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A Novel Steganographic Protocol from Error-correcting Codes

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Abstract. *The idea of "Matrix encoding" was introduced in steganography by Crandall in 1998* [6]*. The implementation was then proposed by Westfeld with steganography algorithm F5* [1]*. The objective is to transmit a message within an image via the modified image, but with the constraint of minimizing the number of coefficients of the image changed. In this paper, new construction of protocol steganography is considered, it is an extension of the error correcting code and steganography construction. The proposed method consists of use the Majority logic decoding introduced in* [5]*, for embedding the message in the cover image, the extraction function is always based on syndrome coding. An asymptoticly tight bound on the performance of embedding schemes is given.*

Keywords: Majority Logic Decoder, Steganography, Error-correcting code, average distortion, matrix encoding, embeding efficient.

1. **Introduction.** The goal of digital steganography is to modify a digital object (cover) to encode and conceal a sequence of bits (message) to facilitate covert communication [12, 13, 14]. The goal of steganalysis is to detect (and possibly prevent) such communication. Often, the cover media correspond to graphics files. Graphics files are the typical choice because of their ubiquitous presence in digital society, but any medium that contains a substantial amount of perceptually insignificant data can be used.
An interesting steganographic method is known as matrix encoding, introduced by Crandall [6]. Matrix encoding requires the sender and the recipient to agree in advance on a parity check matrix H, and the secret message is then extracted by the recipient as the syndrome (with respect to H) of the received cover object. This method was made popular by Westfeld [1], who incorporated a specific implementation using Hamming codes in his F5 algorithm, which can embed t bits of message in $2^t - 1$ cover symbols by changing, at most, one of them.

There are two parameters which help to evaluate the performance of a steganographic method over a cover message of N symbols : the average distortion $D = \frac{R_a}{N}$, where R_a is the expected number of changes over uniformly distributed messages; and the embedding rate $E = \frac{t}{N}$, which is the amount of bits that can be hidden in a cover message [4]. In general, for the same embedding rate a method is better when the average distortion is smaller.
Furthermore, we will also assume that a discrete source produces a sequence $x = (x_1, ..., x_N)$,

where, N is the block length, each $x_i \in \mathbb{F}_2$. The message $s = (s_1, ..., s_M)$, where M is the message length, we want to hide into a host sequence x produces a composite sequence $y = f(x, s)$, where $y = (y_1, ..., y_N)$, and each $y_i \in \mathbb{F}_2$. The composite sequence y is obtained from distorting x, and the distortion will be assumed to be a squared-error distortion. In these conditions, if information is only carried by the least significant bit (LSB) of each x_i, the appropriate solution comes from using binary Hamming codes [1].

In this work we propose a steganography method based on a $[n, k, t]$ codes BCH. While the size of each cover block is $n = 2^m - 1$, $m = 3, 4, 5...,$, and the rate of capacity is $M = m \times t$. Our method uses $t = 2$. Because with $t = 2$ BCH is semi perfect. for this paper is find embedding efficiency=(rate of capacity/number of changes), better than code Hamming used in [1].

The rest of this paper is organized as follows. Section 2 introduces the relations between error-correcting codes and steganography systems. We construct our approach in section3. Section 4 introduces the bound on the performance of embedding schemes. Conclusions and future work are presented in section 5.

2. **Steganography and Error Correcting Code.** R. Crandall [6] introduced the matrix encoding idea to improve the embedding efficiency for steganography. F5 proposed by Westfeld [1] is the first implementation of the matrix encoding concept to reduce modification of the quantized DCT coefficients. Since F5, steganographers take the reduction of embedding capacity sincerely and coding theory into consideration. Basically the matrix encoding technique in F5 modifies at most 1 coefficient among n nonzero coefficients to hide k bits. For example, the matrix encoding method modifies at most one coefficients among seven coefficients to hide three bits like a $[7, 3]$ Hamming code. Thus, distortion of image is reduced at the cost of sacrificing the embedding capacity. Now, not all coefficients have to be modified by using $[n, k, 1]$ code where $n = 2^k - 1$. Modified matrix encoding (MME) [7] uses $[n, k, 2]$ code where one more coefficients may be changed in each group compared with the matrix encoding. Main concept of the matrix encoding technique is "the less number of modification to the DCT coefficients, the less amount of distortion in the image" [8].

Matrix encoding using linear codes (syndrome coding) is a general approach to improving embedding efficiency of steganographic schemes. The covering radius of the code corresponds to the maximal number of embedding changes needed to embed any message. Steganographers, however, are more interested in the average number of embedding changes rather than the worst case. In fact, the concept of embedding efficiency-the average number of bits embedded per embedding change-has been frequently used in steganography to compare and evaluate performance of steganographic schemes.

2.1. **Error Correcting Code in Steganography.** An important kind of steganographic protocols can be defined from coding theory. Error-correcting codes are commonly used for detecting and correcting errors, or erasures, in data transmission. An explicit description of the relations between error-correcting codes and steganographic systems was presented by Zhang and Li in [9] and shows that there is a corresponding relation between the maximum length embeddable (MLE) codes and perfect error correcting codes. The most used codes in steganography are linear. The existence of a parity check matrix helps on designing good steganographic protocols.

Let C be a linear $[n, n-t]$ code over the finite field $\mathbb{F}_q$, equivalently, a linear subspace of $\mathbb{F}_q^n$, where the dimension is $k = n - t$. The covering radius ρ of code C is defined as $\rho = max_{v \in \mathbb{F}_q^n} d(v, C)$, where $d(v, C)$ means the minimum Hamming distance from vector v to the code C. The support of a vector $v = (v_1, v_2, ..., v_n) \in \mathbb{F}_q^n$ is the set $supp(v) = \{i | v_i \neq 0\}$.

Let $\mathbb{F}_q^n$ and H be a parity check matrix of C. The syndrome of any $v \in \mathbb{F}_q^n$ is the vector $r(v) = H \times v^T$, where v^T means the vector v as a column vector. Coset $C + v$ is the set of all vectors in $\mathbb{F}_q^n$ with the same syndrome. A vector $l_{r(v)}$ of the minimum weight in $C + v$ will be called leader of the coset, although it is not necessarily unique. The above syndrome map $r : \mathbb{F}_q^n \to \mathbb{F}_q^t$, such that $r(v) = H \times v^T$, is called the retrieval map of a $[n, t, \rho]$ steganographic protocol, which will be called linear to emphasize that the retrieval map r is a linear map. The embedding algorithm to compute $e(s, v)$ for a linear steganographic protocol works in the following way [10]:

Coset algorithm.

- Compute $u := r(v) - s$,
- define $e(s, v) := v - l_u$, where l_u is a leader of the coset $C + u$ of all the vectors in $\mathbb{F}_q^n$ with the same syndrome u. So, $r(l_u) = u$.

Single-Error Correcting Codes. We now give an example of a protocol steganography constructed from a linear single-error-correcting code. This was also discussed, for example, in [1]. Start from the matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

whose entries are elements of $\mathbb{F}_2$. Extracting Scheme is defined

$$F : \mathbb{F}_2^7 \to \mathbb{F}_2^4.$$

$F(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (y_1, y_2, y_3)$, where

$$y_1 = x_1 + x_4 + x_5 + x_7, y_2 = x_2 + x_4 + x_6 + x_7$$

and

$$y_3 = x_3 + x_5 + x_6 + x_7.$$

This function can be described in terms of matrix H.In fact, y_i, is the dot product of x and the i-th row of H. We claim that F is a extracting function of protocol steganography $(7, 3, 1)$.

Embedding Scheme for example, $F(0, 0, 1, 1, 0, 1, 0) = (1, 0, 0)$. Assume $y = (1, 1, 1)$. We claim that it is possible to replace $x = (0, 0, 1, 1, 0, 1, 0)$ by x' such that $F(x') = (1, 1, 1)$ and $d(x, x') = 1$. In fact, we claim more: the coordinate where x has to be changed is uniquely determined. In our case, this is coordinate number 6, so $x' = (0, 0, 1, 1, 0, 0, 0)$, Here is the general embedding rule: form $F(x) + y$, (in the example this is 011). Find the column of H which has these entries (in our example, this is the sixth column). This marks the coordinate where x needs to be changed to embed payload y. This procedure indicates how H and F were constructed and how this can be generalized: the columns of H are simply all nonzero 3-tuples in some order. In general, we start from our choice

of n and write a matrix H whose columns consist of all nonzero n-tuples. Then H has $N = 2^n - 1$ columns. The extracting function,

$$F : \mathbb{F}_2^N \to \mathbb{F}_2^n$$

is defined by way of the dot products with the rows of H. Finally it is clear that embedding efficiency= 3.

2.2. **Majority Logic Decoder.** Majority logic decoding algorithm is briefly explained below. Let H be the parity check matrix of a $[n, k]$ linear error correcting code C. Majority logic decoding implements a voting scheme among a set of check sums orthogonal on a bit or subset of error bits. The majority logic decoding rule is defined for a set of, J check sums orthogonal on error bit e_j as follows [5]:
Let the estimate $\widehat{e_j}$ of error bit e_j be the value assumed by the majority of the J check sums. In the case of a tie, let $\widehat{e_j} = 0$. The majority logic decoding rule guarantees a correct estimate of e_j as long as there are no more than $\lfloor \frac{J}{2} \rfloor$ errors among the error bits being checked. It is clear that for a block code with minimum distance d_{min}, majority logic decoding will be optimal when $J = d_{min} - 1$. In such a case, the code is said to be completely orthogonalizable [5]. The J orthogonal check sums provide reliability information. In general, the greater the number of check sums which agree, the higher the reliability of the estimate. The lack of an extensive majority among the J orthogonal check sums can be used to generate a retransmission request.

3. **New Protocol Steganography.** In this section we discuss the proposed approach for hiding information within the spatial domain of the gray scale image. The proposed approach works by dividing the cover into blocks of equal sizes. A block of binary data, e.g., LSB values of cover data, $\{v_0, v_1, ..., v_{n-1}\}$ over $\mathbb{F}_2$ can be represented by a polynomial of X over $\mathbb{F}_2^m$ such as $v(X) = v_0 + v_1 X + ... + v_{n-1} X^{n-1}$. Embedding message m into the cover data v produces the stego data r which is represented as $r(X) = r_0 + r_1 X + ... + r_{n-1} X^{n-1}$. The relation between m and r can be expressed as a matrix form as follows:

$$m = r \times H^t \tag{1}$$

Decoder also use Equation (1) to extract message from the stego data. By hiding message, some of the cover data bits are flipped from 0 to 1 and vice versa. Let $e(X)$ be the flip pattern that represents which bit positions are flipped [8]. As a result, stego data is modified according to the flip pattern as follows:

$$r(X) = v(X) + e(X) \tag{2}$$

From Equations (1) and (2), we have

$$m - v \times H^t = e \times H^t \tag{3}$$

The left-hand side of Equation (3) is called syndrome S. In other words, the syndrome is expressed as follows:

$$S = m - v \times H^t \tag{4}$$

or, equivalently

$$S = e \times H^t \tag{5}$$

From the steganographic point of view, our objective is to find a minimal number of flips of e(X) satisfying Equation (5) in order to decrease distortion. This is the syndrome coding. Data hiding by error-correcting code solves Equation (5) based on the vector e.

The solution shows the proper positions of the elements in vector $v(X)$ to be modified in order to hide message m to vector $v(X)$. The stego vector $r(X)$ is calculated according to Equation (2). The hidden message can be recovered from stego vector $r(X)$ using Equation (1).

Proposed Embedding Scheme.

- **Input** : message m, block of image C, distortion maximum allowed t.
- **Output** : stgo-image
- Step(1) : calculate S by equation 4.
- Step(2) : if $S = 0$; $supp(e) = \emptyset$, $e(X) = 0$, then the message is already hiding else go to Step(3).
- Step(3) : $S = (S_0, S_1, ..., S_{n-k-1})$; It seeks to construct a set P systems of equations parity , Such that each system $L \in P$, there is a $i \in supp(e)$; L is orthogonal to e_i.
- Step(4) : solve all the systems belonging to P ; and Removal $e_i = 0$ or 1 ; for all $i \in supp(e(X))$
- Step(5) if $w(e(X)) \leq t$; go to step(6), else go to step (3).
- Step(6) : It calculates the stgo-support $r(X)$ by equation 2, you spin the algorithm for every image blocks

Extracting Scheme. The message is retrieved from the stego-support, with the function of syndrome. The relation between m and r can be expressed as a matrix form as follows:

$$m = r \times H^t$$

Proposed Method. Consider the $[7, 4]$ cyclic code generated by $g(x) = 1 + x + x^3$. This is a Hamming code. The parity-check matrix (in systematic form) is found as follows:

$$H = \begin{pmatrix} h_0 \\ h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

We see that the vectors, h_0 and h_2 are orthogonal on digit position 5 and 6(or X^5 and X^6). We also see that the vectors, $h_0 + h_1$ and h_2, are orthogonal on digit positions 4 and 6. Let $E_1^1 = \{e_5, e_6\}$ and $E_2^1 = \{e_4, e_6\}$ be two selected sets. Let $r = (r_0, r_1, ..., r_6)$ be the received vector. then the parity-check sums formed from h_0 and h_2 are:

$$A_1 = r \times h_0 = e_0 + e_3 + e_5 + e_6$$

$$A_2 = r \times h_2 = e_2 + e_4 + e_5 + e_6$$

and the parity-check sums formed from $h_0 + h_1$ and h_2 are:

$$B_1 = r \times (h_0 + h_1) = e_0 + e_1 + e_4 + e_6$$

$$B_2 = r \times h_2 = e_2 + e_4 + e_5 + e_6$$

the parity-check sums, A_1 and A_2, are orthogonal on the set $E_1^1 = \{e_5, e_6\}$ and the parity-check sums, B_1 and B_2, are orthogonal on the set $E_2^1 = \{e_4, e_6\}$. Therefore, the sum $S(E_1^1) = e_5 + e_6$ can be estimated from A_1 and A_2, and the sum $S(E_2^1) = e_4 + e_6$ can be estimated from B_1 and B_2. the sums $S(E_1^1)$ and $S(E_2^1)$ would be correctly estimated provided that there is no more than on error in the error vector e. Now let $E_1^2 = \{e_6\}$. We see that $S(E_1^1)$ and $S(E_2^1)$ are orthogonal on e_6. Hence, e_6 can be estimated from $S(E_1^1)$ and $S(E_2^1)$. The value of e_6 will estimated correctly provided that there are no more than on error in e. Therefore, the $[7, 4]$ Hamming code can be decoded with two

steps of orthogonalization and it is two-step majority-logic decodable. Since its minimum distance is 3 and $J = 2$, it is two-step completely orthogonalizable.

3.1. **Experimental Results.** A generalization of perfect codes is the following: a t-error-correcting code is said to be quasi-perfect if covring raduis=t+1(or equivalently, if the spheres of radius t+1 around the codewords contains all vectors of $\mathbb{F}_q^n$). For example, all double-error-correcting BCH codes are quasi-perfect (see [11], Chapter 9, Section 8). We can use these codes to construct steganographic protocols. Remark that example of single-error correcting Codes also provides the parameter a for quasi-perfect codes. For every integer $m > 2$, the binary two-errorcorrecting BCH code C_m has parameters $[2^m - 1, 2^m - 2m - 1, 5]$ [11]. Its covering radius is $\rho = 3$. Let S_m be the protocol obtained from C_m, by taking a parity check matrix as described above. It is a $[2^m - 1, 2m, 3]$ protocol. The following tables collects the parameters of S_m and the corresponding version of F5 [1] (obtained from the Hamming code)

S_m					
m	n	k	ρ	$\frac{k}{n}$	$\frac{k}{\rho}$
3	7	6	3	0,857	2
4	15	8	3	0,533	2,66
5	31	10	3	0,322	3,33
6	63	12	3	0,190	4
7	127	14	3	0,110	4,66

$F5$					
m	n	k	ρ	$\frac{k}{n}$	$\frac{k}{\rho}$
3	7	3	1	0,428	3
4	15	4	1	0,266	4
5	31	5	1	0,161	5
6	63	6	1	0,095	6
7	127	7	1	0,055	7

$\frac{k}{n}$, $\frac{k}{\rho}$ measure, respectively, the embeding rate and embeding efficient.

4. **Bound on the Performance of Embedding Schemes.** Since for any given cover-word v only $\sum_{i=0}^{t} C_n^i$ different stego-words can be obtained by changing at most t coordinates of v, then we have the following proposition [3]

proposition. For any embedding scheme of distorting t, using binary words of length n as cover words, the number of different messages $M(n,t)$ that can be embedded is bounded by

$$M(n,t) \leq \sum_{i=0}^{t} C_n^i \tag{6}$$

Recall that $h(n,t) = logM(n,t)$. The right hand side of (6) is upper bounded by

$$2^{nH_2(\frac{t}{n})}$$

for $2t < n$, where $H_2(x) = -xlogx-(1-x)log(1-x)$ is the entropy function [3]. Therefore, we have

$$h(n,t) = nH_2(\frac{t}{n})$$

Note this is a good approximation of (6) when t grows linearly with n. For other important case t fixed and n growing to infinity it follows from

$$h(n,t) = t \times logn - log(t!), tfixed. \tag{7}$$

Fortunately there are known constructions of steganography method very close to the Hamming bound. In the precedent section we give constructions showing the upper bounds are asymptoticly tight.

5. **Conclusions.** In this paper, we have presented a new method for steganography, based on error correcting code. This method use a class of decoding for error correcting code "majority logic decoding". This technique has representation that makes them efficient to work with. Future work will consider doing the following modifications to the proposed method:

- Investigating the proposed method on color images.
- modifying the proposed approach to embed image inside another image.
- Preserve the secret message even if we do some transformations on the image like rotation, scaling compression.
- Relate the encryption process with steganography in which we encrypt the message before embedding it inside the image in order to increase the security of the proposed method.

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