A Coverage Loopholes Recovery Algorithm in Wireless Sensor Networks

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Received April, 2016; revised June, 2016

ABSTRACT. A Wireless Sensor Networks (WSNs) system is composed of sensors, which are capable of producing data by sensing the environment and transmit the data through the network back to the data process center. Because there are various kinds of attacks and energy exhausted, coverage loopholes is easy appeared in wireless sensor networks. Repairing of the coverage hole is essential. Localize the center of the coverage loopholes and restorative method are two important steps for repairing WSNs. In this paper α hull is applied to calculate the Minimum Circumscribed Circle (MCC) of a set of points, which is used for locating the center of the hole. After locating the hole in the WSNs, the Recovery Algorithm of Coverage Hole Based on α -hull (RACH) is proposed for the restorative process. We provide the analysis of correctness of RACH through theoretical proof, and the comparison between our algorithm and others through practical simulation. Experimental results show that the proposed algorithm is capable to restore the WSNs environment with less number of new added nodes.

Keywords: Wireless Sensor Networks, Recovery, Coverage loophole, α -hull

1. Introduction. Wireless sensor networks are a continuation of the evolution of networks toward larger-scale, distributed computing, they are made up of mostly small sensor with limited-resources and capabilities [1]. Recent years, WSNs have been widely applied in different applications. The deployed sensors cover a region to provide network services or sensing capabilities. Coverage problem is one of the most important performance parameter for WSNs [2,3]. It reflects the WSNs' quality of service, which can better fulfill the task of environmental awareness, information collection and transmission. However, because of the unequal random deployment, network energy consumption and attack, which lead to the formation of the coverage loopholes.

The emergence of the coverage loopholes, cause the loss of network coverage and connectivity, which serious impact on the performance of the network. Therefore, for the next step of implementation and deployment of security mechanisms, it is rather essential to find the hole center. However, it is not easy to locate the holes [4]. First of all, since the energy of sensor itself is limited, if the location methods of hole center bring too much communication overhead, it will not be feasible; secondly, the majority of location algorithms, protocols need to depend on special equipment, so the location is not easy to achieve. It is imperative to find an effective hole center location method. Finally, in order to make the network back to normal work, how to repair coverage hole is the most important problem.

The main contributions of this paper are as follows.

(1) An effective and accurate method for Minimum Circumscribed Circle (MCC) detection is established to locate the hole center, which is mainly through finding the least squares circle of the set of points and iteratively approaching the MCC with recursive subdivision. α -hull is applied to calculate the Minimum Circumscribed Circle (MCC) of a set of points. All vertices of the α -hull will be on the same circle, if $1/\alpha$ is equal to the radius of points' MCC.

(2) A Recovery Algorithm of Coverage Hole Based on α -hull (RACH) is proposed, which is based on the honeycomb covering. The algorithm is effective to improve the network coverage. The number of static sensors we added into the network is small, and at the same time reduce the number of the mobile nodes.

(3)We provide the analysis of correctness of our algorithm through theoretical proof, and the comparison between our algorithm and others through practical simulation. Experimental results show this algorithm need less new nodes for restorative.

The rest of this paper is organized as follows: Section II presents the related work. Section III presents Algorithm of Recovery Algorithm of Coverage Hole Based on α -hull (RACH). Simulation results are demonstrated in Section IV. The conclusion is in Section V.

2. Related work. There are a lot of researches focus on coverage hole [2,3,5,6,7,8], which can be divided into three kinds: static, dynamic and hybrid. The recover method by static is to add some sensors into the network. Researched by dynamic network can be divided into the method based on "virtual force" and the method based on geometry. There are many kinds of geometry method, such as the method based on Voronoi diagram, geometric vector method or using topology geometric rules methods. Hybrid network is based on the static network, a small amount of mobile sensor nodes are added into the network as the auxiliary. This method is higher than the method of static network in reliability, and the energy consumption is lower.

In Hybrid network, how to use the static node to realize mobile geometric positioning is very important. Wang et al. [9] proposed a patching algorithm triangle by triangle (PATT), which was to form the third side of the triangle by using the three nodes. Then through the proof of relevant geometric to find the location of the mobile node. However, when the density of the covering nodes around the hole is high, the algorithm will be complex, and made the high redundancy. Furthermore, too much mobile sensor nodes are needed to join the network.

Since the cost of mobile nodes is much higher than ordinary nodes, at the same time of guarantee coverage, the number of mobile node should be reduced as far as possible.

Organizing sensor nodes into clusters has been proven to be an effective strategy to prolong network lifetime [10-14]. In a cluster-based WSN, the cluster head can be constructed as a virtual backbone. Since traffic is only forwarded by virtual backbone, routing is easier and can adapt quickly to topology changes, so it can improve the routing performance and save energy of the networks. Mobile sensors are usually employed to improve the lifetime of the network. They can move to different places on demand. From the studies on sensor node organization, it is obvious that WSNs can indeed efficiently operate for a longer time by organizing sensor nodes into different types of WSN configurations. Cluster heads take important responsibility, whose energy resources may be used up rapidly. Mobile sensors can move to the cluster head on demand.

3. Problem statement.

3.1. **Definitions and notations.** In this paper, communication model will be introduced by using graph theory. Sensor nodes are randomly distributed in the network field and have the same transmission range. The link between any pair of nodes is bidirectional. One network is modeled as a connected bidirectional graph G = (V, E), where Vand E represents the node set and the link set in G, respectively. $\forall u, v \in V$, there exists an edge (u, v) in G if and only if u is in v's transmission range in the network, v is also in u's transmission range, and there is no obstacle preventing radio wave transmission between u and v.

This paper makes the following assumptions:

1) The sensor nodes in the system is classified into two classes, i.e., the static node and the mobile node. All static nodes are equipped with the same amount of energy in the beginning, and the batteries are not rechargeable. Moreover, the communication ability of each node is the same. Unlike the static nodes, the mobile sensor nodes are mobile in the space with the rechargeable batteries.

2) Let symbol R denotes the sensing range of a sensor node, The communication range of all nodes is defined as 2R.



FIGURE 1. The model of a coverage loophole

Definition 1. (Node neighbourhoods) [15] Consider a node u. The set of nodes covered by u is represented by N(u), $N(u) = \{v \mid (v, u) \in E\}$ is called the open neighbor set of u. $N[u] = N(u) \cup \{u\}$ is called the closed neighbor set of u.

Definition 2. (Coverage loophole) coverage loophole is an irregular area, which is not covered by any node sensing range. The coverage loophole is initialized by the edge nodes.

As shown in Fig. 1, nodes A, B, C, D, ... are edge nodes. Polygon ABC...I called coverage loophole polygon.

The area of the coverage loophole can be reduced by adding sensor nodes. Then the static nodes can be gradually put to fill the whole coverage loophole. The choice of the node position mainly considering the follows: Firstly, sensors can't be too far away from the boundary nodes, otherwise, a larger coverage loophole may be formed, and even cause the communication breakdown among the addition sensor nodes and the boundary nodes. Secondly, the static nodes can not to be too close to the boundary nodes. Otherwise, large overlapping will be caused and then increase the redundancy. So, how to determine the location of the sensors? In this paper, the center of the loophole should be determined.

Definition 3 (α -hull) [16]: For the real numbers α and a finite set of points P in a plane, the intersection of all the α - disks that inclusive P defined as the α -hull of P. All the points on the border of the α -hull defined as the α -hull vertex of the set P.

For $\alpha < 0$, the α -hull of P is the complement of the union of all disks of radius $-1/\alpha$ that do not contain any point of P, for $\alpha = 0$, the α -hull is just the convex hull and for $\alpha > 0$, the α -hull of P is the intersection of all disks of radius $1/\alpha$ that contain P. Fig. 2 illustrates α -hull of P for three values of α .

Positioning algorithm can be divided into two categories according to the positioning mechanism: the Range-based and the Range-free positioning technologies. In this paper α -hull is applied to calculate the Minimum Circumscribed Circle (MCC) of a set of points. All vertices of the α -hull will be on the same circle, if $1/\alpha$ is equal to the radius of points' MCC. Based on this nature, an effective and accurate method for MCC detection is established, it is mainly through finding the least squares circle of the set of points and iteratively approaching the MCC with recursive subdivision.

4. Recovery Algorithm of Coverage Hole Based on α -hull (RACH). The Recovery Algorithm of Coverage Hole Based on α -hull (RACH) is divided into 3 steps: Firstly, the center of the loophole should be determined. Secondly, the static nodes can be gradually put to fill the whole coverage loophole. Finally, Set the mobile nodes' mobile route.

4.1. Location of the center. A rather commonplace WSN model will be adopted in this paper. The sensors are deployed in the area of interest, which is a huge physical environment. The distribution of sensors is modeled by the Poisson distribution.

All vertices of the α -hull will be on the same circle, if $1/\alpha$ is equal to the radius of points' minimum circumscribed circle (MCC). And then based on the number of the vertices, the radius of the minimum circumscribed circle can be calculated.

Algorithm 1. Algorithm of α -MCC

Input: $P = \{p_1, p_2, p_3, \dots, p_n\}$, which is a set of points which are the disturbed cells. Output: the coordinates of centre O and r.

- 01: choose three points which are non-collinear.
- 02: calculating the center O of the circle that the three points are all in it.
- 03: Calculating the distances between the points of P and the center O.
- 04: find the farthest distance rb and the closest range ra, let r = rb.
- 05: if $(ra \ge rb)$, then do

06: ra is the radius of the minimum circumscribed circle,

07: calculating the center O, and the radius r

08: else find the α -hull $(1/\alpha = r)$ of the set P, the P_1 set up by the α -hull vertexes; 09: if $(P_1 \subset \phi)$ then do

10: r = (ra + rb)/2; ra = (ra + rb)/2; goto 5

11: else goto 12.

12: if $(\forall A(x, y) \in P_1)$, and the points of a circle) then do

13: if the center of the circle inside the α -hull, then

14: calculating the center O, and the radius r,

15: else find the biggest obtuse angle of the trianglethe edge which obtuse angle corresponding to is the diameter of the minimum circumscribed circle

16: then calculating the center O calculating the radius r, return r. (It is define that the biggest obtuse angle of the triangle the triangle with the edge which obtuse angle corresponding to is the biggest.)

17: if $(\forall A(x, y) \in P_1)$, the points are not on a circle) goto 18. 18: r = (ra + rb)/2; ra = (ra + rb)/2, $P = P_1$ goto 5.

4.2. Recovery Algorithm of Coverage Hole Based on α -hull (RACH). The WSN is governed by some hexagons, which like honeycomb. Sink labeled by (0,0) is at the center honeycomb cell of the network. As shown in Fig. 3, each honeycomb cell has its own label (x, y), where x express the layer of the cell, x express the serial number of the cell in its layer. The center cell is called as the *sublayer* -0, the cell labeled by (x, y) is called as *sublayer* -x. The labels marking steps are as follows:



FIGURE 2. The basic structure of the hierarchical WSN

(1) the cell in sublayer -0 is labeled by (0,0), and initialization $x \leftarrow 0$;

(2) set $x \leftarrow x + 1$, until meet regional boundary;

(3) found the cell in sublayer -x, which is near the cell (x - 1, 0) and unlabeled, this cell is labeled by (x, y), and initialization $x \leftarrow 0$;

(4) set $x \leftarrow x + 1$, until meet regional boundary or y = x, go to step (2);

(5) according to the clockwise direction, found the unlabeled sell, which is near (x, y-1) and in the sublayer -x, labeled it as (x, y), then go to step (4).

The label of the cells has two properties:

(1) for the cell labeled by (x, 0)(1 < x < n - 1), the cells'labels around it are $\{(x - 1, 0), (x, x - 1), (x, 1modx), (x + 1, 0), (x + 1, 1), (x + 1, x)\}$, if x = n - 1, the cells'labels around it are

$$\{(x-1,0), (x,x-1), (x,1modx)\}.$$

(2) for the cell labeled by (x, y)(0 < y < x - 1), the cells' labels around it are $\{(x - 1, y - 1), (x - 1, ymod(x - 1)), (x, y - 1), (x, (y + 1)modx), (x + 1, y), (x + 1, y + 1)\}$.

Theorem 1: The communicate radius R_c is two times of the sensing radius R, every two neighbour nodes can be communication, and the coverage ratio is 100%.

Proof: As shown in Fig. 4, nodes A and B are neighbours, $d(AB) = \sqrt{3}R < 2R$. Since the communicate radius R_c is two times of the sensing radius R, nodes A and B can communicate. In a similar way, nodes A, B and C can communicate with each other.

Since the hexagonal coverage is seamless connection, it is only needed to consider the hexagon whether can be covered by sensing range. From Fig. 4, it is obviously that the sensing circle is the circum-circle of each hexagon. So the coverage ratio is 100%.

Algorithm 2. Recovery Algorithm of Coverage Hole Based on α -hull (RACH)



FIGURE 3. The coverage of the neighbour sensors

01. generate coverage loopholes polygon;

02. calculation of the hole center according to Algorithm 1.

03. the first node put in the center.

04. while the coverage loophole is still exist

05. for each new static node

06. locate the place where $d = \sqrt{3}R$ away from the center node. Place a static node.

07. find the location: 60 degrees clockwise and $d = \sqrt{3}R$ away from the center node. Place a static node.

08. end for

09. end while

10. end

4.3. **Dynamic node.** Dynamic nodes have two functions, collecting data and move it to where sensors want it. Firstly, the communicate radius is two times of the sensing radius, so the cluster head can be chosen for each cluster to collect the data. Then dynamic node can move to the cluster head for data collection. Secondly, each coverage loophole just place one dynamic node. It is needed to move when there are same necessaries, such as some information need to be forward. How to build the mobile routing? The DS-based routing will be constructed. A Dominating Set of a graph G = (V, E) is a set of nodes $V' \subseteq V(G)$ such that for every $(u, v) \in E(G)$, $u \in V'$ or $v \in V'$. We found that the two cells, which are shown in Fig. 4, will be selected as the Cluster Head cell (CH-cell).

After the numerical value analysis, the CH-cells' labels (x, y) are met Formula (1). For example, when $x = 17 = 5 + 3 \times 4$, then $y_0 = (4 + 4)mod7 = 1$, $y_1 = y_0 + 7 = 8 \le 17$ and $y_2 = y_0 + 14 = 15 \le 17$. So that the labels (17, 1), (17, 8)and(17, 15) are the CHcells. Then the Cluster head broadcasts his message, his cluster member will receive this message. If someone can't receive any CH's message, itself will be a cluster head.



FIGURE 4. The cluster head

$$\begin{cases} x = 3 + 3k, y_0 = (k+1) \mod 7, y_l = y_0 + 7l \le x \\ x = 5 + 3k, y_0 = (k+4) \mod 7, y_l = y_0 + 7l \le x \\ x = 7 + 3k, y_0 = k \mod 7, y_l = y_0 + 7l \le x \\ (l = 1, 2, 3, ...). \end{cases}$$
(1)

The DS-based routing, which is called as backbone tree. There are some parent clusters and son clusters in the tree. Set the center (0,0) as the root, and the dynamic node is placed here initialization. The routing searches as follows:

First, when the CH-cells are determined by the Formula (1) at l = 0. For the cell labels for k = i, the sells label for k = i + 1 are his son cluster. For example, for the formula x = 3 + 3k, $y_0 = (k + 1)mod$ 7, when k = 0, there is a cell labeled by (3, 1). When k = 1, the cell's label is (6, 2), and when k = 2 the cell's label is (9, 3). So the sell labeled by (6, 2) is (9, 3)'s father cluster.

Second, if it is unable to find the parent clusters by the first way, the cells (x, y)'s parents, which are labeled by (x_1, y_1) , can be found by the following way:

 $if(y == 0), x_1 = x - 1; y_1 = y + 2;$ else $x_1 = x - 2; y_1 = y - 3;$



FIGURE 5. One example of the repairing of the coverage hole in WSN

Finally, the basic structure of the tree from DS, which is shown in Fig. 5. After the DS has been constructed, DS-based routing will be constructed. For example, when there are some information for the cell (9,5) need the dynamic node, the routing $(0,0) \rightarrow (3,1) \rightarrow (5,4) \rightarrow (8,5) \rightarrow (9,5)$ is DS-based mobile routing.

5. **evaluation.** In the algorithm of α -MCC, it is use the method of α -hull to solving the position of the jammer. So this problem is transformed to find the minimum circumscribed circleso that it can cover all the interference nodes. The center can be estimated to be the location of the jammer. The following theorems are to prove its correctness.

Theorem 2: For a finite set of points P in a plane, there is only one minimum circumscribed circle, r0 is the radius of the minimum circumscribed circle, then all vertices of the α -hull $(1/\alpha = r0)$ will be on the same circle.

Proof: If assume there is a point p in set P which is in the minimum circumscribed circle. We can do with r0 as the radius of the circle and p is in the circle, and this circle inclusive of any points that is in set P, because of r > r0, therefore, there must be a circle

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with r as the radius which contains all the points in the set P, and also p is in the circle. According to the concept of α -hull, so this point must be the α -hull $(1/\alpha = r)$ vertex of the set P;

Anther, assumption that the α -hull $(1/\alpha = r)$ of the set P is existing, so there are at least one point which is the α -hull vertex. According to the concept of α -hull, there is at least one circle C cross the point p with r as the radius which contains all the points in the set P, like that $r \geq ro$. This is the original assumptions of which 0 < r < r0 contradict, so the assumption does not hold. This completes the proof.

Last, for any points p which is the α -hull $(1/\alpha = r0)$ vertex of the set, there are at least one circle C with r0 as the radius which contains all the points in the set P. circle C is the minimum circumscribed circle. So point p is in the circle. Empathy may permit, all of the α -hull $(1/\alpha = r0)$ vertices of the set P are on the minimum circumscribed circle. This completes the proof.

By the above theorem reasoning, for the already determined attack graph, the interference node set with the minimum circumscribed circle radius is r0 in the graph, for $\forall r$ which is positive real number, the number of the α -hull $(1/\alpha = r)$ vertices of the set P, n and r related, and satisfy the laws as follows:

1) For r < r0, n = 0;

2) For r = r0, all vertices of the α -hull $(1/\alpha = r)$ are on the same circle;

3) For r > r0, n gradually decreases with the decrease of the r;

4) For $r = \infty$, n is the number of vertices of the convex hull of the set P.

It can be seen from the above rules, For a finite set of points P in a plane, it can according the number of the α -hull $(1/\alpha = r)$ vertices to judge the relation of the rand the minimum circumscribed circle radius r0. By constantly projected r, when all vertices of the α -hull $(1/\alpha = r)$ are on the same circle, vertices are located points on the minimum circumscribed circle, then calculate its radius. The correctness of the algorithm has demonstrated.



FIGURE 6. simulation network diagram

As shown in Fig. 6, a size of 1000 * 1000 (unit length) rectangular area of wireless sensor networks has been simulated by VC++6.0, the upper left corner coordinate is (0,0), lower right corner coordinates is (1000, 1000). Sensor nodes are randomly and uniformly distributed in this area. Assuming that the jammer is on the center of the simulated regional, and its coordinate is (500, 500), the length of the 375 units for the radio interference radius. Fig. 7 gives the error value of the position of the center. It obvious that the error is very small. The error value is almost lower than 0.04.



FIGURE 7. The error value



FIGURE 8. The number of added nodes by RACH and PATT algorithms

Now, we set the sensing radius as 2.5 unit and the communicate radius as 5 unit. N sensors are needed for the repair of the coverage loophole. The coverage ratio is set as $cov = S_{cov}/S_{hole}$, where S_{cov} representative the area have been covered by the added sensor, S_{hole} is the area of the coverage loophole. Our RACH algorithm is compared with the PATT algorithm, which is shown in Fig. 8.

From Fig. 8, it is obvious that as the number of the added node increases, the coverage ratio increases linearly. That is to say each node can decrease the area of the coverage loophole. For the RACH algorithm, when 4278 nodes are added into the hole, the repair is completed. The coverage ratio is almost 100%. However, 6000 new nodes are need for the PATT algorithm. RACH is greater than PATT.



FIGURE 9. The number of cluster head

As shown in Fig. 9, the number of cluster-head is small, so the mobile nodes only need to move those positions.

6. Conclusion. In order to repairing of the coverage hole. Localize the center of the coverage loopholes and restorative method are two important steps. this paper bring the concept of α -hull, which is in the computation geometry applied to α -MCC (Algorithm for Minimum Circumscribed Circle Detection Based on α -hull). According to α -hull concept and their properties, it is proved that all vertices of the α -hull will be on the same circle, if $1/\alpha$ is equal to the radius of points' MCC. Later, Recovery Algorithm of Coverage Hole Based on α -hull (RACH) is proposed for restorative. We provide the analysis of correctness of RACH through theoretical proof, and the comparison between our algorithm and others through practical simulation. Experimental results show this algorithm need less new nodes for restorative.

Acknowledgment. Acknowledgment. The authors wish to thank National Natural Science Foundation of China (Grant NO: 61072080, 61572010), Fujian Normal University Innovative Research Team (No. IRTL1207), the support by the key project in Fujian Provincial Education Bureau (JA15323, JAT160328, JA14217, JA15329), the scienti

c research project in Fujian University of Technology (GY-Z160066), Natural Science Foundation of Fujian Province of China (2014J01218) and Fujian provincial key project of science and technology (2014H0008, 2015H0009, 2014-G-83).

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