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Bimodal Multi-Feature Fusion Based on Quaternion Fisher Discriminant Analysis

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Abstract. *Because of its higher reliability, wider applicability and stronger security, multimodal biometrics has become a polar research direction of biometric recognition and attracts more and more research groups focusing on this area. Along with other fusion level of multimodal biometrics, feature level can reduce the redundant information to avoid calculation consumption, and simultaneously acquire the discriminative information to improve the system performance. The traditional methods of fusion level can only fuse two single feature modalities or one modality with two kinds of feature. This paper imported quaternion concept and proposed quaternion Fisher discriminant analysis that can fuse two modalities with four different features. Face and palm are selected as the experimental object and extracted the linear feature and the non-linear feature by PCA and KPCA respectively. Experimental results show the proposed algorithm achieves much better performance than four single feature recognition algorithms.*
Keywords: Quaternion field, Multimodal biometrics, Orthogonal eigenvectors, QFDA.

1. **Introduction.** Unimodal biometric recognition has drawn extensive attention during the past decades for its huge potentials in many applications[1, 2, 3, 4]. However, the performance of unimodal biometrics systems has to contend with a variety of problems such as background noise, non-universal applicability,stable stability,inter-class similarities. Therefore, multimodal biometric systems are proposed to solve the above mentioned problems. Because of its higher reliability, wider applicability and stronger security, multimodal biometrics has become a polar research direction of biometric recognition and attracts more and more research groups focusing on this area.

According to the fusion location, multimodal biometric system has four fusion levels: pixel level, feature level, matching score level and decision level. The lowest level is pixel level[5, 6] which fuses the sample image derived from the sensors and retains as much of the data information. But it has to face some problems, such as large computation, poor anti-interference ability and long processing time. Matching score level[7, 8] combined the matching score derived from the unimodal biometric system to get the final matching score. Because of easy operability, it is the most widely used. Decision level fusion[9, 10] synthetically considers the decisions of each unimodal biometric system to do the final decision. Matching score level and decision level both depend on the individual characteristics of the recognition performance, the space to be improved is limited. Comparing with three levels, feature level can derive the most discriminative information from original multiple feature sets and eliminate the redundant information resulting from the correlation between different feature sets.

In general, there are two basic modes for feature level fusion: serial rule and weighted sum rule [11, 12]. However, the former consumes large computational resources. For the latter, The choice of weights value is a controversial issue. Besides, Yang [13] proposed a novel method which avoid the large amount of computation and the selection of the weighted value. This method takes two features as real part and imaginary part of a complex vector. However, these methods mentioned above can only fuse two single feature modalities or one modality with two kinds of feature. This paper proposed a new fusion algorithm based on quaternion that can fuse two modalities with four different features. And our algorithm involved the linear feature and the non-linear feature of one modality at the same time.

The rest of this paper is organized as follows: section 2 gives the related concept of quaternion and quaternion matrix; Our algorithm is presented in section 3; In section 4, experimental results are illustrated. Finally, section 5 concludes this paper.

2. Quaternion and quaternion matrix.

2.1. The relate concepts. If $a, b, c, d \in R$ and i, j, k meet $i^2 = j^2 = k^2 = -1, ij = -ji = k, jk = -kj = i, ki = -ik = j$, then $q = a + bi + cj + dk$ is named a quaternion. Quaternion q is consist of the real part a and imaginary part $bi + cj + dk$ respectively. We can also rewrite quaternion q as $q = (a + ib) + (c + di)j$ according to imaginary multiplication rule. Let $q_1 = a_1 + b_1 i + c_1 j + d_1 k, q_2 = a_2 + b_2 i + c_2 j + d_2 k$, then we can define the basal operation principles of quaternion by the following equations:

- Equality $q_1 = q_2 \Leftrightarrow a_1 = a_2, b_1 = b_2, c_1 = c_2, d_1 = d_2$
- Addition and subtraction $q_1 \pm q_2 = (a_1 a_2) + (b_1 b_2)i + (c_1 c_2)j + (d_1 d_2)k$
- Multiplication $q_1 \cdot q_2 = (a_1 a_2 - b_1 b_2 - c_1 c_2 - d_1 d_2) + (a_1 b_2 + b_1 a_2 + c_1 d_2 - d_1 c_2)i + (a_1 c_2 + a_2 c_1 + b_2 d_1 - d_2 b_1)j + (a_1 d_2 + d_1 a_2 + b_1 c_2 - c_1 b_2)k$
- Square $q^2 = (a^2 - b^2 - c^2 - d^2) + 2abi + 2acj + 2adk$
- Conjugate$\bar{q} = a - bi - cj - dk$
- Rules $N(q) = q\bar{q} = \bar{q}q = a^2 + b^2 + c^2 + d^2 \geq 0$
- Modulus $|q| = \sqrt{(N(q))} = \sqrt{a^2 + b^2 + c^2 + d^2}$

Let Q be a set of quaternion, and suppose $A \in Q^{n\times n}$ be a matrix, then the following concepts and propositions are used in our algorithm:

- If $A^H = A$, then A is named self-conjugate quaternion matrix. The set of n order self-conjugate quaternion matrix write $SC_n(Q)$;
- If $lambda \in Q$ and $0 \neq a \in Q^{n\times 1}$ satisfy $Aa = a\lambda$(or $Aa = \lambda a$), then λ is called the right(or left) eigenvalue of A and a is the eigenvector belong to right(or left) eigenvalue of A. If λ is both the right and left eigenvalue of A, then name λ the eigenvalue of A;
- If $U \in Q^{n\times n}$and $UU^H = U^H U = I$, then U is extended unitary matrix. It means $U^H = U^{-1}$ if U is an extended unitary matrix;
- We mentioned that quaternion matrix can expressed as the sum of two plural matrices in the first paragraph. So $A = A_1 + A_2 j$, then the plural matrix $A^\sigma = \begin{pmatrix} A_1 & -A_2 \\ \bar{A_2} & \bar{A_1} \end{pmatrix}$ is the induced matrix of quaternion matrix A;
- If A is self-conjugate quaternion matrix, then A^σ is Hermite matrix;

- Eigenvalues of quaternion matrix A are the same as eigenvalues of its induced matrix A^σ;
- If A^σ is the induced matrix of quaternion matrix A, and λ is eigenvalue of A^σ, the corresponding eigenvector is $\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$, then the eigenvector of the matrix A eigenvalue λ is $\alpha_1 + \bar{\alpha_2} j$;

The trigonometric expression of q=a+bi+cj+dk can be written as the following form:

$$q = |q|(\cos\theta + I\sin\theta) \tag{1}$$

$$I = \frac{1}{\sqrt{b^2+c^2+d^2}}(bi+cj+dk) \tag{2}$$

$$\theta = \arctan\frac{\sqrt{b^2+c^2+d^2}}{a} \tag{3}$$

Then the formula of quaternion n-th root expresses as

$$\sqrt[n]{q} = \sqrt[n]{|q|}(\cos\frac{\theta+2k\pi}{n} + I\sin\frac{\theta+2k\pi}{n}), k = 0, 1, \cdots, n \tag{4}$$

If Q is a quaternion vector, the Euclidean norm of Q expresses as

$$\|Q\|_2 = (QQ)^{\frac{1}{2}} = (\sum_{i=1}^{n} q_i^2)\frac{1}{2} \tag{5}$$

2.2. **Orthogonal eigenvectors.** It is easy to verify the associative law and the commutative law for quaternion addition, and the associative law for quaternion multiplication. But quaternion multiplication does not satisfy the commutative law which makes quaternion matrix calculation much more complex than real or plural matrix. It is necessary to compute the orthogonal eigenvectors of quaternion matrix for our algorithm. In theory, the above section shows a way to get the eigenvectors of quaternion matrix. Though the eigenvectors of the induced matrix can be figured out and build the eigenvectors of the quaternion matrix by Matlab, it is uncertain that these eigenvectors of quaternion matrix are mutually orthogonal. In this paper, we adopt a reasonable way proposed by the reference[16] to solve this problem.

A random $n \times n$ self-conjugate quaternion matrix A(the train sample scatter matrix in this multimodal fusion recognition application is a self-conjugate quaternion matrix), its induced matrix is A^σ. $A^\sigma X = \lambda X$ is the feature equation of A^σ, that is $(A^\sigma - \lambda I)X = 0$. Let $\lambda_i \in R$ be the eigenvalues and the corresponding eigenvectors of A^σ are $X_i, i = 1, 2, \cdots, n$. Build the following expressions with different λ_i, λ_j:

$$I - (A^\sigma - \lambda_i I)^+(A^\sigma - \lambda_i I) \tag{6}$$

$$I - (A^\sigma - \lambda_j I)^+(A^\sigma - \lambda_j I) \tag{7}$$

If $\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$ and $\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$ respectively are the first column vector of the above two expressions, and $\alpha_1, \alpha_2, \beta_1, \beta_2$ have the same dimensions. Then λ_i, λ_j are the eigenvalues of quaternion matrix A, and $\alpha_1 + \bar{\alpha_2} j, \beta_1 + \bar{\beta_2} j$ are the corresponding eigenvectors.

3. **Proposed algorithm.** We reference the principle of Fisher discriminant analysis(FDA) and proposed quaternion Fisher discriminant analysis(QFDA). Face and palm are selected as the experimental objects. Figure 1 display the framework of the proposed algorithm.

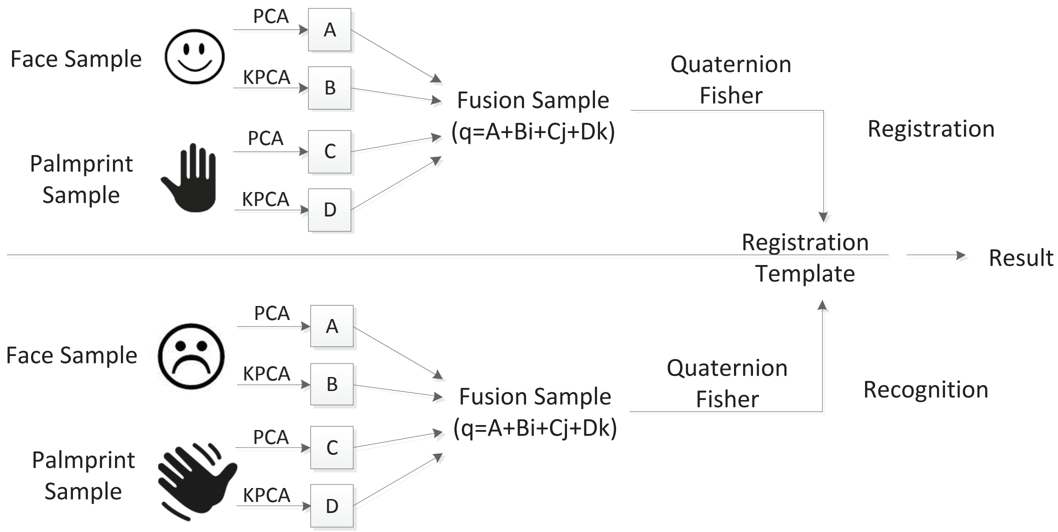

FIGURE 1. The framework of the proposed algorithm

3.1. **Multi-feature extraction.** Our algorithm fuses different biological features at the feature level, so features must be extract at the first. A successful recognition algorithm highly depends on the extract and choice of biological feature. It aims at extracting obviously differentiable features and reducing dimension of pattern space at the same time to improve the running speed.In this paper, we respectively extract the face feature with PCA and KPCA, and then extract palm feature with PCA and KPCA. In this way, we get four different features of two kinds of modalities that can be fused.

As you know, the Principal Component Analysis(PCA)[14] and the Kernel Principal Component Analysis (KPCA)[15] both are widely used in the research of identity authentication. PCA reconstitutes the modal sample to find the principal projection vector. It gives linear features. Suppose there are n training samples $x_1, x_2, \cdots, x_n$, we construct the total scatter matrix S_t as follows:

$$S_t = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(x_i - \bar{x})^t \tag{8}$$

where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ is the mean value of the total samples. The objection function of PCA is defined as follows:

$$J_{PCA} = \arg\max_{W} W^t S_t W \tag{9}$$

The eigenvectors of S_t compose the projection of W. In order to unify the dimension of feature vectors, we can select m largest eigenvalues and their corresponding eigenvectors to compose the projection W. Then the feature vector $y_i = W^t x_i$ can be obtained.

KPCA introduces the kernel function on the base of PCA and projects the input space to high-dimensional feature space by the nonlinearity projection. Then run the process of PCA in the high-dimensional feature space. First of all, a nonlinear mapping ϕ is used to map the sample space R^d into the nonlinear space F

$$\begin{array}{rccc} \phi: & R^d & \rightarrow & F \\ & x & \mapsto & \phi(x) \end{array} \tag{10}$$

Then perform PCA in the nonlinear space F. At the same condition, there are n training samples $x_1, x_2, \cdots, x_n$, we construct the total scatter matrix in the nonlinear space F

$$S_t = \frac{1}{n}\sum_{i=1}^{n}(\phi(x_i) - \bar{\phi})(\phi(x_i) - \bar{\phi})^t \tag{11}$$

where $\bar{\phi} = \frac{1}{n}\sum_{i=1}^{n}\phi(x_i)$. However, it is difficult to do so directly because of the high dimension of the nonlinear space F. Fortunately, kernel tricks can avoid this computation by the following rule

$$k(x_i, x_j) =< \phi(x_i), \phi(x_j) >= \phi(x_i)^t\phi(x_j) \tag{12}$$

where $k(\cdot)$ denotes the kernel function. In this method, the explicit mapping ϕ is not required. Three classes of kernel functions widely used in kernel classification are polynomial kernels, Gaussian kernels and sigmoid kernels. Our algorithm adopts polynomial kernel function which can be represented as

$$k(x_i, x_j) = (x_i \cdot x_j)^r \tag{13}$$

where r is a constant.

Our goal is to compute the eigenvalues and the corresponding eigenvectors of S_t. Singular value decomposition (SVD) technique is also adopted to reduce computational effort derived from the high dimensional nonlinear space F. Define the matrix $Q = [\phi(x_1), \phi(x_2), \cdots, \phi(x_n)]$, and form the matrix $\tilde{R} = Q^tQ$ whose elements can be computed as follows

$$\tilde{R}_{ij} = \phi(x_i)^t\phi(x_j) = k(x_i, x_j) \tag{14}$$

Then centralize $\tilde{R}$ as

$$R = \tilde{R} - 1_n\tilde{R} - \tilde{R}1_n + 1_n\tilde{R}1_n \tag{15}$$

where 1_n denotes a $n \times n$ matrix whose elements are all equal to $1/n$. Suppose $u_1, u_2, \cdots, u_m$ be the eigenvectors of R corresponding to m largest eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m$. Then, by SVD technique, the eigenvectors $w_1, w_2, \cdots, w_m$ of S_t corresponding to m largest eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$ are

$$w_i = \frac{1}{\sqrt{\lambda_i}}Qu_i \; i = 1, 2, \cdots, m \tag{16}$$

By projecting the mapped vector $\phi(x)$ onto the eigenvector w_i, we can obtain the ith projection component

$$\begin{aligned} y_i &= w_i^t\phi(x) = \tfrac{1}{\sqrt{\lambda_i}}u_i^tQ^t\phi(x) \\ &= \tfrac{1}{\sqrt{\lambda_i}}u_i^t[k(x_1, x), k(x_2, x), \cdots, k(x_n, x)]^t \end{aligned} \tag{17}$$

So the CKPCA-transformed projection vector Y of the sample x is composed as $Y = (y_1, y_2, \cdots, y_m)^t$.

3.2. **Quaternion Fisher discriminant analysis.** As you know, quaternion q is consist of four parts which are one real part a and three imaginary parts $bi+cj+dk$. Four different feature extracted in the above section will be taken as the four parts of quaternion. For example, we name the face feature extracted with PCA is A, and similarly there are B, C and D. Let A be the real part of quaternion and B be the first imaginary part, C be the second and D be the last. In this way, We can get an quaternion matrix consist of these four different features. Training and testing sample set of fused of multi-feature can be built in this method. Then we expand FDA to quaternion field and deal with the sample set. It is worth mentioning that different features may have different dimensions, so we add zeros to make sure that they are in the same dimension.

We reduce the dimension of the sample set before apply QFDA. Suppose there are a total of n classes, each class of m samples. In the unitary space, the within-class scatter matrix S_w, the between-class scatter matrix S_b and the total scatter matrix S_t can be defined as follows:

$$S_w = \sum_{i=1}^{n} p(\omega_i) E\{(x - \bar{x}_i)(x - \bar{x}_i)^H | \omega_i\} \tag{18}$$

$$S_b = \sum_{i=1}^{n} p(\omega_i)(\bar{x}_i - \bar{x})(\bar{x}_i - \bar{x})^H \tag{19}$$

$$S_t = S_w + S_b = E\{(x - \bar{x})(x - \bar{x})^H\} \tag{20}$$

where $p(\omega_i)$ denotes the prior probability of class i; $\bar{x}_i$ and $\bar{x}$ denote the mean vector of class i and the mean of all training samples respecively. $E\{\cdot\}$ denotes the expectation operation. In the actual operation, we should convert the defination of S_w, S_b, S_t into discrete version. For any sample x, the prior probability $p(\omega_i)$ is equal to $\frac{1}{n}$, then

$$S_w = \frac{1}{n} \sum_{i=1}^{n} E\{(x - \bar{x}_i)(x - \bar{x}_i)^H | \omega_i\} \tag{21}$$

where

$$\begin{aligned} & E\{(x - \bar{x}_i)(x - \bar{x}_i)^H | \omega_i\} \\ = \; & E\{(x - \bar{x}_i)(x - \bar{x}_i)^H\}_{x \in \omega_i} \end{aligned} \tag{22}$$

We denote $x \in \omega_i$ as x_i^j which represents sample j of class i, then

$$\begin{aligned} & E(x_i^j - \bar{x}_i)(x_i^j - \bar{x}_i)^H \\ = \; & \tfrac{1}{m} \textstyle\sum_{j=1}^{m} (x_i^j - \bar{x}_i)(x_i^j - \bar{x}_i)^H \end{aligned} \tag{23}$$

So,

$$\begin{aligned} S_w &= \tfrac{1}{n} \textstyle\sum_{i=1}^{n} \tfrac{1}{m} \sum_{j=1}^{m} (x_i^j - \bar{x}_i)(x_i^j - \bar{x}_i)^H \\ &= \tfrac{1}{mn} \textstyle\sum_{i=1}^{n} \sum_{j=1}^{m} (x_i^j - \bar{x}_i)(x_i^j - \bar{x}_i)^H \end{aligned} \tag{24}$$

In general, the constant $\frac{1}{mn}$ is ignored. According to the similar process, we can get

$$\begin{aligned} S_b &= \textstyle\sum_{i=1}^{m} (\bar{x}_i - \bar{x})(\bar{x}_i - \bar{x})^H \\ S_t &= \textstyle\sum_{i=1}^{n} \sum_{j=1}^{m} (x_i^j - \bar{x}_i)(x_i^j - \bar{x}_i)^H \end{aligned} \tag{25}$$

And the follow quaternion Fisher is similar to general Fisher in the field of real number. Its worth noting that the generating matrix structured with above method is a self-conjugate quaternion matrix. With these above definitions, we can achieve our algorithm in quaternion field.

4. **Experiment results and analysis.** Aiming at the superiority and feasibility of the algorithm, we performed an experiment. Yale face database is build by Yale university center for computational vision and control, it involves 15 volunteers, 165 pictures with different illumination, expression and posture. PolyU palm print database is build by The Hong Kong Polytechnic University and involves 100 volunteers with 6 pictures each. In the experiment, modal images are provided by these two databases. We ran the algorithm on these two databases to verify multi-feature recognition.

False match rate (FMR) and false non-match rate (FNMR) are more suitable to evaluate the performance of the algorithms in an off-line technology test [17] and therefore are used as the performance parameters of the proposed algorithm in this letter. Furthermore, equal error rate (EER) is taken as another parameter to evaluate the performance of our algorithm. EER unify FMR and FNMR to one parameter to measure the overall performance of algorithm. In the same coordinate, FNMR and FMR have an intersection that means they have the same value. The value of this print is EER. FMR and FNMR both should be smaller in same threshold value for a better algorithm. Figure 2 shows the comparing DET curves. From figure 2, we find that the proposed algorithm outperforms four single feature recognition algorithms: face with PCA[14], face with KPCA [15], palm with PCA [18] and palm with KPCA [19].

Table 1. EER comparison of different methods

Algorithm	EER(%)
Face with PCA [14]	13.3
Face with KPCA [15]	15.8
Palm with PCA [18]	17.7
Palm with KPCA [19]	19.7
The proposed algorithm	6.6

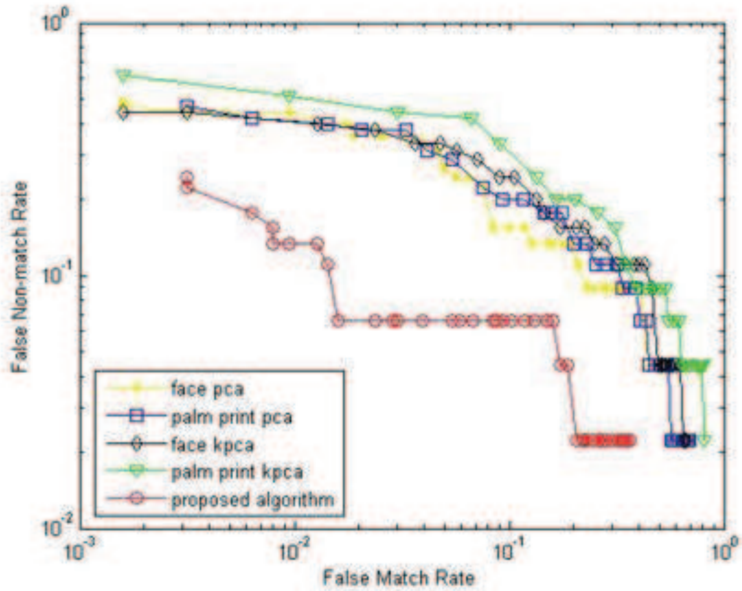

Figure 2. The comparing results of DET curves

Table 1 gives the data of EER of four single biological features recognition and the fused feature with the algorithm we proposed. From the table, we can clearly see that the lowest EER of these single features is Face with PCA and its 13.3%. And the highest is Palm with KPCA, it even be 19.7%.However, it has a huge change after fuse. The EER of fused feature with the algorithm we proposed is just 6.6% and it is only half of the lowest single feature. This is an acceptable improvement.

5. **Conclusions.** Comparing with unimodal biometrics, multimodal biometric technology has the higher reliability, wider applicability and stronger security. So it has draw more and more attention for its huge potentials. For multimodal biometrics, feature level fusion can derive the most discriminantivef informations from original multiple feature

sets and eliminate the redundant information resulting from the correlation between different feature sets. This paper fused different features into quaternion, and extracted the feature of the quaternion matrix. Compare with these existing algorithms, we enhance the features from two to four. And the recognition rate have greatly improved than single feature. Experiment results show that the proposed algorithm is feasible and accurate.

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