

# Face Recognition Based on LDA and Improved Pairwise-Constrained Multiple Metric Learning Method

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**ABSTRACT.** *The accurate alignment for different face images acquired in different conditions is very difficult because face appearance and noise are complex and variable. For solving this question, an Improved Pairwise-constrained Multiple Metric Learning method (IPMML) is proposed as a classification metric, which can obtain a Mahalanobis matrix by PMML between a testing face image and a training face image instead of all training face images. However, the calculation of the IPMML is complicated and time consuming for the data with high dimension. Therefore, Linear Discriminant Analysis (LDA) is selected to decrease the feature dimension and enhance recognition accuracy. Thus, a novel face recognition method based on LDA and IPMML is proposed. First, the fisher features of a face image are extracted using the LDA method. Then, the extracted features are divided into a number of blocks according to the feature dimension and every block is changed to a column vector. Third, the IPMML is used to compute the Mahalanobis matrices between the testing sample blocks and every training sample blocks. Fourth, the final discriminative distance is obtained using the optimum Mahalanobis matrices. Finally, the Nearest Neighborhood Classifier (NNC) is performed to recognize face. The experimental results based on Yale, Extended Yale B, and AR Databases showed that the proposed method can recognize face effectively and reliably, especially for the little training sample number and misalignment conditions.*

**Keywords:** Face Recognition; LDA; IPMML; NN.

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1. **Introduction.** In identification, face recognition technology has been used more and more because of its natural characters and no harmful [1]. Feature extraction and classification is two important parts of face recognition because the performance of the feature

and the classifiers decides the final classification results directly. However, face images still contains various noise signals because of the difference of image acquisition environment even if the preprocessing methods have improved the image quality before recognition. So it is a key to select an appropriate classifier. NNC [2] was the most commonly used classification method which can classify face images correctly and effectively. Support Vector Machine (SVM) [3] had many special advantages, such as solving small samples, nonlinear and high dimensional pattern recognition, and so on. Back-Propagation neural network [4] had strong nonlinear mapping capacity and adaptive learning. Zhou et al. [5] used the SRC and the multi-wavelet method to recognize face. But these methods are effective mainly for the alignment images. The pairwise-constrained Multiple Metric Learning (PMML) proposed by Cui et al[6], which could give an effective metric between the testing sample and the training samples by computing an optimum Mahalanobis matrix between the testing face image and all training face images. Although the PMML method has improved recognition rate than the above methods when the face images are not aligned precisely, but it can not still recognize face effectively for the misaligned face image set.

By analyzing the PMML algorithm, it can be found that the optimum Mahalanobis matrix is obtained by all training images, which depresses the difference among the training face images. For solving this question, an Improved Pairwise-constrained Multiple Metric Learning (IPMML) is proposed, which can obtain the discriminate distance by computing the Mahalanobis matrix between a testing face image and every training face image. Compared with PMML, IPMML can preserve the detail characteristics of a face image and effectively recognize the misalignment face images.

However, the complexity of the calculation of IPMML is very high because the optimum Mahalanobis matrix between a testing face image and every training face image should be computed. So the dimension reduction is considered in this paper. Because LDA [7] can decrease the feature dimension and maintain image category information more effectively than Whitened Principle Component Analysis (WPCA) used in [6], LDA is used to extract the face fisher features and decrease feature dimensions before using the IPMML method.

Thus, a face recognition method based LDA and IPMML is proposed in this paper. First, a face fisher features are extracted using the LDA method, which can effectively reduce the data dimension at the same time. And then, the extracted features are blocked according to the feature dimension and every block is reshaped to a column vector. Third, the optimum Mahalanobis matrices between the testing sample blocks and every training sample blocks are obtained using the IPMML method. Fourth, the final discriminative distance can be computed using the optimum Mahalanobis matrices. Finally, the face classification is realized using the NNC. The experiment results show that the method proposed in this paper can recognize face effectively, especially for misalignment face images.

The rest of this paper is organized as follows. In Section 2, we give a review of the LDA face feature extraction method. IPMML is introduced as a metric computation method in Section 3. In Section 4, we illustrate the details of the proposed method. Then, we perform some experiments on Yale, Extended Yale B, and AR Databases and analyze the results in Section 5. Finally, some conclusions are concluded in Section 6.

**2. Linear Discriminant Analysis (LDA).** LDA [6] is to project the high-dimensional samples into a low-dimensional subspace using linear mapping, which has the maximum inter-class distance and minimum intra-class distance between the projected samples in the low-dimensional subspace through searching for an optimized projection matrix. Suppose

face image  $\mathbf{A}_{ij}$  ( $i = 1, \dots, C, j = 1, \dots, N$ ) is the  $j$ -th sample of the  $i$ -th class, where  $C$  is the total class number and  $N$  is the number of the samples per class.  $\mathbf{S}_W$  is the intra-class dispersion of samples, and  $S_B$  is the inter-class dispersion of samples.

$$\mathbf{S}_W = \frac{1}{N} \sum_{i=1}^C \sum_{j=1}^N (\mathbf{A}_{ij} - \mu_i)(\mathbf{A}_{ij} - \mu_i)^T, \mathbf{S}_B = \sum_{i=1}^C (\mu_i - \mu)(\mu_i - \mu)^T \quad (1)$$

where  $\mu_i = \frac{1}{N} \sum_{j=1}^N \mathbf{A}_{ij}$  is the mean vector of  $i$ -th sample,  $\mu = \frac{1}{C} \sum_{i=1}^C \mu_i$  is the mean vector of all samples.

For enhancing the role of the intra-class and inter-class dispersions between samples, Fisher criterion function [7] is defined as follows,

$$J_F = \frac{w^T \mathbf{S}_B w}{w^T \mathbf{S}_W w} \quad (2)$$

where  $w$  is the projection matrix. Therefore the maximum inter-class dispersion and minimum intra-class dispersion can be obtained at the same time by searching a maximum  $J_F$ . For obtaining the maximum  $J_F$ , the question can be translated into searching the eigenvector corresponding to the maximum eigenvalue of equation 3.

$$\mathbf{S}_B w = \lambda \mathbf{S}_W w \quad (3)$$

where  $\lambda$  is Lagrange multiplier. Because  $\mathbf{S}_W$  is nonsingular, both sides of equation (3) can be left multiplied by  $\mathbf{S}_W^{-1}$ .

$$\mathbf{S}_W^{-1} \mathbf{S}_B w = \lambda w \quad (4)$$

thus  $\lambda$  is the eigenvalues of  $\mathbf{S}_W^{-1} \mathbf{S}_B$ , its corresponding eigenvector is  $w$ . All nonzero eigenvalues is ordered from large to small. The  $P$  eigenvectors corresponding to the front  $P$  ( $P \leq C - 1$ ) eigenvalues in  $(C - 1)$  nonzero eigenvalues of  $\mathbf{S}_W^{-1} \mathbf{S}_B$  are selected to construct a feature subspace  $\mathbf{W}$ , which can be expressed as follows.

$$\mathbf{W} = [w_1, w_2, \dots, w_P] \quad (5)$$

where  $w_i$  ( $i = 1, 2, \dots, P$ ) is the optimum projection direction of  $i$ -th sample.

**3. IPMML.** PMML [5] computes the optimum Mahalanobis matrix only based on all training face images, so it can not recognize the face effectively for misalignment images. Therefore, the IPMML method is proposed, in which the optimum Mahalanobis matrix is computed for the testing face image and every training face image for improving the recognition.

Assume  $\mathbf{B}_i^k$  ( $i = 1, \dots, N, k = 1, \dots, K$ ) and  $\mathbf{B}_j^k$  ( $j = 1, \dots, M, k = 1, \dots, K$ ) represent the  $k$ -th block of the  $i$ -th training sample and the  $k$ -th block of the  $j$ -th testing sample separately, where  $N$  is the number of training samples,  $M$  is the number of testing samples. The distance between  $\mathbf{B}_i^k$  and  $\mathbf{B}_j^k$  is defined as follows.

$$d(\mathbf{B}_i, \mathbf{B}_j) = \frac{1}{K} \sum_{k=1}^K (\mathbf{B}_i^k - \mathbf{B}_j^k)^T \mathbf{W}_k (\mathbf{B}_i^k - \mathbf{B}_j^k) \quad (6)$$

where  $\mathbf{W}_k$  is Mahalanobis matrix. In general, the two samples can be thought as the same class if the distance  $d(\mathbf{B}_i, \mathbf{B}_j)$  between two given samples  $\mathbf{B}_j$  and  $\mathbf{B}_i$  is less than a certain threshold  $\rho$ . Otherwise, the two samples belong to different class. Thus, we should find an optimized matrix  $\mathbf{W}_k$  ( $k = 1, \dots, K$ ) to compute the distance.

To obtain optimal Mahalanobis matrix  $\mathbf{W}_k$ , we define the objective function of IPMML as follows.

$$\min_{\mathbf{W}_k, \xi_{ij}} \frac{1}{K} \sum_{k=1}^K H(\mathbf{W}_k, \mathbf{W}_0) + \frac{\gamma}{n} \sum_{i,j} l(\xi_{ij}, \delta_{ij}\rho - \tau) \quad (7)$$

$$\text{s.t. } \frac{\delta_{ij}}{K} \sum_{k=1}^K d_{\mathbf{W}_k}(\mathbf{B}_j^k, \mathbf{B}_i^k) \leq \xi_{ij} \quad (8)$$

where  $n$  is the number of training sample pairs,  $\gamma$  is a tradeoff parameter, the initial matrix  $\mathbf{W}_0$  is set as an identity matrix, if the two samples belong to the same people,  $\delta_{ij} = 1$ , otherwise,  $\delta_{ij} = -1$ .  $l(\cdot, \cdot)$  is the first order continuous differentiable hinge loss function and it is defined as follows.

$$l(x, x_0) = \begin{cases} 0 & x \leq x_0 \\ (x - x_0)^2 & x > x_0 \end{cases} \quad (9)$$

The detail process of IPMML is as follows.

Step 1: Set  $t = 1$ ,  $\mathbf{W}_k^1 = \mathbf{W}_0$ ,  $\eta_{ij} = 0$ ,  $\xi_{ij}^t = \delta_{ij}\rho - \tau$ ,  $\rho = 1 \times 10^4$ ,  $\tau = 1.0$ ,  $\gamma = 0.1$ .

Step 2: Compute the distance  $d_{\mathbf{W}_k^t}(\mathbf{B}_j^k, \mathbf{B}_i^k)$  between the  $k$ -th block of testing sample  $\mathbf{B}_j^k$  and the  $k$ -th block of training sample  $\mathbf{B}_i^k$  ( $k = 1, \dots, K$ ).

Step 3: Solve  $\alpha$  according to the equation (10) and set  $\alpha = \min(\alpha, \eta_{ij})$ ,  $\eta_{ij} = \eta_{ij} - \alpha$ .

$$\frac{\delta_{ij}}{K} \sum_{k=1}^K \frac{d_{\mathbf{W}_k^t}(\mathbf{B}_j^k, \mathbf{B}_i^k)}{1 - \delta_{ij}\alpha d_{\mathbf{W}_k^t}(\mathbf{B}_j^k, \mathbf{B}_i^k)} - \left( \xi_{ij}^t - \frac{n}{2\gamma}\alpha \right) = 0 \quad (10)$$

Step 4: Update  $\mathbf{W}_k^{t+1}$  according to the equation (11) and set  $\mathbf{W}_k = \mathbf{W}_k^{t+1}$ , ( $k = 1, \dots, K$ ),

$$\mathbf{W}_k^{t+1} = \mathbf{W}_k^t + \mu \left( \mathbf{W}_k^t (\mathbf{B}_i^k - \mathbf{B}_j^k) (\mathbf{B}_i^k - \mathbf{B}_j^k)^T \mathbf{W}_k^t \right) \quad (11)$$

where,  $\mu = \delta_{ij}\alpha / \left( 1 - \delta_{ij}\alpha d_{\mathbf{W}_k^t}(\mathbf{B}_j^k, \mathbf{B}_i^k) \right)$ .

Step 5: Update  $\xi_{ij}^{t+1}$  according to the equation (12).

$$\xi_{ij}^{t+1} = \xi_{ij}^t - \frac{n}{2\gamma}\alpha \quad (12)$$

Step 6: Stop the iteration and output  $\mathbf{W}_k$  if  $\frac{\delta_{ij}}{K} \sum_{k=1}^K d_{\mathbf{W}_k}(\mathbf{B}_j^k, \mathbf{B}_i^k) \leq \xi_{ij}$  or  $t > (N + M) \times 10$ , otherwise, set  $t = t + 1$  and go to Step 3.

Step 7: Compute the distance between  $\mathbf{B}_j$  and  $\mathbf{B}_i$  using the optimal Mahalanobis matrix  $\mathbf{W}_k$ .

**4. The proposed method.** LDA can extract features and reduce image dimension effectively. The IPMML can preserve the detail characteristics of a face image and perform better in recognizing misalignment face images. So an face recognition approach combining LDA and IPMML is proposed. The diagram is as shown in Figure 1. First, the LDA method is performed to obtain the fisher features of a face image. Second, the extracted features are divided into several blocks and every block is changed to a column vector. Third, the Mahalanobis matrices between the testing sample blocks and every training sample blocks are computed using the IPMML method. Fourth, the final discriminative

distance can be calculated using the optimum Mahalanobis matrices. Finally, the classification of the training sample corresponding to the minimum distance is the classification of the testing sample. The detail process of the proposed approach is as follows.

Step 1: Apply LDA on training samples  $\mathbf{A}_i$  ( $i = 1, \dots, N$ ) to search a group of optimal eigenvectors and the eigenvectors are used to obtain low-dimension training images  $\mathbf{B}_i$ .

Step 2: Divide the extracted features into  $K$  blocks and every block is changed to a column vector,  $K$  is selected according to the feature dimension.

Step 3: Process testing samples  $\mathbf{A}_j$  ( $j = 1, \dots, M$ ) according to step 1,2 to obtain low-dimension testing images  $\mathbf{B}_j$ .

Step 4: Compute the distance of testing sample  $\mathbf{B}_j$  and training samples  $\mathbf{B}_i$  ( $i = 1, \dots, N$ ) according to the algorithm proposed in Section 3.

Step 5: If the distance of two samples  $d(\mathbf{B}_j, \mathbf{B}_l) = \min_i d(\mathbf{B}_j, \mathbf{B}_i)$ , and  $\mathbf{B}_l \in c$ ,  $1 \leq c \leq C$  ( $C$  is the number of training samples categories), then  $\mathbf{B}_j \in c$ .

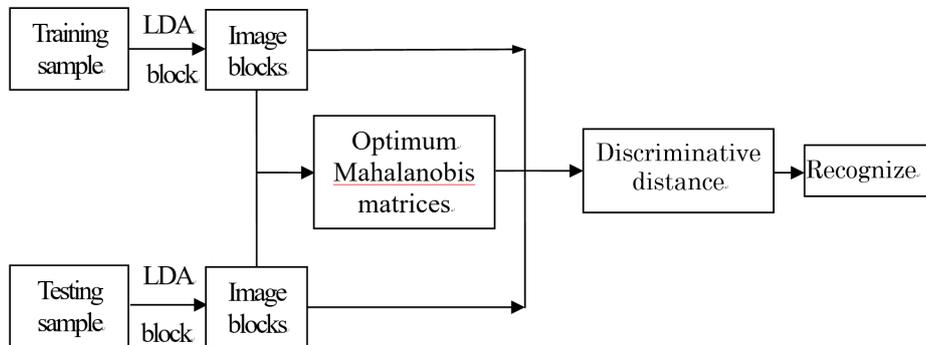


FIGURE 1. The face recognition block diagram based on LDA and IPMML

**5. Experimental results and analysis.** In this paper, Yale face database, Extended Yale B (EYB) database and AR database is used to evaluate the proposed approach. Yale face database is a database with variable illuminations which includes 165 images of 15 individuals under various facial expressions, gaits and lighting conditions. The EYB database contains 38 people and the first 10 people were selected and every one has 64 images with different facial expressions and illumination conditions. AR face database includes 126 individuals and each one has 26 images under variable illuminations and different expressions. In this paper, the number of image blocks is selected according to the image dimension. For convenient calculation, the size of the images in the experiments is transformed to  $64 \times 64$ . In addition, we choose the first 3 images of a person as the misalignment face images by moving 10 pixels away for check the performance of the proposed. We take a person as an example to illustrate, as shown in Fig. 2 and Fig. 3. The same process is performed for all experimental database in this paper.



FIGURE 2. Six original face images of a person of the EYB database



FIGURE 3. The images with the first 3 misalignment face images of figure 2

5.1. **Yale dace database.** In the experiment, we randomly choose 2, 3,  $\dots$ , 7 images as training samples, the others as testing samples. The extracted features using LDA are divided into 7 blocks and the extracted features using WPCA are divided into 15 blocks. In this paper, we do experiments using LDA + improved PMML (LIP), LP, LDA + NNC (LN), WPCA + improved PMML (WIP) on original images and LDA + improved PMML (LIP), LDA + PMML (LP), LDA + NNC (LN) on misalignment face images. Every experiment is performed 5 runs. The average recognition accuracy (ARA) and the standard deviation (SR) are calculated. The results are shown in Table 1 and Table 2. From Table 1 and 2, the proposed approach can recognize face image better than the other method whatever for the alignment and misalignment conditions, especially when the number of the training samples is small. In addition, the proposed method is effective when the face images are under various facial expressions, gaits and lighting conditions.

For comparing the complexity, we take 2 images as the training face images. 13.6s is used to recognize a testing face only by the PMML method. Whereas, it needs 17.71s only by IPMML method. But 0.136s and 0.181s are needed to recognize a testing face by the LP and LIP method respectively. Although the recognition time using the proposed method is longer than the LP method, the recognition rate is improved obviously, especially for misalignment images.

TABLE 1. The ARAs on the Yale database (%)

$n^*$	Original alignment Images				Misalignment Images		
	LIP	LP	LN	WIP	LIP	LP	LN
2	<b>82.72</b>	78.02	77.78	57.78	<b>62.96</b>	58.77	58.27
3	<b>88.33</b>	85.28	85.00	73.89	<b>72.50</b>	68.33	67.78
4	<b>94.60</b>	92.06	92.06	80.00	<b>83.17</b>	76.19	75.87
5	<b>95.19</b>	93.70	93.33	83.70	<b>85.56</b>	81.85	81.85
6	<b>96.00</b>	90.67	91.11	82.22	<b>88.44</b>	82.22	82.67
7	<b>98.89</b>	98.33	98.89	94.44	<b>85.56</b>	80.56	80.00

$n^*$ : The number of training samples per person

TABLE 2. The SRs on the Yale database (%)

$n^*$	Original alignment Images				Misalignment Images		
	LIP	LP	LN	WIP	LIP	LP	LN
2	<b>3.50</b>	2.60	2.67	11.50	<b>13.17</b>	12.67	11.91
3	<b>7.41</b>	8.55	8.46	7.09	<b>4.33</b>	6.51	6.14
4	<b>2.91</b>	2.40	1.45	3.43	<b>3.85</b>	2.86	2.91
5	<b>2.80</b>	3.21	2.94	1.28	<b>5.09</b>	5.70	5.70
6	<b>3.53</b>	9.33	9.83	5.39	<b>6.01</b>	10.01	10.07
7	<b>2.89</b>	1.92	6.74	8.55	<b>14.56</b>	15.28	0.96

**5.2. EYB face database.** In the experiment, we randomly choose 2, 3,  $\dots$ , 7 images as training samples, the others as testing samples. The extracted features using LDA are divided into 3 blocks and the extracted features using WPCA are divided into 10 blocks. In this paper, we do experiments using LIP, LP, LN, WIP on original images and misalignment face images. Every experiment is performed 5 runs as section 5.1. The results are shown in Table 3 and Table 4. From Table 3 and 4, the same results with section 5.1 can be obtained. At the same time, the proposed method is effective for face images with different facial expressions and illumination condition.

TABLE 3. The ARAs on the EYB database (%)

$n^*$	Original alignment Images				Misalignment Images		
	LIP	LP	LN	WIP	LIP	LP	LN
2	<b>55.81</b>	31.34	31.88	38.39	<b>51.56</b>	27.69	28.12
3	<b>77.76</b>	49.95	52.51	61.75	<b>64.97</b>	36.39	37.87
4	<b>80.56</b>	64.28	64.06	65.17	<b>70.17</b>	50.11	50.44
5	<b>89.72</b>	81.69	81.36	74.24	<b>72.82</b>	60.28	60.06
6	<b>87.93</b>	79.60	79.83	83.51	<b>81.67</b>	68.33	68.56
7	<b>89.36</b>	83.16	83.27	84.97	<b>83.39</b>	74.21	74.33

TABLE 4. The SRs on the EYB database (%)

$n^*$	Original alignment Images				Misalignment Images		
	LIP	LP	LN	WIP	LIP	LP	LN
2	<b>18.67</b>	7.45	8.15	17.51	<b>14.25</b>	4.67	5.06
3	<b>6.80</b>	20.78	15.59	2.94	<b>12.89</b>	11.71	9.88
4	<b>5.95</b>	10.46	10.08	4.59	<b>9.82</b>	11.57	11.53
5	<b>4.26</b>	4.61	5.04	4.18	<b>7.12</b>	11.08	10.90
6	<b>1.64</b>	1.64	1.30	11.66	<b>2.85</b>	5.96	5.44
7	<b>2.81</b>	3.70	3.34	7.39	<b>6.34</b>	4.48	4.28

**5.3. AR face database.** In the experiment, we randomly choose 2, 5, 7 images as training samples, the others as testing samples. The extracted features using LDA are divided into 13 blocks and the extracted features using WPCA are divided into 15 blocks. The experiments are performed using the method based on LIP, LDA + PMML (LP), LN, WIP on original images and misalignment face images. Every experiment is performed 5 runs as above experiments. The results are shown in Table 5 and Table 6. From Table 5 and 6, the same results with section 5.1 and 5.2 can be obtained. Like section 5.2, the proposed method is also effective for face images with different facial expressions and illumination condition.

TABLE 5. The ARAs on the AR database (%)

$n^*$	Original alignment Images				Misalignment Images		
	LIP	LP	LN	WIP	LIP	LP	LN
2	<b>63.99</b>	54.86	54.97	55.21	<b>53.16</b>	43.75	43.92
5	<b>82.90</b>	82.14	81.90	<b>87.82</b>	<b>77.02</b>	73.37	73.25
7	<b>96.58</b>	96.36	96.49	<b>97.85</b>	<b>84.43</b>	84.12	84.25

TABLE 6. The SRs on the AR database (%)

$n^*$	Original alignment Images				Misalignment Images		
	LIP	LP	LN	WIP	LIP	LP	LN
2	<b>2.99</b>	4.26	4.12	4.38	<b>2.68</b>	4.47	4.01
5	<b>10.05</b>	9.34	9.47	5.40	<b>6.25</b>	7.47	7.70
7	<b>1.39</b>	1.67	1.57	1.69	<b>2.43</b>	4.28	4.10

From Table 1, 2,  $\dots$ , 6, we can find that the proposed method can achieve higher recognition rates than the other three method both on original face images and misalignment face images with different facial expression, gait and illumination condition.

**6. Conclusions.** In this paper, a face recognition method based on LDA and IPMML is proposed, in which LDA can decrease dimension effectively and IPMML based on image blocks can reduce the affects because of image misalignment better than PMML. The experiment results on Yale face database, EYB face database and AR database show that the proposed method can recognize face with variable facial expressions, gait and illumination condition effectively and reliably, especially for the little training sample number and misalignment conditions.

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