

A Modified Thresholding Method based on Relative Homogeneity

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ABSTRACT. *Image binarization method focusing on objects can overcome some shortcomings of famous Otsu's method. In this paper, applying with the gray and neighborhood average gray information, a more detailed description of thresholding algorithm is presented. Using relative homogeneity information both of foreground and background, a new threshold discriminant criterion is proposed. For better adaptability, valley point neighborhood histogram information is introduced, a modified threshold criterion is constructed. The results show that, comparing with Otsu's and focusing on objects methods, the proposed method has better segmentation accuracy, especially, for images with large gray distribution difference of classes, the adaptability is much better.*

Keywords: Image segmentation; Gray histogram; Otsu's method; Focusing on objects methods; Relative homogeneity.

1. **Introduction.** In image analysis, image segmentation is one of the most fundamental and important tasks. It is often a necessary step towards any further processing and analysis^[1-4]. Thresholding method is a main image segmentation method, there are 40 applications^[5], which are categorized into the six groups based on the type of information used: histogram shape-based methods, clustering-based methods, entropy-based methods, object attribute-based methods, the spatial methods and local methods.

In earlier thresholding research, Otsu^[6] proposed maximum between-class variance criteria to select the best threshold, which is regarded as one of the classic techniques and clustering criterion^[5]. Nonetheless, when gray level distribution functions have either unequal variances or populations, Otsu's method will provide a biased threshold. As an attempt to overcome the inherent defect of Otsu's method, Hou^[7] proposed minimum class variance thresholding (MCVT) method, a generalized Otsu's method. Chen^[8] analyzed the limitations of Otsu's criteria and developed a new binarization method by defining a discriminant criterion. Extensive research^[9-12] has been already conducted to introduce new and more robust thresholding techniques based on class variance thresholding segmentation.

In this paper, we have a research based on Chen's^[8] method, a more detailed description of threshold calculation is presented using histogram information of image gray distribution. In addition, we analyzed the shortcoming of Chen's method, introduced relative homogeneity information between foreground and background, developed a new discriminant function. Taking into account the specific histogram distribution images, a valley point neighborhood histogram information was introduced, a modified discriminant criterion is proposed, which improves the accuracy and adaptability. The experiments results on real images demonstrate that the proposed approaches can perform not only visually better segmentation but also can better adapt to the images of different gray distribution characteristics.

The rest of this paper is organized as follows: In Sect.2 image binarization focusing on objects based on image histogram is described. In Sect.3 modified thresholding method based on relative homogeneity between classes is proposed. In Sect.4. the experiment results are discussed, and finally, conclusions are presented in Sect.5.

2. Image binarization focusing on objects based on image histogram.

2.1. Binarization focusing on objects. Assume the gray levels of given image ranges in $\{0,1,2,\dots,L-1\}$, L is the total gray levels number, $M \times N$ is the size. The gray value of (x,y) is expressed by i , $f(i)$ is the total pixels number of gray level i , then $p(i)$ is the probability of gray level.

Otsu's binarizing method selects an optimal threshold t for a given image by a within-class discriminant function^[6]:

$$\begin{aligned}\sigma_W^2(t) &= P_1(t)\sigma_1^2(t) + P_2(t)\sigma_2^2(t) \\ &= \sum_{i=0}^{t-1} p(i)(i - \mu_1(t))^2 + \sum_{i=t}^{L-1} p(i)(i - \mu_2(t))^2\end{aligned}\quad (1)$$

Where, $\sigma_1^2(t) = \sum_{i=0}^{t-1} (i - \mu_1(t))^2 \frac{p(i)}{P_1(t)}$, $\sigma_2^2(t) = \sum_{i=t}^{L-1} (i - \mu_2(t))^2 \frac{p(i)}{P_2(t)}$, $P_1(t) = \sum_{i=0}^{t-1} p(i)$, $P_2(t) = \sum_{i=t}^{L-1} p(i)$ is the prior probability of two classes, $\mu_1(t) = \frac{\sum_{i=0}^{t-1} ip(i)}{P_1(t)}$, $\mu_2(t) = \frac{\sum_{i=t}^{L-1} ip(i)}{P_2(t)}$ is the mean of two classes.

In nature, Otsu's method views both object and background as having uniformity or homogeneity of gray levels. However, for some images, object pixels may have more uniformity or homogeneity in gray level distribution than background pixels, it means that background possesses more likely heterogeneous and non-uniform distribution, and naturally produces many different and diverse gray levels. Therefore, a biased threshold estimate will possibly be resulted with adopting a single mean to represent background.

In order to remedy the shortcoming of both Otsu's method, Chen^[8] defined an alternative discriminant criterion, which focus primarily information on the object segmented, and only assumes object has gray level homogeneity.

The criterion $J_{LC}(t)$ is:

$$J_{LC}(t) = \left(\frac{P_1(t)}{P_2(t)} \right)^\alpha \frac{\sum_{(x,y) \in O} [\lambda(g(x,y) - m)^2 + (1 - \lambda)(\bar{g}(x,y) - m)^2]}{\sum_{(x,y) \notin O} [\lambda(g(x,y) - m)^2 + (1 - \lambda)(\bar{g}(x,y) - m)^2]} \quad (2)$$

Then, the optimal threshold t^* is:

$$t^* = Arg \min_{0 \leq t \leq L-1} J_{LC}(t) \tag{3}$$

Here, $m = \frac{1}{|O|} \sum_{(x,y) \in O} g(x,y)$, $P_1(t) = \frac{1}{|O|}$, $P_2(t) = \frac{1}{N-|O|}$, O is the set of pixels belonging to the object, $g(x,y)$ denotes the gray level at (x,y) , $\bar{g}(x,y)$ is the neighboring average gray level, m is the mean of gray levels, N is the total number of pixels, α ($\alpha \geq 0$) and λ ($0 \leq \lambda \leq 1$) are adjustable parameters to trade off. In formula (2), the numerator only measures the object-class similarity or scatter degree. The more similar (compact) the pixels in object class, the smaller the scatter and thus the smaller the numerator value is. And the denominator measures the background-class dissimilarity to the object class. A larger value of the denominator implies that the two classes are better separated even when background is heterogeneous.

This criterion more focuses on both the similarity of object class itself and the dissimilarity of background to object, for better avoiding the problem probably incurred by the heterogeneity of background. Even if background is not consistent, object pixels can be better separated.

2.2. Application based on image histogram information. The discriminant(2) gives a new criterion for threshold selection of images. Based on this criterion, the threshold value is selected focusing on the homogeneity object, and local neighborhood mean is introduced. It is not only better in segmentation but also has a relatively obvious noise removal ability. In addition, two parameters can be adjusted to optimize the threshold value.

In discriminant (2), it is implied that m can represent all, and t can also represent all. If we take into account the difference of gray histogram and neighborhood average histogram, a more detailed description can be given, a thresholding criterion based on histogram information is obtained, i.e.

$$J_o(t) = \left(\frac{P_1(t)}{P_2(t)} \right)^\alpha \frac{[\sum_{i \in O} \lambda \cdot p(i)(i - \mu(t))^2 + (1 - \lambda) \cdot \bar{p}(i)(i - \bar{\mu}(t))^2]}{[\sum_{i \notin O} \lambda \cdot p(i)(i - \mu(t))^2 + (1 - \lambda) \cdot \bar{p}(i)(i - \bar{\mu}(t))^2]} \tag{4}$$

Where, $P_1(t) = 1/\sum_{i \in O} p(i)$, $P_2(t) = 1/\sum_{i \notin O} p(i)$ are the prior probability, $\mu(t)$ denotes the object area mean value of original image, $\bar{p}(i)$ and $\bar{\mu}(t)$ denote the probability and area mean value of neighborhood average image.

However, in application, we found limitation of criterion(4). To simplify, we use $(\sigma_1(t))^2$ represent the numerator and $(\sigma_2(t))^2$ represent the denominator, then $J_O(t)$ is expressed as:

$$J_O(t) = \left(\frac{P_1(t)}{P_2(t)} \right)^\alpha \frac{(\sigma_1(t))^2}{(\sigma_2(t))^2} \tag{5}$$

Rice image is selected to express the limitation, as shown in Fig.1. With the increase of threshold t , $(\sigma_1(t))^2$ increases monotonically, while $(\sigma_2(t))^2$ decreases monotonically about t . In this way, extreme value will be meaningless. Naturally, the applicability of discriminant function is limited.

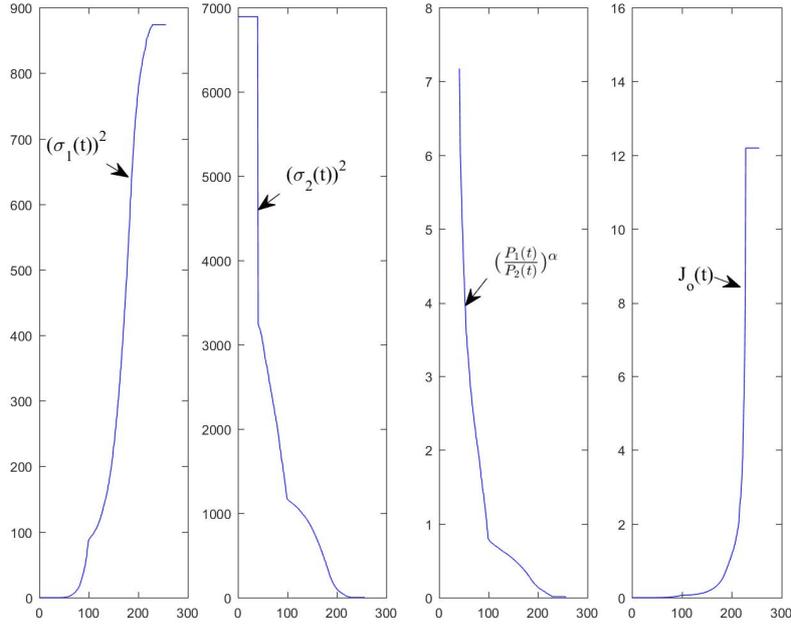


FIGURE 1. Experimental data curve of Rice image

$J_O(t)$ only includes the consistency degree of object (background) with respect to the other pixels, the less the pixels, the better the internal relative uniformity. That is, with the increasing of pixels number, the relative uniformity will be more and more poor, so with the increase of t , $J_O(t)$ shows monotonous.

3. Modified thresholding method based on relative homogeneity.

3.1. Thresholding method based on relative homogeneity between classes. From section 2, the optimal threshold value can not be obtained only by considering the unilateral homogeneity of object (background). In addition, the pixels with good homogeneity may be located in low gray value region or in high value region, therefore, the above discriminant function can be described as the following:

$$J_{O1}(t) = \left(\frac{P_1(t)}{P_2(t)} \right)^\alpha \frac{\sum_{i=0}^{t-1} [\lambda \cdot p(i) \cdot (i - \mu_1(t))^2 + (1 - \lambda) \cdot \bar{p}(i) \cdot (i - \bar{\mu}_1(t))^2]}{\sum_{j=t}^{L-1} [\lambda \cdot p(j) \cdot (j - \mu_1(t))^2 + (1 - \lambda) \cdot \bar{p}(j) \cdot (j - \bar{\mu}_1(t))^2]} \quad (6)$$

$$J_{O2}(t) = \left(\frac{P_2(t)}{P_1(t)} \right)^\alpha \frac{\sum_{i=t}^{L-1} [\lambda \cdot p(i) \cdot (i - \mu_2(t))^2 + (1 - \lambda) \cdot \bar{p}(i) \cdot (i - \bar{\mu}_2(t))^2]}{\sum_{j=0}^t [\lambda \cdot p(j) \cdot (j - \mu_2(t))^2 + (1 - \lambda) \cdot \bar{p}(j) \cdot (j - \bar{\mu}_2(t))^2]} \quad (7)$$

Where, $P_1(t) = 1/\sum_{i=0}^{t-1} p(i)$, $P_2(t) = 1/\sum_{i=t}^{L-1} p(i)$ are the prior probability of object and background. $\mu_1(t)$ and $\mu_2(t)$ denotes the object and background area mean value of original image, $\bar{\mu}_1(t)$ and $\bar{\mu}_2(t)$ denotes the object and background area mean value of neighborhood average image.

$J_{O_1}(t)$ denotes the criterion function in which homogeneity is better in low gray level region, $J_{O_2}(t)$ denotes the criterion function in which homogeneity is better in high gray level region. For an image, the information will be lost if only using formula(6) or formula(7), for taking into account the regional internal uniform information, we can combine the two formulas and construct a new threshold formula as follows:

$$J_{OB}(t) = J_{O_1}(t) + J_{O_2}(t) \quad (8)$$

Then, the optimal thresholding value is:

$$t^* = Arg \min_{0 \leq t \leq L-1} J_{OB}(t) \quad (9)$$

In formula (8), it is adaptive for object is not only in low but high gray level region, and the uniformity and integrity of two classes can be taken.

When $\lambda = 1$, the numerator of (9) is same as criterion(1), the denominator of (9) is used to measure relative degree of uniformity. For two classes, although there is greater uniformity difference, against to the other class, there is a better homogeneity characteristics, it seems more realistic. According to the form, we defined this method as the thresholding method based on relative homogeneity between classes.

3.2. A neighborhood histogram modified method based on relative homogeneity. In section 2, using the relative homogeneity between object and background classes, the rationality of threshold selection can be improved. However, for some images with complex background or specific distributions, the applicability is still limited. The threshold formula can be adjusted combining with image spatial information.

According to gray distribution histogram of image, optimal threshold should be located at the valley point, therefore, taking into account the neighborhood information of valley point^[13], a modified thresholding method based on relative homogeneity between classes is proposed.

Assuming the original image histogram is denoted as $f(g)$, then the modified formula using histogram is:

$$\bar{f}(g) = [f(g-h) + f(g-h+1) + \dots + f(g) + f(g+1) + \dots + f(g+h)] \quad (10)$$

This formula can be viewed as a filter, it can be simplified as:

$$\bar{f}(g) = \sum_{i=-h}^h f(g+i) \quad (11)$$

Generally, h can be valued 1,2,3,4,5. The greater h , the longer the filter.

Since the optimal threshold should be located at the valley point of histogram, the gray probability and the neighborhood gray level probability should be taken as the minimum. So a new threshold formula is constructed as:

$$J_{OBM}(t) = \bar{f}(t) * [J_{OB}(t)] \quad (12)$$

The optimal thresholding value t^* is

$$t^* = Arg \min_{0 \leq t \leq L-1} J_{OBM}(t) \quad (13)$$

4. Experiment results and analysis. The thresholding methods are implemented in MATLAB 2009a for different images.

4.1. The results of method focusing on objects. Fig.2 is the segmented results of Color image. Fig.2(a) is original image, Fig.2(b) is 1d histogram, Fig.2(c) is the segmented result by $J_O(t)$ (denoted as $1d_O$), Fig.2(d) and Fig.2(e) are the segmented results by 1d and 2d Otsu's methods(denoted as $1d_Otsu$ and $2d_Otsu$) respectively. From the results, we can know that the object cant be extracted using $1d_Otsu$ and $2d_Otsu$ methods,while the extraction result by $J_O(t)$ is complete, as shown in Fig.2(c). Here, the parameters α and λ were calculated by 0.5 according to the paper[8].

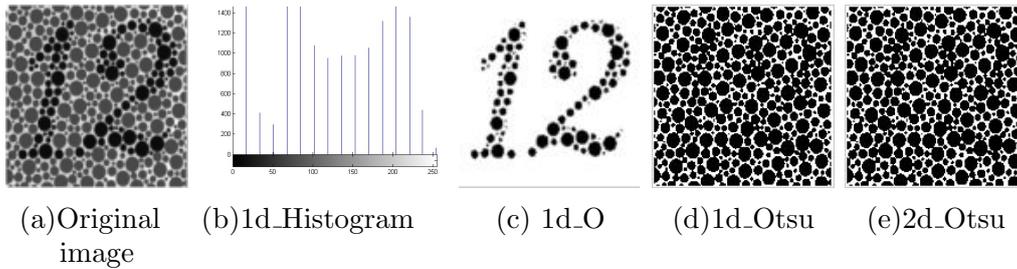


FIGURE 2. The thresholded results of Color image

4.2. The results of relative homogeneity method. The Letter and Medicine images are selected to show the effectiveness of the method based on the relative homogeneity between classes, as shown in Fig.3-4. Fig.3-4(a) are original images, Fig.3-4(b) are the segmented result by $J_{O1}(t)$ (denoted as $1d_O1$), Fig.3-4(c) are the segmented result by $J_{O2}(t)$ (denoted as $1d_O2$), Fig.3-4(d) are the segmented results by $J_{OB}(t)$ (denoted as $1d_OB$), Fig.3-4(e) are the segmented results by $1d_Otsu$.The thresholding value are shown in Table 1. Comparing with the results of four methods, we can see that the proposed relative homogeneity method between classes can obtain the most complete object extraction results.

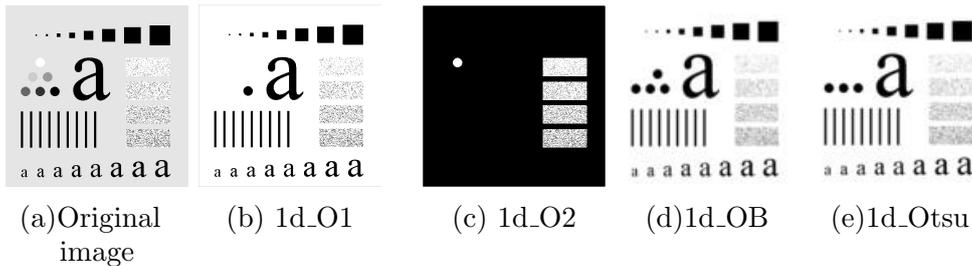


FIGURE 3. The results of Letter image

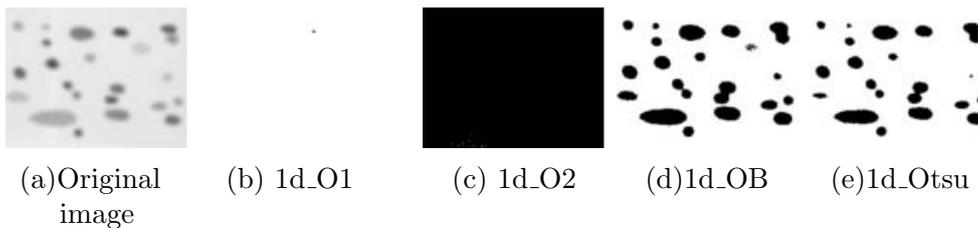


FIGURE 4. The results of Medicine image

TABLE 1. Comparison of segmentation results for 4 methods

Method	$J_{O1}(t)$	$J_{O2}(t)$	1d_OB	1d_Otsu
Letter	2	253	155	150
Medicine	71	241	205	195

4.3. **The results of histogram modified method.** In Fig.5-6, the experiments of Lymp and Aerial images are used to show the effects, the modification method is simplified as $1d_OBM$, is shown in Fig.5-6(d). Fig.5-6(a) are original images, Fig.5-6(b) are 1d histogram, Fig.5-6(c) and Fig.5-6(e) are the results by $1d_OB$ and $2d_Otsu$. The thresholded values are listed in Table 2. The results show that the integrity and clarity of object extraction are obviously improved by using modified method.

For the parameter h of $\bar{f}(t)$, which is used to indicate the length of filter, the threshold results of different h are shown in Table 2. When $h = 2$, thresholding results are the best, thus, we take $h = 2$.

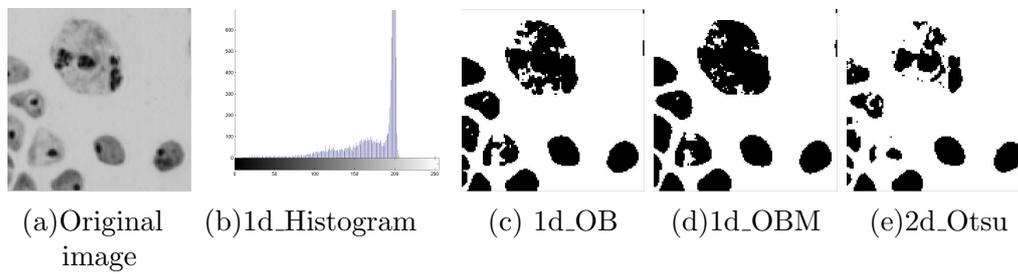


FIGURE 5. The thresholded results of Lymp image

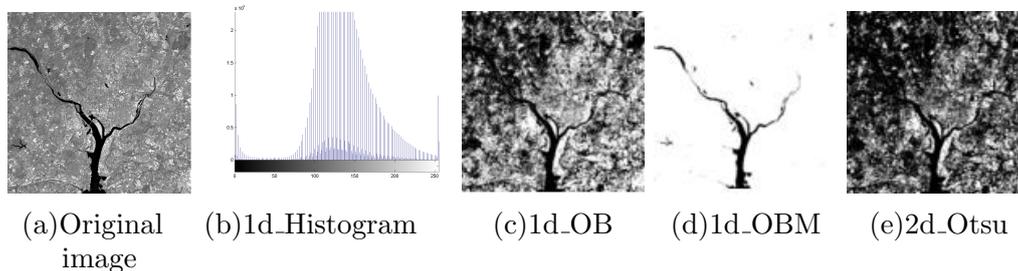


FIGURE 6. The thresholded results of Aerial image

TABLE 2. The threshold results of 3 methods

Method	$1d_OB$	$1d_OBM$	2d_Otsu
Lymp	169	h=1	148
		h=2	175
		h=3	171
		h=5	170
Aerial	135	h=1	48
		h=2	48
		h=3	51
		h=5	48

5. **Conclusion.** In this paper, we analyzed the limitations of Otsu’s thresholding method, discussed the thresholding method based on within-class classification criterion,. Based on the method focusing on objects, using gray and neighborhood average gray histogram of pixels, a more detailed description of threshold discriminant function is developed. Taking into account the homogeneity information both of foreground and background,

a new threshold discriminant criterion based on relative homogeneity between classes is proposed.

Comparing with the criterion focusing on objects, the new criterion introduced relative information of uniformity, the adaptability is better. For some images with large classes distribution difference. The modified method deduced the deviation, the segmented result is more reasonable. The application of test images illustrates the adaptability, it is better than $1d_Otsu$, Chen's method and $2d_Otsu$. In addition, for better effects, the weight coefficient of $J_{O1}(t)$ and $J_{O2}(t)$ can be considered according to the distribution. If the expected segment number is achieved, this method can be used in multilevel thresholding problem.

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