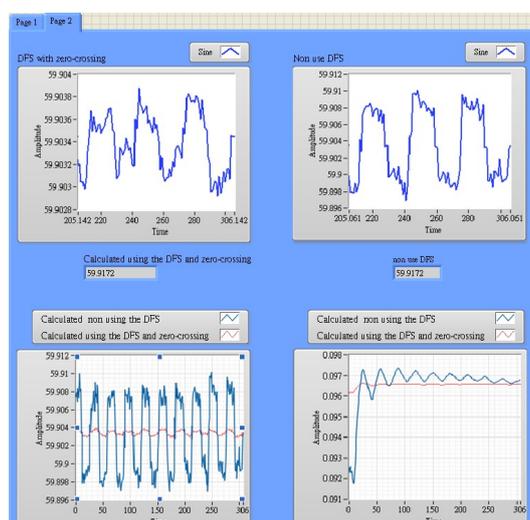


FIGURE 13. User interface II of Labview

Prior to the real measurement, Labview is used to produce a variety of simulation signals and stimulate the signal from the analog output port of NI-myDAQ [14], then the stimulated signal is acquired from the analog input port of NI-9225 [15] to processing the input signal and verifying the proposed algorithm. Since the real power signal of general household electricity shows a voltage magnitude of about 155 volts, NI-9225 which possesses a voltage acceptability of about 300 volts is selected and used to measure these power signals.

The user interface of Labview is designed as shown in figure 12 and 13, then figure 14 is the hardware setup of this research where wiring setup for port AI0 of NI-9225 is wired to the real power signal, port AI1 the Signals sent by the NI-myDAQ, port AI2 is Signal sent by the function-generator. The function-generator can generate a specific signal to test the measurement accuracy of NI-9225 and also can be correlated with the signal output from the NI-myDAQ. Figure15 shows the fundamental frequency determined by our measurement system and shown on a mobile device though the data sharing technology (data dashboard) of Labview.

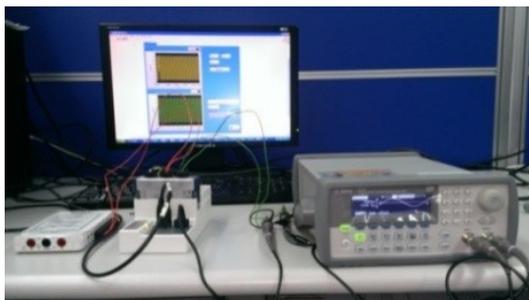


FIGURE 14. Hardware setup of the research

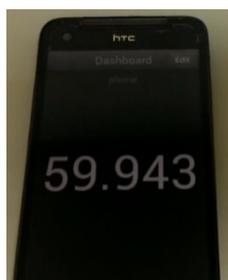


FIGURE 15. The fundamental frequency measure as shown by a cellphone

By taking advantage of the LabVIEW Web Publishing Tool, you can quickly publish the program as a webpage to mobile devices. And you can not only achieve the real-time monitoring remotely but also control the instrument to achieve some specific purposes. We can also utilize the Data Dashboard APP on the mobile devices to monitor the calculated values from the Labview program without spending time and extra-cost to develop the other APP.

In this paper, the NI cDAQ-9174 [16] is used for power signal acquisition, and LabVIEW is used to develop programs, where the NI cDAQ-9174 is a 4 slot USB chassis which is sold with the NI 9225 and other analog and digital modules. The proposed Fourier algorithm is used to get fundamental signals by filtering out signal noise and high harmonic signals. And then the zero-crossing algorithm is used to calculate the frequency of measured signals. LabVIEW and NI-DAQ are confirmed to effectively integrate hardware and software, and can be used to implement a high-performance, portable, and real-time monitoring platform of electrical signals.

**5. Conclusion.** In this paper, the digital Fourier series (DFS) and the zero-crossing of the two algorithms are matched to validate whether it can achieve and solve the shortcomings and problems of only the zero-crossing algorithm.

Therefore, by using the virtual instrument technology of Labview, we set up a few simulation conditions to verify the proposed algorithms which proves the reliability and accuracy of the proposed hardware configuration, finally, the NI cDAQ-9174 & NI 9225 are used to measure real signals.

Verified by the above experiments, it can be found that the proposed method can eliminate many problems of the only zero-crossing algorithm. The problems ruled out by the research methods can be used in actual measurement.

In this study, LabVIEW programming technology is utilized to implement both the digital Fourier series and zero crossing algorithm. Then, the Fourier transform algorithm with zero-crossing algorithms is proved being superior compared with the only zero-crossing

algorithm in many experimental conditions. At last, the Data Dashboard app for the LabVIEW applications are also implements to build a remote power frequency monitoring system.

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