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Research on Coordinated Multi-point Precoding Algorithm Based on QR-SVD

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Abstract. *ABSTRACT. In LTE-A system, the existing coordinated multi-point (CoMP) multiuser precoding algorithms can not suppress multi-user, intra-user and noise interference at the same time, and its computational complexity is high. To overcome the conventional precoding algorithms limitations this paper proposes a novel linear precoding algorithm based on QR-SVD suitable for CoMP transmission system. Firstly, a noise factor is introduced into the complementary channel matrix to exclude the impacts of multi-user and noise interference on the system. Besides, the introduction of low-complexity matrix decomposition method (QR) reduces computational complexity. Finally, by applying singular value decomposition (SVD) to equivalent channel matrix, intra-user interference is supressed, meanwhile, complete precoding matrix and decoding matrix is obtained. Simulation results show that the proposed algorithm has certain advantages in terms of computational complexity, system capacity and bit error rate (BER), compared with other precoding algorithms.*

Keywords: Coordinated multi-point, Precoding algorithm, Matrix decomposition, Computational complexity, Singular value decomposition

1. **Introduction.** As a key technology in Long Term Evolution-Advanced (LTE-A) system, coordinated multi-point (CoMP) communication technology can suppress inter-cell interference, increase system throughput, and improve cell-edge users service quality through mutual cooperation among several base stations (BSs)[1-2]. Coordinated multipoint precoding algorithm is a typical signal preprocessing technology at the transmitter, when the transmitter obtains complete channel state information (CSI). Through precoding operation, the interference, caused by transmitting signal going through wireless channel, can be eliminated in advance, so as to reach the purposes of ensuring communication reliability and improving system performance [3]. As the key technology of LTE-A physical layer, precoding technology can in advance suppress interference between users or data streams at the transmitter, but there are a variety of defects in typically precoding algorithms. In literature [4], zero-forcing (ZF) precoding algorithm is simple and intuitive, but it does not directly consider the influence of noise, which causes the noise power to be amplified. In addition, although ZF can handle the received signal from its own antennas and differentiate data streams from multiple transmitter antennas, its not ideal to eliminate interference from other users. Hence multi-user interference still exists. In literature [5-6], the main step of block diagonalization (BD) precoding algorithm is to

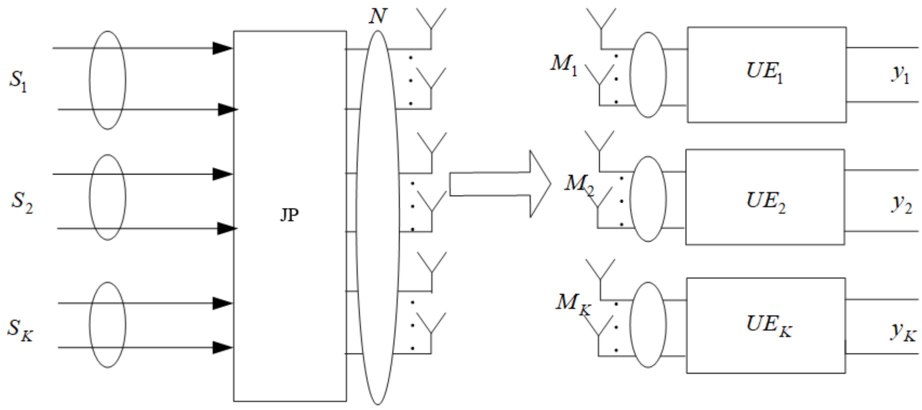

FIGURE 1. . MU-CoMP communication system in JP mode

execute singular value decomposition (SVD) twice for each user. For multi-user CoMP communication systems, its computational complexity is very high. In literature [7], regularized block diagonalization (RBD) precoding algorithm introduces regularization factor, and takes the noise into account to reduce the influence of noise on the system. However, like BD, RBD still has high computational complexity. Moreover, both of BD and RBD precoding algorithms have not yet eliminated intra-user multi-antenna interference. Combined with the above analysis, we know that ZF precoding algorithm can not effectively suppress multi-user and noise interference, BD and RBD precoding algorithms have extremely high computational complexity. In order to overcome these defects and improve system performance, a novel linear precoding algorithm based on QR-SVD is proposed, which extends the channel matrix to suppress multi-user and noise interference, introduces QR matrix decomposition scheme to reduce computational complexity, and executes SVD to suppress intra-user interference and obtain decoding matrix. In theory, the proposed algorithm can reduce computational complexity, improve system capacity, reduce bit error rate (BER), and effectively improve system performance. The rest of this paper is organized as follows. The system model is given in Section 2. The proposed QR-SVD precoding algorithm is described in detail in Section 3. Section 4 presents computational complexity analysis and simulation. Algorithm performance analysis and simulation is displayed in Section 5. Finally, in Section 6, we summarize our main works.

2. **System Model.** Figure 1 shows multi-user coordinated multi-point (MU-CoMP) communication system in joint processing (JP) mode [8]. First, the transmitted data is precoded. Then the precoded data is transmitted to the user equipment (UE) via BSs. We consider a MU-CoMP system, where the BS is equipped with N transmit antennas and thek_{th}UE,UE_k, is equipped with receiving antennas. It is assumed that there are K users in the MU-CoMP system.$\mathbf{W} = [\mathbf{W}_1, \mathbf{W}_2, ..., \mathbf{W}_K]$is the transmit precoding matrix, where$\mathbf{W}_k \in^{N \times l_k}$.$l_k$data streams are transmitted toUE_k. The received signal at$UE_k$is given by:

$$y_k = \mathbf{H}_k \mathbf{W}_k s_k + \mathbf{H}_k \sum_{i \neq k} \mathbf{W}_i s_i + n_k \tag{1}$$

where$y_k$denotes the received signal at UE_k.$\mathbf{H}_k \in^{M_k \times N}$ is the channel matrix from the BS to UE_k.$s_k \in^{l_k \times 1}$ $(k = 1, 2, ..., K)$is the transmitted signal toUE_k.$n_k$is the corresponding additive Gaussian noise with zero mean and$\sigma_k{}^2$variance.

3. QR-SVD precoding algorithm. Assuming that the QR-SVD precoding matrix ofUE_kcan be expressed as$\mathbf{W}_{QR-SVD-k} = \mathbf{W}_k^1\mathbf{W}_k^2$ and $l_k = M_k$. We define complement channel matrix$\bar{\mathbf{H}}_k$ as the channel matrix of all users except user k. The complement channel matrix can be expressed as:

$$\bar{\mathbf{H}}_k = \left[\mathbf{H}_1^T, ..., \mathbf{H}_{k-1}^T, \mathbf{H}_{k+1}^T, ..., \mathbf{H}_K^T\right]^T \in^{\bar{M}_k \times N} \tag{2}$$

where $\bar{M}_k = \sum\limits_{i=1, i\neq k}^{K} M_i$. QR-SVD precoding algorithm consists of the following steps. Step 1: Firstly, we extend complement channel matrix to obtain extended channel matrix, and its expression can be expressed as

$$\tilde{\bar{\mathbf{H}}}_k = \left[\sigma_k \mathbf{I}, \bar{\mathbf{H}}_k\right] \tag{3}$$

where is the $\bar{M}_k \times \bar{M}_k$ unit matrix. Thus, the dimension of $\tilde{\bar{\mathbf{H}}}_k$is $\bar{M}_k \times \left(\bar{M}_k + N\right)$. The number of columns for $\tilde{\bar{\mathbf{H}}}_k$ is greater than the number of rows, which meets the dimension requirement of obtaining its null-space. Therefore this scheme is suitable for MU-CoMP system with users having arbitrary receiving antennas. As can be seen from the expression (3),$\tilde{\bar{\mathbf{H}}}_k$ takes into account both the noise factor and the influence factor$\bar{\mathbf{H}}_k$, which causes multi-user interference. Step 2: Then, the Hermitian transpose of extended channel matrix is denoted by$\tilde{\bar{\mathbf{H}}}_k{}^H$. QR decomposition of $\tilde{\bar{\mathbf{H}}}_k{}^H$ can be expressed as

$$\tilde{\bar{\mathbf{H}}}_k{}^H = \mathbf{Q}_k \mathbf{R}_k \tag{4}$$

where $\mathbf{Q}_k \in^{\left(\bar{M}_k+N\right)\times\left(\bar{M}_k+N\right)}$ is an orthogonal matrix and $\mathbf{R}_k \in^{\left(\bar{M}_k+N\right)\times\bar{M}_k}$ is an upper triangular matrix.

After transforming expression (4), we can obtain expression (5), as shown below.

$$\mathbf{Q}_k{}^H \tilde{\bar{\mathbf{H}}}_k{}^H = \begin{bmatrix} \mathbf{Q}_{k,1}, \mathbf{Q}_{k,2} \\ \mathbf{Q}_{k,3}, \mathbf{Q}_{k,4} \end{bmatrix} \begin{bmatrix} \sigma_k \mathbf{I} \\ \bar{\mathbf{H}}_k^H \end{bmatrix} = \mathbf{R}_k = \begin{bmatrix} \mathbf{R}_{k,1} \\ \mathbf{R}_{k,2} \end{bmatrix} \tag{5}$$

where$\mathbf{Q}_{k,1} \in^{\bar{M}_k \times \bar{M}_k}$, $\mathbf{Q}_{k,2} \in^{\bar{M}_k \times N}$, $\mathbf{Q}_{k,3} \in^{N \times \bar{M}_k}$,$\mathbf{Q}_{k,4} \in^{N \times N}$, $\mathbf{R}_{k,1} \in^{\bar{M}_k \times \bar{M}_k}$, $\mathbf{R}_{k,2} \in^{N \times \bar{M}_k}$. The following is the proof of$\mathbf{W}_k^1 = \mathbf{Q}_{k,4}$. From equation (5), we can deduce $\sigma_k \mathbf{Q}_{k,3} + \mathbf{Q}_{k,4}\bar{\mathbf{H}}_k^H = 0$, that is

$$\sigma_k \mathbf{Q}_{k,3}{}^H + \bar{\mathbf{H}}_k \mathbf{Q}_{k,4}{}^H = 0 \tag{6}$$

For unitary matrix, there is

$$\mathbf{Q}_{k,3}\mathbf{Q}_{k,3}{}^H + \mathbf{Q}_{k,4}\mathbf{Q}_{k,4}{}^H = \mathbf{I} \tag{7}$$

$$\begin{aligned} \mathbf{Q}_{k,4}\bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k \mathbf{Q}_{k,4}{}^H &= \left(\bar{\mathbf{H}}_k \mathbf{Q}_{k,4}{}^H\right)^H \left(\bar{\mathbf{H}}_k \mathbf{Q}_{k,4}{}^H\right) \\ &= \left(-\sigma_k \mathbf{Q}_{k,3}\right)\left(-\sigma_k \mathbf{Q}_{k,3}{}^H\right) = \sigma_k{}^2 \mathbf{Q}_{k,3}\mathbf{Q}_{k,3}{}^H \end{aligned} \tag{8}$$

From equations (7) and (8), we can get

$$\mathbf{Q}_{k,4}\left(\bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k + \sigma_k{}^2 \mathbf{I}\right)\mathbf{Q}_{k,4}{}^H = \sigma_k{}^2 \mathbf{I} \tag{9}$$

According to minimum mean square error (MMSE) optimization criterion in the literature [9], we can get the following formula.

$$\mathbf{W}_1 = \min E\left\{\sum_{k=1}^{K} \left\|\bar{\mathbf{H}}_k \mathbf{W}_{1k}\right\|^2 + \frac{\|n\|^2}{\beta^2}\right\} \tag{10}$$

Referring to the appendix of literature [9], we can see that$\mathbf{Q}_{k,4}$ is the optimal solution to formula (10), that is,$\mathbf{W}_k^1 = \mathbf{Q}_{k,4}$. This completes the proof. Step 3: After Step 2, we

TABLE 1. In (6, 2, 3, 2) case, RBD precoding algorithm computational complexity

Steps	*Operations*	*flops*	*(6,2,3,2)case*
1	$\overline{\mathbf{H}}_k = \overline{\mathbf{U}}_k \overline{\mathbf{\Sigma}}_k \overline{\mathbf{V}}_k^H$	$32K\left(N\overline{M}^2 + 2\overline{M}^3\right)$	*6912*
2	$\mathbf{W}_k^1 = \overline{\mathbf{V}}_k\left(\overline{\mathbf{\Sigma}}_k^T \overline{\mathbf{\Sigma}}_k + \alpha \mathbf{I}_N\right)^{-\frac{1}{2}}$	$K\left(8N^3 + 18N + \overline{M}\right)$	*3678*
3	$\mathbf{H}_k^{equ} = \mathbf{H}_k \mathbf{W}_k^1$	$8KMN^2$	*1728*
4	$\mathbf{H}_k^{equ} = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^H$	$8K\left(4N^2M + 8NM^2 + 9M^3\right)$	*17712*
			Total 30030

can define equivalent channel matrix for each user as $\mathbf{H}_k{}^{equ} = \mathbf{H}_k\mathbf{Q}_{k,4}$. Therefore, SVD decomposition of $\mathbf{H}_k{}^{equ}$can be expressed as

$$\mathbf{H}_k{}^{equ} = \mathbf{U}_k\mathbf{\Sigma}_k\mathbf{V}_k{}^H = \mathbf{U}_k\mathbf{\Sigma}_k[\mathbf{V}_{k,1}, \mathbf{V}_{k,2}]^H \tag{11}$$

Where $\mathbf{U}_k$is a$M_k \times M_k$unitary matrix and$\mathbf{\Sigma}_k$ is a$M_k \times N$ diagonal matrix. The dimension of $\mathbf{V}_k$ is $N \times N$.$\mathbf{V}_{k,1}\in^{N\times M_k}$ is composed of the first M_k columns of $\mathbf{V}_k$, and is the standard orthogonal basis of $\mathbf{H}_k{}^{equ}$ row space, so we have$\mathbf{W}_k^2 = \mathbf{V}_{k,1}$.

Step 4: Finally, the QR-SVD precoding matrix of can be expressed as

$$\mathbf{W}_{QR-SVD-k} = \mathbf{W}_k^1\mathbf{W}_k^2 = \mathbf{Q}_{k,4}\mathbf{V}_{k,1}, 1 \le k \le K \tag{12}$$

The decoding matrix for each user at the receiver is . After precoding and channel transmission, the received signal at becomes:

$$\begin{aligned} y_k{}' &= \mathbf{U}_k{}^H y_k \\ &= \mathbf{U}_k{}^H \left(\mathbf{H}_k\mathbf{W}_k s_k + \mathbf{H}_k \sum_{i\neq k}\mathbf{W}_i s_i + n_k\right) \\ &= \mathbf{\Sigma}_k s_k + \mathbf{U}_k{}^H\mathbf{H}_k \sum_{i\neq k}\mathbf{W}_i s_i + \mathbf{U}_k{}^H n_k \end{aligned} \tag{13}$$

The above equation shows that$\mathbf{U}_k{}^H\mathbf{H}_k\mathbf{W}_k = \mathbf{\Sigma}_k$, where $\mathbf{\Sigma}_k$ is a diagonal matrix. And it means that each data stream is respectively transmitted on a separate sub-channel, which can eliminate the intra-user interference. Besides, the extended channel matrix takes into account both multi-user and noise interference. So QR-SVD precoding scheme can effectively suppress intra-user, inter-user and noise interference at the same time. Thus, QR-SVD precoding scheme can reduce system BER.

4. Computational complexity analysis and simulation.

4.1. Computational complexity analysis.

This part analyzes and compares the complexity of QR-SVD precoding scheme with the existing RBD precoding scheme. The total number of floating-point operations (flops) is used to measure the precoding algorithm computational complexity. Literature [10-11] gave the number of flops required by the real matrix QR decomposition and SVD decomposition. The number of flops needed for SVD decomposition of$m \times n$complex matrix is equivalent to that of $2m \times 2n$real matrix. Here are the numbers of flops needed to perform various operations on complex matrix. For a $m \times n\,(m \ge n)$ complex matrix, its QR decomposition complexity is$16\,(m^2n - mn^2 + n^3/3)$flops. The complexity of complex matrix multiplied by complex matrix is $8mnp$. For a $m \times n\,(m \le n)$ complex matrix where only $\mathbf{\Sigma}$ and $\mathbf{V}$ are obtained, its SVD complexity is $32\,(nm^2 + 2m^3)$ flops. For a$m \times n\,(m \le n)$ complex matrix

TABLE 2. In (6, 2, 3, 2) case, QR-SVD precoding algorithm computational complexity

Steps	*Operations*	*flops*	*(6,2,3,2)case*
1	$\tilde{\tilde{\mathbf{H}}}_k^H = \mathbf{Q}_k \mathbf{R}_k$	$16K\left(N^2\bar{M} + N\bar{M}^2 + \frac{1}{3}\bar{M}^3\right)$	*5472*
2	$\mathbf{H}_k^{equ} = \mathbf{H}_k \mathbf{Q}_{k,4}$	$8KMN^2$	*1728*
3	$\mathbf{H}_k^{equ} = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^H$	$8K\left(4N^2M + 8NM^2 + 9M^3\right)$	*17712*
			Total 24912

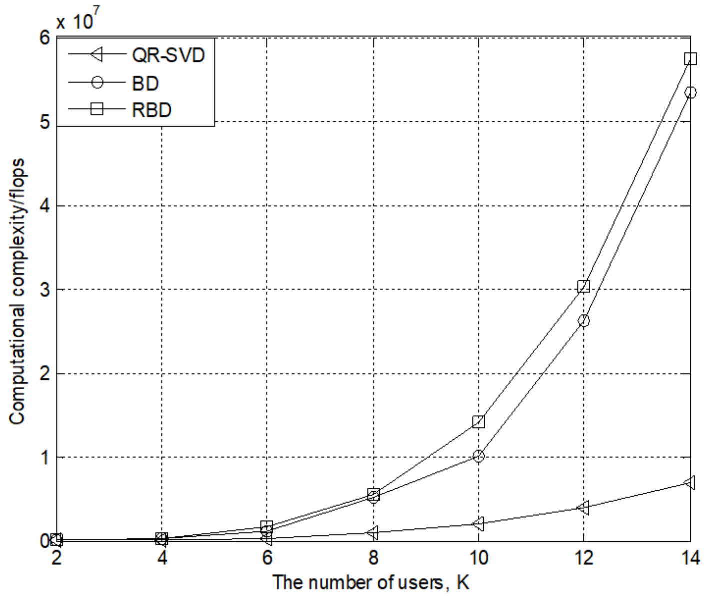

FIGURE 2. Various precoding algorithms computational complexity

where$\mathbf{U}$,$\mathbf{\Sigma}$and$\mathbf{V}$are obtained, its SVD complexity is $8\left(4n^2m + 8nm^2 + 9m^3\right)$ flops [12-13]. Assuming that each user has the same number receive antennas, denoted as $M_k = M, \bar{M} = (K-1)M$. (6, 2, 3, 2) case indicates that the number of transmit antennas is , the system has 2 cells, each user receiving antennas number is and the users number is . In (6, 2, 3, 2) case, Table 1 is RBD precoding algorithm computational complexity and Table 2 is QR-SVD precoding algorithm complexity.

As can be seen from Table 1 and Table 2, the computational complexity of the precoding algorithm is an increasing function with respect to the number of users K, if the number of receiving antennas of each user and the total number of transmit antennas are unchanged, that is, M and N are unchanged. Compare Table 1 and Table 2, in (6, 2, 3, 2) case, QR-SVD algorithm computational complexity is far lower than RBD algorithm, providing a theoretical basis for practical application.

4.2. **Computational complexity simulation.** This part is the computational complexity simulation result of QR-SVD, BD and RBD precoding algorithms. Figure 2 shows various precoding algorithms complexity varies with the number of users. From Figure 2, with and fixed $(M = 3, N = 6)$, BD and RBD precoding algorithms computational complexity grows relatively faster than QR-SVD algorithms with the increasing of K. The reason is that the SVD operation of BD and RBD precoding algorithms is implemented K times on$\bar{M}_k \times N$dimension .

Table 3. Simulation parameters

Total number of transmit antennas	*6*
Cell number	*2*
Each user receive antennas number	*3*
Users Number	*2*
Modulation mode	*QPSK*
Channel mode	*Flat Rayleigh*

The proposed QR-SVD precoding algorithm shows the lowest computational complexity in Figure 2. The reason is that we use a less complex QR decomposition to replace the SVD operation in BD and RBD precoding algorithms.

5. Algorithm performance analysis and simulation.

5.1. System capacity analysis and simulation. This part is the analysis and simulation of the system capacity of QR-SVD, ZF, BD and RBD precoding algorithms. System capacity is calculated using the following formula.

$$C = \sum_{k=1}^{K} \log_2 \left(1 + \frac{\|\mathbf{H}_k \mathbf{W}_k\|^2}{\sum\limits_{i \neq k} \|\mathbf{H}_k \mathbf{W}_i\|^2 + {\sigma_k}^2} \right) (bps/HZ) \tag{14}$$

The simulation is carried out in MATLAB 7.0 simulation environment. Assuming that cell number of multi-user CoMP model is 2, the number of each BS transmission antennas is 3, so the total number of transmission antennas at the BSs is 6. User number is 2, each users receiving antennas number is 3, therefore the total number of receiving antennas at the UE is 6. Modulation mode adopts quadrature phase shift keying (QPSK). The channel model employs flat Rayleigh channel, and Rayleigh distribution is a zero mean and${\sigma_k}^2$variance stationary narrowband Gauss process. Simulation parameters are shown in Table 3[14]. Figure 3 shows various precoding algorithms system capacity performance. It illustrates that the system capacity performance of RBD precoding algorithm is almost same as that of BD precoding algorithm. However, the system capacity of ZF precoding algorithm is the lowest one, comparing with other precoding algorithms. The proposed QR-SVD precoding algorithm has the best system capacity performance. The reason is that QR-SVD precoding algorithm can suppress intra-user, inter-user and noise interference at the same time.

5.2. Bit error rate analysis and simulation. This part is QR-SVD, ZF, BD and RBD precoding algorithms BER analysis and simulation. BER is calculated using the following formula.

$$BER = 1 - \left[1 - \frac{1}{2} erfc \left(\sqrt{\frac{E_b}{2N_0}} \right) \right] \tag{15}$$

where$E_b$is the average bit energy, is the noise power spectral density. The BER has the same simulation settings as system capacity. Figure 4 illustrates various precoding algorithms BER performance. The proposed QR-SVD precoding algorithm shows the best BER performance in the SNR range of 0 30dB. The reason is that QR-SVD precoding algorithm can suppress intra-user, inter-user and noise interference at the same time. However, ZF and BD precoding algorithms can not eliminate noise interference. And RBD precoding algorithm can not eliminate intra-user interference.

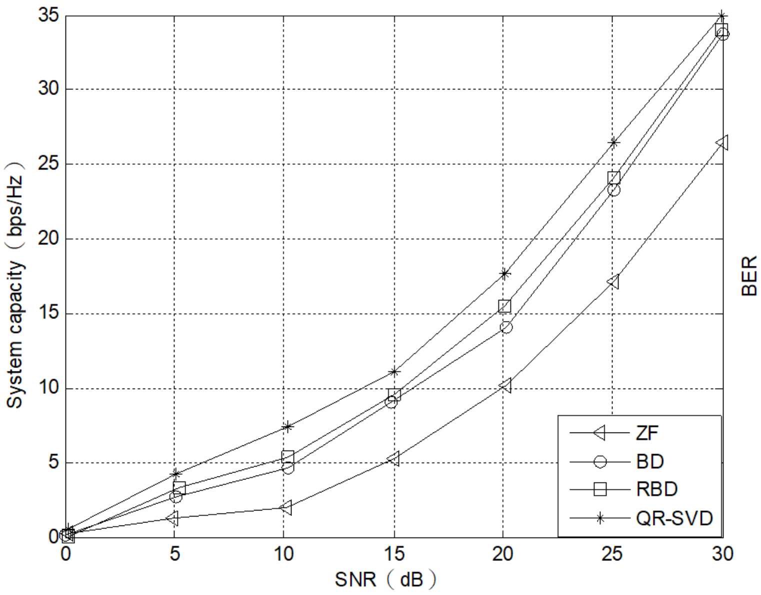

FIGURE 3. Various algorithms system capacity

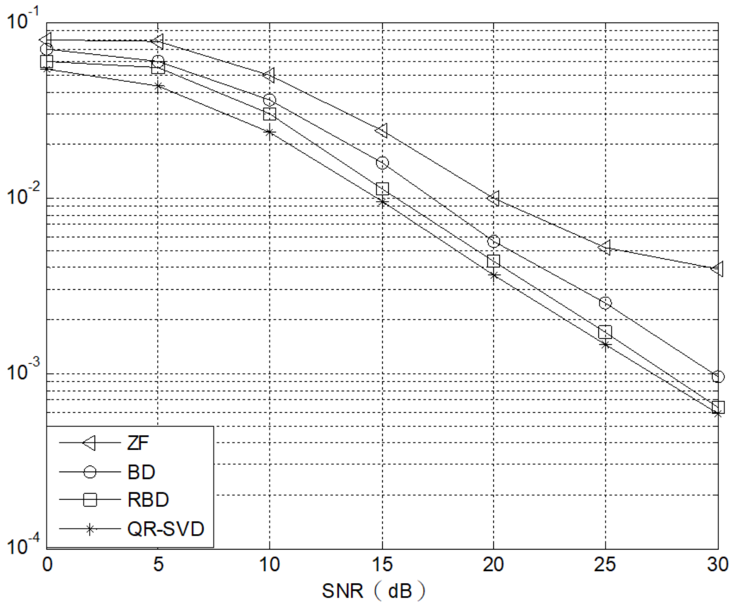

FIGURE 4. Various algorithms BER

6. **Conclusion.** This paper proposes a new CoMP precoding algorithm based on QR-SVD. The proposed algorithm extends channel matrix to suppress multi-user and noise interference, then introduces matrix decomposition method QR, enabling a much lower computational complexity, and finally applies SVD to equivalent channel matrix to obtain complete precoding matrix and decoding matrix. The proposed algorithm has certain advantages over other algorithms in terms of computational complexity, system capacity and bit error rate. In LTE-A system, there are many CoMP precoding algorithms. This paper just describes a part of linear precoding algorithms in CoMP application. Future research could focus on better linear and nonlinear precoding algorithms, obtaining

more outstanding performance and providing more solid theoretical basis for practical applications.

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