

# Outdoor 3D Illumination in Real Time Environments: A Novel Approach

Hoshang Kolivand

Department of Computer Science, Liverpool John Moores University, Liverpool, UK, L3 3AF

Mohd Shahrizal Sunar

MaGIC-X (Media and Games Innovation Centre of Excellence)  
Universiti Teknologi Malaysia 81310 Skudai Johor MALAYSIA

Amjad Rehman

College of Computer and Information Systems Al-Yamamah University Riyadh 11512 KSA

Abdulaziz S. Almazayad

College of Computer and Information Systems Al-Yamamah University Riyadh 11512 KSA  
College of Computer and Information Sciences King Saud University Riyadh 11451 KSA

Tanzila Saba

College of Computer and Information Sciences Prince Sultan University Riyadh 11586 KSA

Received February, 2016; revised September, 2016

---

**ABSTRACT.** *Comprehensive enlightenment is one of the fundamental components that virtualize the real environment. Accordingly, sky shading is one of the important components considered in the virtualization process. This research introduces the Dobashi method of sky luminance; additionally, Radiosity Caster Culling is applied to the virtual objects as the second thought for outside illumination. Pre-Computed Radiance Transfer is connected to ascertain the division of patches. Moreover, for real sky shading, the Perez model is utilized. By pre-ascertaining sky shading vitality and outside light, the vitality of the entire open air is figured ahead of time. The open air vitality is shared on virtual articles to make the situations more practical. Commercial videos and cartoon creators could utilize the strategy to produce real outside situations.*

**Keywords:** Illumination, Radiosity, Outdoor rendering, Virtual reality.

---

1. **Introduction.** In outdoor rendering, natural phenomena such as sky illumination, shadows, and clouds are very attractive topics for researchers. At the top of everything in sky illumination, the modelling of atmospheric particles is a fascinating area to make outdoor rendering as realistic as possible.

Natural illumination is the key part of outdoor rendering, that is lighted by the sun and sky colour that creates shadows correctly. An extremely important part of natural illumination can be revealed with respect to shadows. The fact that shadows move around the occluders and are soft enough appreciated factor to make the outdoor scene more realistic. Indeed, several tools are available to illuminate scenes such as Lighting etc but are patch-oriented and so are not suitable for real-time rendering [1].

**2. Research Background.** The sun and sky play an important role in natural illumination. Light from the sky is the significant source of outdoor illumination for realistic scenes [1]. In 1986, Nishita proposed a new technique to calculate the sky light luminance [2]. They created photorealistic images of outdoor scenes by this technique. In the method, the part of the sky that can be seen by the observer affects the image. They proposed some methods such as creating many sample points in the sky and setting the proportional weighting factor to the sky luminance of each point. The sky dome was divided into many finite components such as rectangles or triangles. The sky dome was divided into many bands. However, they selected a third method to reduce the cost of calculations. An extension of their method is proposed by [3]. They developed it by adding specular reflection of sky light. Obviously, the calculation of specular reflection of sky light is expensive, and it is necessary to reduce the calculations. As a result, in 1993, Tadamura proposed a quick method using graphic hardware [4]. This method can generate the image rapidly, but it requires pre-calculation of the image every time. Whenever there are multi-lights, the luminance of all light sources can be calculated as the sum of all of the individual luminances [5].

Nimeroff [6] proposed a quick method to calculate the luminance of the sky when the position of the sun changed. In the method, many parameters of the sun's position are extracted analytically with respect to the sky intensity distribution. It takes a lot of time to calculate the basis luminance. In the method proposed by [1], the integrated distribution was expanded to many other basic functions to exactly intensity distribution of the sky, and as a result the number of distributions was reduced. One of the advantages of the method is that the sky light luminance is calculated when the position of the sun changes. In 1987, Cabral et al. proposed a similar method [7] with the ability to handle the occlusion effect, but the difference is that basis luminance.

The main issue with regard to realistic virtual objects in virtual environments and augmented reality systems is maintaining the balance between realism and FPS. Increasing the realism is not a big issue if the speed of rendering can be ignored. In 2010, Noh and Sunar tried to generate realistic soft shadows in AR using the addition of soft projection shadows [18, 24]. In 2013, Madsen and Lal considered the sky illumination and volume shadows in AR to enhance the realism but no care about FPS [19,25]. Many other works have been employed to overcome the issue of realism in AR, which is discussed in the section 4. In 2012, Lensing and Broll considered indirect illumination in an AR system for dynamic objects. Their approach is an image-space global illumination based on reflective shadow maps to reveal diffuse indirect illumination [20,26,26]. In this work, neither real direct illumination onto virtual objects nor shadows are taken into account.

### 3. Methods and Materials.

**3.1. Lighting.** The process of light rendering in a virtual environment approximates physical based lighting. In outdoor rendering, the colour of each pixel must be calculated with respect to the position of sun and the sky colour during the rendering. "In this collision (light with surface), many complicated dynamic processes occur that determine the pixel's color. To focus more requires knowing about photons, which is electromagnetic wave modeling under the physics of light. In this case the brightness of light and the color of the light can be controlled by the number of photons and energy contained in photons respectively" [1].

**3.2. Light Scattering in Outdoor Scenes.** In real-time outdoor rendering, sky colour, terrain, and shadows are the most visually dominant parameters. Atmospheric scattering in outdoor scenes is one of the most important effects in real-time outdoor rendering. The

colour of the sun changes from light red at sunrise to yellow at noon and then back to light red at sunset due to atmospheric scattering. During the day, the colour of the sky changes not only due to the time of day and light scattering but also because of pollution, weather, and other factors [28,29,30].

There are two kinds of illumination: direct and indirect. "Direct illumination arrives at the object directly from light sources, while indirect illumination reaches the point by reflecting light from other objects. The main direct light sources are the sun and sky, but indirect illumination can arrive from  $n - 1$  objects, where the number of objects in the scene is  $n$ .

In the case of direct light sources, although the sun is far away, it cannot be considered as a point light source (about 0.5 degree in angular diameter). This is the main reason why outlines of shadows in outdoor rendering should be soft. However, the sky light is the result of solar illumination; it should be considered as an independent light source in the outdoor rendering. Indirect illumination is a more subtle effect for making scenes realistic" [1]. Absorption is a concept of lighting that should not be forgotten. Some parts of light are absorbed by particles in the air. This means that increasing the amount of pollution will lead to an increase in the amount of absorption.

Out-scattering and in-scattering are other concepts related to the incident irradiance that scatters in somewhere else. Scattering is slightly more complicated than absorption because when a ray is absorbed by a particle, we can forget about it, but in scattering we must consider new directions. When light is scattered by an object that is smaller than the wavelength of light, it is called Rayleigh scattering. Rayleigh is the main parameter for determining sky colour. High frequencies scatter more while low frequencies scatter by fewer degrees. During the day, the sunlight is scattered by the atmosphere in the blue range, while during twilight, it is scattered in the orange range because of long traveling. In these cases, waves of long wavelength are received by the human eyes [31,32].

For outdoor scattering such as atmospheric scattering, the number of particles must be infinite and the atmosphere is supposed to be a semi-transparent object.

**3.3. Pre-Computed Radiance Transfer.** In 2000, Sloan et al. proposed a wonder technique to pre-compute (PRT) some parts of the huge real-time calculation to speed up the rendering [8]. The method is convenient for various effects such as illumination in different directions and shadows. Lehtinen et al. proposed a technique for glossy objects efficiently [9]. Ng [10, 11] proposed a method of pre-calculation using wavelet transform to represent low- and high-frequency shadows under environmental illumination. A drawback of the method is that it is only able to deal with static scenes. Kautz [12] proposed a method for deformable objects, but it can only be used for low-frequency shadows. In 2005, Tamura [13] proposed a PRT technique where an object could be translated or rotated because they considered each object separately. In this technique, two kinds of occlusion have been considered: self-occlusion and other-occlusion. Radiance can be calculated due to self-occlusion and other-occlusion separately. The algorithm has three steps:

---

**Step 1:** Render the whole scene with the radiance taking into account only self-occlusion ( $L_s(p)$ )

**Step 1:** Compute the radiance obscure just for visible areas ( $L_o(p)$ )

**Step 1:** Compute the subtraction of both two renders ( $L(p)=L_s(p)-L_o(p)$ ).

---

To reduce the calculation of the occlusion ratio that reveals the parts of the distant area light sources that are invisible from  $p$ , they proposed an occlusion map for each object separately with respect to each light source.

**4. Daylight.** “Daylight is a combination of all direct and indirect light from the sun and diffusion from other objects, especially the earth. In other words, daylight includes direct sunlight, diffuse sky radiation, and both reflected from the earth and terrestrial objects. The intensity of skylight or sky luminance is not uniform and depends on the clarity of the sky” [23,33,34]. “Multiplying the luminance from a light source by a factor  $w$  is equivalent to multiplying the intensity of the light source by  $w$ . In general, the luminance of sky light can be calculated by the following formula:

$$I_{Skylight} = \int_{\Omega} L(S)\kappa(s)ds \quad (1)$$

where

$\Omega$  is the hemispherical integral domain above the surface.

$S$  is a unit vector toward an arbitrary direction in the hemisphere above the surface.

$L(s)$  is an intensity distribution of the sky.

$k(s)$  is a factor determined by the reflectance function of the calculation point” [1].

**5. Sky Light Luminance calculations.** As we mentioned before, the sky dome is a hemisphere with a large radius. Deem that in a local coordinate system with the z-axis as normal and with the assumption of the sky dome generation section, the luminance of the sky can be calculated as [2]:

$$F_{\lambda}(\alpha, \delta; n, v, L_0^{(\lambda)}) = \int_0^{\alpha} \int_0^{\delta} L_{\lambda}(\alpha^*, \delta^*; n, L^{(\lambda)})\rho_{\lambda}(\alpha, \delta; v)\sin^2\alpha^*\sin^2\delta^*d\alpha^*d\delta^* \quad (2)$$

In 1996, Dobashi et al. [1] improved the formula and proposed the following formula to reduce the number Fourier cosine series:

$$F_{\lambda}(\alpha, \delta; n, v, L_0^{(\lambda)}) \approx \sum_{i=1}^N \sum_{j=1}^N w_{ij}^{(\lambda)}(n, v, L_0^{(\lambda)})\cos(i\alpha)\cos(j\delta) \quad (3)$$

where

$N$  is the degree of cosine functions

$w_{ij}^{(\lambda)}$  stands for weight function for each cosine function and  $w_{ij}^{(\lambda)}$  can be calculated by:

$$w_{ij}^{(\lambda)}(n, v, L_0^{(\lambda)}) = 2/\pi \int_0^{\pi} \int_0^{\pi} F_{\lambda}(\alpha, \delta; n, v, L_0^{(\lambda)})\cos(i\alpha)\cos(j\delta)d\alpha d\delta \quad (4)$$

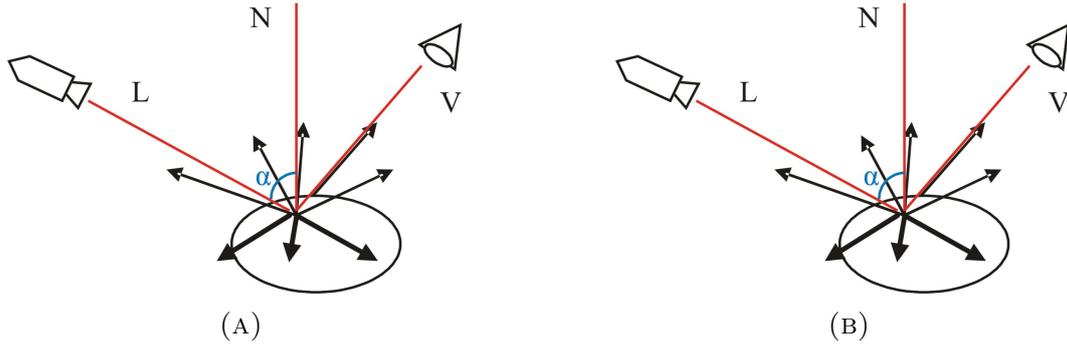
Now the sky luminance can be calculated as:

$$I_{Skylight}^{(\lambda)}(p, n, v, L_0^{(\lambda)}) \approx \sum_{i=1}^N \sum_{j=1}^N w_{ij}^{(\lambda)}(n, v, L_0^{(\lambda)})\Theta_{ij}(p) \quad (5)$$

where

$$\Theta_{ij}(p) = \sum_{i=1}^{N_{band}} \sum_{m=1}^{N_{vis}^{(t)}} (\cos(ia_m^{(t)})\cos(j\delta_l^+) - \cos(ia_{m+1}^{(t)})\cos(j\delta_l^+) - \cos(ia_{m+1}^{(t)})\cos(j\delta_l^-) + \cos(ia_m^{(t)})\cos(j\delta_l^-)) \quad (6)$$



FIGURE 2. Lighting theory;  $V$  is direction of observer

$$I_s = k_s I (V * R)^n \quad (9)$$

where

$K_s$  : coefficient of specular reflection

$I$  : intensity of the light source

$V$  : vector direction of the viewer

$R$  : vector direction of the reflected ray

$n$  : specularity

Specularity is amount of sharpness and can be measured between 0 and 100 or more.

Finally the total local intensity is:

$$I = I_a + I_d + I_s \quad (10)$$

$$I = K_a I_a C + K_d I_d C (L * N) + k_s I_s (V * R)^n \quad (11)$$

**7. Sky Color.** “In contrast to indoor rendering, outdoor rendering consists of more components such as the sun’s position, sky colour, shadows, rainbows, haze, trees and grass. This chapter begins by attempting a working definition of some more important parameters for outdoor rendering. The sun’s position, sky colour, shadow, and interaction between sky colour and objects are the highlighted components in outdoor environments. The purpose of this chapter is to provide an overview of the position of sun, sky colour, shadows, and interaction between the sky colour and virtual objects in augmented reality environments” [23,35].

**8. Radiosity.** “Radiosity is a finite-element approach to the problem of global illumination [15,36]. In order to deal with large-area elements such as outdoor augmented reality environments, it is necessary to integrate over the area of the element explicitly” [16, 17]. Radiosity is another technique for soft shadow generation that was proposed by Goral et al. [14]. This algorithm calculates diffuse inter-reflections between surfaces by determining the energy balance in a closed environment. Radiosity is expensive and can only be used for polygonal surfaces. Each surface is divided into elemental surface patches. The intensity of the surface distribution then depends on the quality of surfaces and is approximated as a constant value. The hemicube approach and the Nishita and Nakamae approach [17] are two different methods that employ radiosity for soft shadow generation. The difference between them is related to the position of the radiosity value of each patch. In the hemicube approach, radiosity values are calculated at the centre of each patch, but in the Nishita and Nakamae approach, calculation at the vertices of the patches is used.

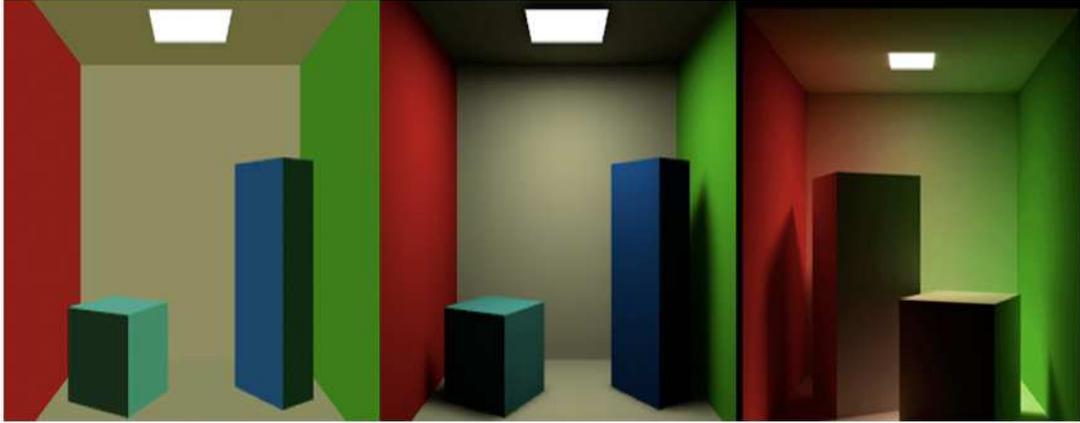


FIGURE 3. Left: simple scene. Center: direct illumination. Right: Radiosity

Radiosity is a special approach for global illumination [15]. It splits scenes into several fragments and calculates the energy of each part that can be transferred between the other parts or other objects. The fraction that determines how much energy could be transferred between elements is called the form factor.

This formula, as well as radiosity in general, is accurate enough if the fragment parts are small enough. On the contrary, small elements will have a low rendering speed. The form factor will be as [16].

“Radiosity is another technique to create soft shadows. This kind of algorithm can calculate diffuse inter-reflection between surfaces by determining the energy balance in a closed environment. These kinds of algorithms are rather expensive and can be used just for polygonal surfaces. Each surface is divided into some elemental surface patches. It must be ensured that the patches are small enough. Then, the intensity distribution of the surface depends on the quality of the surface, and it is approximated a constant value” [22].

---

The algorithm that we propose is the following, which we call Radiosity Caster Culling (RCC).

- Step 1.** Capture the environment from the camera point of view put in the z-buffer.
  - Step 2.** Convert the close partition of objects into the small partition.
  - Step 3.** For each path of the sky dome and for each path of the object.
  - Step 4.** If the object patch is visible from the point of view (camera point of view), carry out the following step; else irritate the process for other patches.
  - Step 5.** Calculate the form factor for the patch.
  - Step 6.** Apply the effect of the sky patch to the object patch.
- 

$$F_{i \rightarrow j} = \frac{\cos\theta_i \cos\theta_j}{\pi r^2} V(i, j) \quad (12)$$

To generate photorealistic images, many aspects must be taken into account. The fact that, which object can be influenced by the current lighting is one such aspect that can be solved using global illumination. The results are illustrated in Figure 4. In general, creating a mask of visible radiosity caster patches performs recognition of visible patches. Radiosity casters are the parts of the scene which receive the energy emitted by the sky colour. This mask is required to limit the radiosity casters which are more extensively

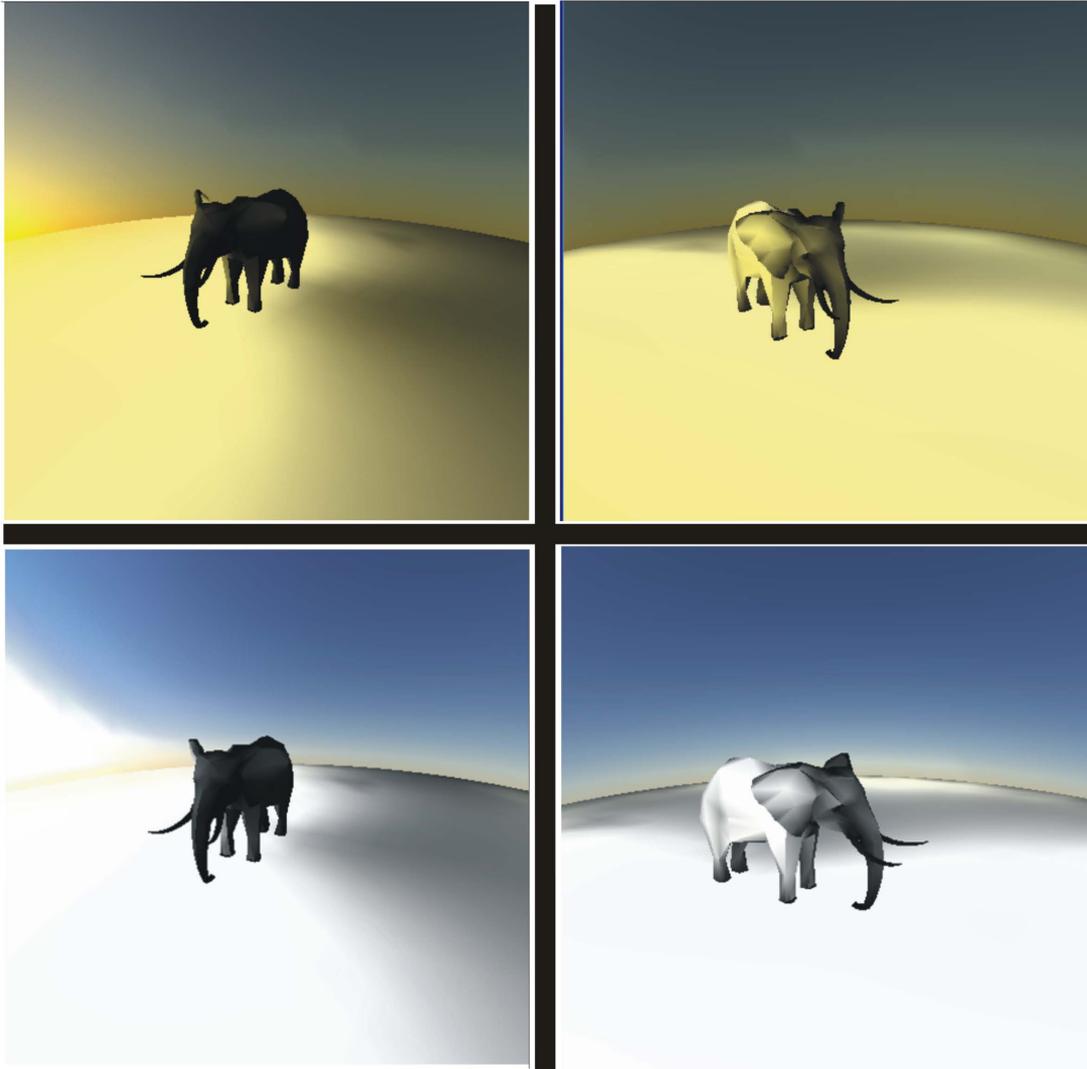


FIGURE 4. Top left and top right: An outdoor environment in the morning. Bottom left and bottom right: The outdoor environment at noon.

consumed on CPU and GPU time. It becomes more efficient as the complexity of the scene or the number of patches increases [17].

**9. Conclusions and future works.** This paper aims to reveal the outdoor illumination onto the virtual objects. To do this, some old but widely used techniques are employed. A new idea called Radiosity Caster Culling is applied to exert the energy of outdoor environments, of which the main part is the sky colour, onto the virtual objects.

Pre-Computed Radiance Transfer (PRT) is applied to calculate the division of patches in advance. PRT allows the radiosity technique to enhance the speed of rendering in real time. The luminance is calculated based on the Dobashi method. The sky colour is generated based on Perez's model [21], which can be used for clear sky. Finally, Radiosity Caster Culling is employed to share the global illumination by considering the sky colour onto the virtual objects.

This work can be used for global illumination in outdoor environments in which the sky colour is highlighted. There are many extensions that can be applied to this method, such as the employment of Augmented Reality, which is a task with which we will deal in the future.

**Acknowledgment.** Authors are thankful to Al-Yamamah University Riyadh 11512 Saudi Arabia for their full financial support to conduct and publish this research. Authors are also thankful to Universiti Teknologi Malaysia, Skudai 81310 Johor Malaysia for their technical support in line of research activities.

## REFERENCES

- [1] H. Kolivand, et al., Real-Time Light Shaft Generation for Indoor Rendering Intelligent Software Methodologies, *Tools and Techniques of the series Communications in Computer and Information Science*, vol. 532, pp 487-495, 2015
- [2] T. Nishita, and E. Nakamaei, Continuous Tone Representation of Three- Dimensional Objects Illuminated by Sky Light *J. Computer Graphics* , vol. 20, no 4, pp.112-118, 1985.
- [3] K. Kaneda et al., Photorealistic Image Synthesis for Outdoor Scenery Under Various Atmospheric Conditions., *The Visual Computer* , vol. 7, no. 6, pp. 247-258, 1991.
- [4] K. Tadamura et al., Modeling of Skylight and Rendering of Outdoor Scenes, *Computer Graphics Forum* , vol. 12, no. 7. pp. 189-200, 1993.
- [5] J. T. Kajiya, The rendering equation, *Computer Graphics* , *Proceedings of ACM SIGGRAPH*, vol. 20, no. 4, pp. 143-150, 1986.
- [6] Nimeroff, J et al., Efficient Re-rendering of Naturally Illuminated Environments, *Proceedings of Eurographics Rendering Workshop*, pp. 359-374, 1994.
- [7] B. Cabral et al., Bidirectional reflection functions from surface bump maps , *Proceedings of ACM SIGGRAPH Computer Graphics*, vol. 21, no. 4, pp. 273-281, 1987.
- [8] P. P. Sloan, et al., Precomputed radiance transfer for real-time rendering in dynamic, low-frequency lighting environments , *Proceedings of SIGGRAPH*, pp. 527-536, 2002.
- [9] Lehtinen, J. and Kautz, J. Matrix radiance transfer , *Proceedings of Symposium on Interactive 3D Graphics* pp. 59-64., 2003.
- [10] R. R. Ramamoorthi, and P. Hanrahan, All frequency shadows using nonlinear wavelet lighting approximation , *ACM Transactions on Graphics*, vol. 22, no. 3, pp. 376-381, 2003.
- [11] R. R. Ramamoorthi, and P. Hanrahan, Triple product wavelet integrals for all-frequency relighting, . *ACM Transactions on Graphics* , vol. 23, no. 3, pp. 477-487, 2004.
- [12] J. Kautz, et al., Hemispherical rasterization for self-shadowing of dynamic objects , *Proceedings of Eurographics Symposium on Rendering*, pp. 179-184, 2004
- [13] N. Tamura, , et al., Deferred Shadowing for Real-Time Rendering of Dynamic Scenes Under Environment Illumination , *Computer Animation and Virtual World*, vol 16, no. 3, pp. 475-486, 2005.
- [14] C. Goral et al., Modeling the interaction of light between diffuse surfaces , *Computer Graphics*, vol. 18, no. 3,pp. 213-222, 1984.
- [15] M. Cohen, and J. R. Wallace, Radiosity and Realistic Image Synthesis , *Morgan Kaufmann.*, pp. 213-222, 1993.
- [16] J. R. Wallace, Et al., A Ray Tracing Algorithm for Progressive Radiosity , *Proceedings of Computer Graphics (Proceedings of SIGGRAPH)*, vol . 23, no.4, pp. 315-324, 1989.
- [17] H. Kolivand, and M. S. Sunar, Covering photo-realistic properties of outdoor components with the effects of sky color in mixed reality , *Multimedia Tools and Applications*, vol. 72, no. 3, pp. 2143-2162, 2014.
- [18] Z. Noh, and M.S. Sunar, Soft Shadow Rendering based on Real Light Source Estimation in Augmented Reality , *Advances in Multimedia - An International Journal (AMIJ)*, vol. 1, no. 2, pp. 26-36, 2010.
- [19] C. B. Madsen, and B. B. Lal, Estimating Outdoor Illumination Conditions Based on Detection of Dynamic Shadows , *Computer Vision, Imaging and Computer Graphics, Theory and Applications*, pp.33-52, 2013.
- [20] P. Lensing, and W. Broll, Instant indirect illumination for dynamic mixed reality scenes, *Mixed and Augmented Reality (ISMAR)* , *IEEE International Symposium*, pp. 109-118, 2012.
- [21] H. Kolivand, , and M. S. Sunar, Real-Time Sky Color with Effect of Sun Position, *International Journal of Scientific and Engineering Research*, vol. 2, no. 7, pp. 2229-5518, 2011.
- [22] H. Kolivand, and M.S. Sunar, An Overview on Base Real-Time Shadow Techniques in Virtual Environments *TELKOMNIKA*, vol. 10, no.1, , pp. 171-178, 2010
- [23] H. Kolivand and M. S. Sunar Realistic Real-Time Outdoor Rendering in Augmented Reality, *PLoS ONE*, vol. 9, no. 9, doi:10.1371/journal.pone.0108334, 2014

- [24] A. Rehman, , and T. Saba, An intelligent model for visual scene analysis and compression, *International Arab Journal of Information Technology*, vol. 10, no. 13, pp. 126-136, 2013.
- [25] AM. Ahmad,G. Sulong, A. Rehman,MH. Alkawaz, T. Saba, Data Hiding Based on Improved Exploiting Modification Direction Method and Huffman Coding, *Journal of Intelligent Systems*, vol. 23, no. 4, pp. 451-459, doi. 10.1515/jisys-2014-0007,2014
- [26] A. Nodehi, G. Sulong, M. Al-Rodhaan, A. Al-Dhelaan, Rehman, A. Saba, T. Intelligent fuzzy approach for fast fractal image compression, EURASIP, *Journal on Advances in Signal Processing*, doi. 10.1186/1687-6180-2014-112, 2014
- [27] M. S. M. Rahim, S. Isa, A. Rehman, T. Saba, Evaluation of adaptive subdivision method on mobile device, *3D Research*, vol. 4, no. 4, pp. 1-10, doi:10.1007/3DRes.02(2013)4, 2013
- [28] M. Mundher, D. Muhamad, A. Rehman, T. Saba, F. Kausar, Digital watermarking for images security using discrete slantlet transform, *Applied Mathematics and Information Sciences*, vol. 8, no. 6, pp: 2823-2830, doi.10.12785/amis/080618, 2014.
- [29] Z.F. Muhsin, A. Rehman, A. Altameem, ; A. Saba, M. Uddin, Improved quadtree image segmentation approach to region information, *the imaging science journal*, vol. 62, no. 1, pp. 56-62, doi. <http://dx.doi.org/10.1179/1743131X13Y.0000000063>, 2014.
- [30] H. Haron, A. Rehman, Adi, DIS, S.P. Lim, and T. Saba, Parameterization method on B-Spline curve, *J. mathematical problems in engineering*, vol. 2012, doi:10.1155/2012/640472, 2012.
- [31] Rad, A.E., Rahim, M.S.M, Rehman, A. Saba, T. Digital dental X-ray database for caries screening, *3D Research*, vol. 7, no. 2, pp. 1-5, doi. 10.1007/s13319-016-0096-5, 2016.
- [32] A. H.Basori, M. H. Alkawaz, T. Saba, A. Rehman, An overview of interactive wet cloth simulation in virtual reality and serious games, *Computer Methods in Biomechanics and Biomedical Engineering: Imaging and Visualization*, doi. 10.1080/21681163.2016.1178600, 2016.
- [33] S. Jadooki, D.Mohamad, T. Saba, A. S. Almazyad, A. Rehman, Fused features mining for depth-based hand gesture recognition to classify blind human communication, *Neural Computing and Applications*, pp. 1-10, doi. 10.1007/s00521-016-2244-5, 2016.
- [34] Fadhil, MS. Alkawaz, MH., Rehman, A., Saba, T. Writers identification based on multiple windows features mining, *3D Research*, vol. 7 (1), pp. 1-6, doi.10.1007/s13319-016-0087-6, 2016.
- [35] S. R. Waheed, M. H. Alkawaz, A. Rehman, A. S. Almazyad,T. Saba, Multifocus watermarking approach based on discrete cosine transform, *Microscopy Research and Technique*, vol. 79, no. 5, pp. 431-437, , doi. 10.1002/jemt.22646, 2016.
- [36] N. A.Husain, M. Rahim, A. R. Khan, M. Al-Rodhaan, Al-Dhelaan, A., Saba, T. Iterative adaptive subdivision surface approach to reduce memory consumption in rendering process (IteAS), *Journal of Intelligent & Fuzzy Systems*, vol. 28, no. 1, pp. 337-344, doi. 10.3233/IFS-14130, 2015.