## On the Security of a Chaotic Maps-based Three-party Authenticated Key Agreement Protocol

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ABSTRACT. Chaotic map has been receiving increasing attention in the cryptographic literature. There are various scholars working on a particular type of authenticated key exchange protocol using chaotic map in the recent years. Very recently, Li et al. proposed a new three-party-authenticated key agreement protocol based on chaotic maps without storing a password table in a server. Compared with previous works, their protocol can completely resistant to password guessing attacks because of using no passwords. In this paper, we demonstrate that this protocol is still vulnerable to a user impersonation attack. Keywords: Cryptanalysis, Three-party Authenticated key agreement, Smart card.

1. Introduction. Information transmitted in an open network is required to be encrypted for confidentiality; thus, an Authenticated Key Exchange (AKE) scheme is necessary. An AKE scheme helps two communication entities to authenticate each other and establish a common session key. This session key is utilized for encrypting and decrypting their following communication. AKE schemes have been studied over the past few decades. So far, a huge number of AKE schemes have been proposed for many environments and applications [1, 2, 3, 4, 5].

Originally, an AKE scheme was designed for two-party situation. In most existing two-party authenticated key exchange (2AKE) schemes, two entities need to pre-share

a common secret, e.g., a password. More specifically, each entity needs to remember different secrets for different entities who communication with. It means that 2AKE is not suitable for large-scale environments because of storage overhead. For this reason, AKE in a three-party (3AKE) setting is described. In a 3AKE scheme, a trusted server is involved. Any two entities establish a common session key with a trusted server's help. Compared with 2AKE, 3AKE is more practical in large-scale environments since each entity only requires to pre-share a secret with a trusted server.

The exiting AKE schemes are normally developed based on different cryptographic operations, such as modular exponentiation operations [6], bilinear pairing [7, 8, 9], etc. Recently, a chaotic system has received increasing attention, especially in designing cryptosystems [10, 11, 12]. Chaos is the well-defined universal, random-like and robust phenomenon in nonlinear system. With the property of better performance, chebyshev chaotic maps have been utilized in designing AKE schemes. So far, various chaotic maps-based AKE schemes have been proposed.

In 2012, Lai et al. [13] proposed a password-based 3AKE scheme using an enhanced Chebeshev chaotic map. This scheme uses a smart card to store sensitive information. However, Zhao et al. [14] demonstrated that this scheme is vulnerable to an off-line password guessing attack and a privileged insider attack. Zhao et al. then proposed an improved scheme which also uses a smart card. In 2013, Lee et al. [15] and Xie et al. [16] proposed their chaos-based 3AKE protocol without using smart cards respectively. Later, Farash et al. [17] proposed another chaos-based 3AKE scheme without using smart cards. Compared with Lee et al. [15] and Xie et al.'s schemes [16], Farash et al.'s scheme uses neither server's public key nor symmetric cryptosystems. Farash et al. [17] also provide a formal proof to demonstrate the security of this scheme. Hu et al. [18] further showed that Lee et al. [15] is vulnerable to a man-in-the-middle attack and a user anonymity attack. In 2015, Lee et al. [19] demonstrated that Xie et al.'s scheme [16] also suffers from an on-line password guessing attack. They also describes a new chaotic-maps 3AKE scheme. Besides, Li et al. [20] proposed another chaotic maps-based 3AKE protocol. In these two schemes [19, 20], a server does not need to store a password table.

After reviewing the above schemes, we found that Li et al.'s protocol [20] has a security flaw. In their design, each user does not need to remember a password since using a low-entropy password easily suffers form password guessing attacks. Instead, each user is required to securely store a hashed value. It is obviously that remembering a hashed value is a difficult task for human beings. For this reason, users may store their hashed values in a smart card issued by a trusted server. However, smart cards could be stolen by a malicious attackers. Although the smart card may come with a tamper-resistant property, sensitive information stored in a smart card can be still extracted by some side channel attacks [21, 22]. In this paper, we will demonstrate that Li et al.'s protocol [20] is vulnerable to a user impersonation attack under the above assumptions.

2. Review of Li et al.'s protocol. In this section, we briefly review Li et al.'s protocol. The detailed steps can refer to [20].

In a registration phase, every client participant requires to register himself to a trusted server S. Assume a participant  $U_i$  desires to register himself to S,  $U_i$  selects his identity  $ID_i$  and submits  $ID_i$  to S. S then computes  $h(ID_i||x)$  and submits it to  $U_i$  through a secure channel.  $U_i$  stores  $h(ID_i||x)$  as a secret. Note that x is a master secret key of S and h() denotes a secure one-way hash function.

Assume there are two client participants  $U_A$  and  $U_B$  desire to establish a session through S, they perform the authentication and key agreement phase described as follows.

Step 1.  $U_A$  generates a nonce a and computes (1)(2)(3).

$$K_A = T_a(X) \tag{1}$$

$$H_A = h(ID_A||ID_B||K_A) \tag{2}$$

$$C_1 = E_{h(ID_A||x)}(ID_A||ID_B||K_A||H_A)$$
(3)

Note that X means a public seed of Chebyshev chaotic maps generated from x and  $E_{key}()$  denotes a symmetric encryption algorithm with key key. Then,  $U_A$  transmits  $(ID_A, C_1)$  to  $U_B$ .

Step 2. While receiving  $(ID_A, C_1)$  from  $U_A, U_B$  generates a nonce b and calculates (4)(5)(6).

$$K_B = T_b(X) \tag{4}$$

$$H_B = h(ID_B||ID_A||K_B) \tag{5}$$

$$C_2 = E_{h(ID_B||x)}(ID_B||ID_A||K_B||H_B)$$
(6)

B then transmits  $(ID_A, C_1, ID_B, C_2)$  to S.

Step 3. S first verifies if  $ID_A$  and  $ID_B$  are valid, then S decrypts  $C_1$  with  $h(ID_A||x)$  and  $C_2$  with  $h(ID_B||x)$  respectively. S now verifies the validity of  $ID_A$  and  $ID_B$  again. S also checks the integrity of  $H_A$  and  $H_B$ . If both hold, S computes (7)(8)(9).

$$H_{SAB} = h(ID_A||ID_B||K_A||K_B) \tag{7}$$

$$C_3 = E_{h(ID_B||x)}(ID_B||ID_A||K_B||K_A||H_{SAB})$$
(8)

$$C_4 = E_{h(ID_A||x)}(ID_A||ID_B||K_A||K_B||H_{SAB})$$
(9)

S then transmits  $(C_3, C_4)$  to  $U_B$ .

Step 4.  $U_B$  first decrypts  $C_3$  received from S with  $h(ID_B||x)$  and then verifies the integrity of  $H_{SAB}$ . If it holds, B computes (10)(11).

$$SK = T_b(K_A) = T_{ab}(X) \tag{10}$$

$$H_{BA} = h(ID_B||ID_A||SK||K_A) \tag{11}$$

 $U_B$  then transmits  $(C_4, H_{BA})$  to  $U_A$ .

Step 5. While receiving the messages from  $U_B$ ,  $U_A$  first decrypts  $C_4$  with  $h(ID_A||x)$  and then verifies the integrity of  $H_{SAB}$ . If it holds,  $U_A$  calculates  $SK = T_a(K_B) = T_{ab}(X)$  and utilizes SK to verify the integrity of  $H_{BA}$ . After that,  $U_A$  computes  $H_{AB} = h(ID_A||ID_B||$  $SK||K_B)$  and transmits  $H_{AB}$  to  $U_B$ .

Step 6.  $U_B$  first checks the integrity of  $H_{AB}$  received from  $U_A$ . Finally,  $U_A$  and  $U_B$  has established a common session key SK.

3. Cryptanalysis on Li et al.'s protocol. In this section, we demonstrate that Li et al.'s protocol [20] is vulnerable to the user impersonate attack.

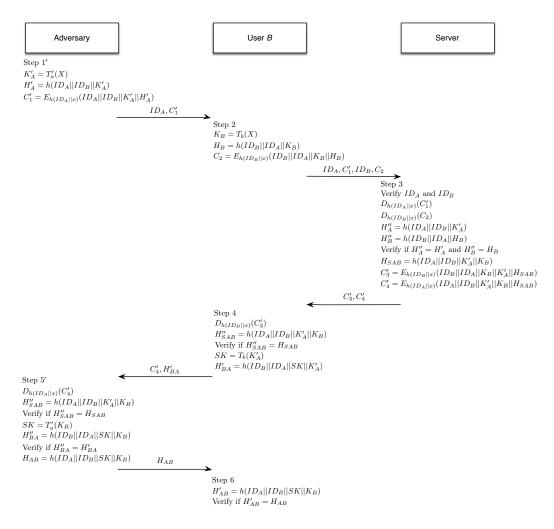


FIGURE 1. The Detailed Steps of Our Attack

3.1. The secrecy of  $h(ID_i||x)$ . In Li et al.'s protocol [20], the authors assume that any client participate  $U_i$  must stores  $h(ID_i||x)$  as a secret. Certainly, keeping this hashed value in mind may be the securest may, but remembering it is indeed a infeasible task for human beings. Consequently, storing this hashed value in a smart card seems a reasonable and acceptable choice. Unfortunately, smart cards could be stolen by a malicious attackers. Although the smart card may come with a tamper-resistant property, sensitive information stored in a smart card can be still extracted by some side channel attacks [21, 22].

3.2. Our Attack. Assume a malicious adversary  $U_E$  steals  $U_A$ 's smart card and extracts  $h(ID_A||x)$  stored in this smart card. Figure 1 illustrates the detailed procedures of our attack. The steps of the proposed attack are listed as follows.

Step 1'.  $U_E$  generates a nonce a' and computes  $K'_A = T'_a(X)$ ,  $H'_A = h(ID_A||ID_B||K'_A)$  and  $C'_1 = E_{h(ID_A||x)}(ID_A||ID_B||K'_A||H'_A)$ . Then,  $U_E$  submits  $(ID_A, C'_1)$  to  $U_B$ .

Step 2. After receiving  $(ID_A, C'_1)$ ,  $U_B$  believes that  $U_A$  desires to establish a session key with him. Then,  $U_B$  generates a nonce b and computes  $K_B = T_b(X)$ ,  $H_B = h(ID_B||ID_A||K_B)$  and  $C_2 = E_{h(ID_B||x)}(ID_B||ID_A||K_B||H_B)$ . After that,  $U_B$  submits  $(ID_A, C'_1, ID_B, C_2)$  to S.

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Step 3. According to the received messages, S believes that  $U_A$  and  $U_B$  attempt to authenticate to each other and further establish a session key. S first assures that  $U_A$  and  $U_B$  are both legitimate user, S then decrypts  $C'_1$  with  $h(ID_A||x)$  and  $C_2$  with  $h(ID_B||x)$  respectively. S further verifies the validity of  $ID_A$  and  $ID_B$  again and checks the integrity of  $H'_A$  and  $H_B$ . At last, S calculates  $H_{SAB} = h(ID_A||ID_B||K'_A||K_B)$ ,  $C'_3 = E_{h(ID_B||x)}(ID_B||ID_A||ID_B||K'_A||K_B)$ ,  $K'_3 = E_{h(ID_B||x)}(ID_B||ID_A||ID_B||K'_A||K_B||H_{SAB})$ . Now S transmits  $(C'_3, C'_4)$  to  $U_B$ .

Step 4. Upon obtaining  $(C'_3, C'_4)$ ,  $U_B$  decrypts  $C'_3$  with  $h(ID_B||x)$  and then verifies the integrity of  $H_{SAB}$ . If it holds, B calculates  $SK=T_b(K'_A)$  and  $H'_{BA}=h(ID_B||ID_A||SK||K'_A)$ .  $U_B$  then submits  $(C'_4, H'_{BA})$  to  $U_A$ .  $U_B$  believes  $(C'_4, H'_{BA})$  are sent to  $U_A$ ; however, these messages are sent to  $U_E$ .

Step 5'.  $U_E$  decrypts  $C'_4$  with  $h(ID_A||x)$  and verifies the integrity of  $H_{SAB}$ . Then,  $U_E$  computes  $SK = T'_a(K_B)$  and utilizes SK to verify the integrity of  $H_{BA}$ . Now  $U_E$  computes  $H_{AB} = h(ID_A||ID_B||SK||K_B)$  and transmits  $H_{AB}$  to  $U_B$ .

Step 6.  $U_B$  first checks the integrity of  $H_{AB}$ . At this time,  $U_E$  and  $U_B$  has established a common session key SK.

4. **Conclusion.** There are many three-party-authenticated key agreement schemes in the literature which apparently seem to work but they have been shown to be insecure. However, the very recent Li et al.'s protocol is shown to suffer from a user impersonation attack in this paper.

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