Journal of Information Hiding and Multimedia Signal Processing ©2016 ISSN 2073-4212
Ubiquitous International Volume **7**, Number **5**, September **2016**

Optimal Beamforming Design for Secure Transmission with Channel Covariance Uncertainty

Yonghong Tan

Experimental Training Center
School of Electronics and Information Engineering
Hunan University of Science and Engineering
YongZhou City, Hunan Province, China
hunantanyonghong@sina.com

Xuhui Wang

Henan Institute of Engineering
Henan, Zhengzhou 451191, China

Xiangdong Yin

School of Electronics and Information Engineering
Hunan University of Science and Engineering
YongZhou City, Hunan Province, China
yinxiangdongxz@163.com

Received March, 2016; revised June, 2016

Abstract. *Physical layer security has attracted substantial research interests in recent years. The introduction of cooperative jamming (CJ) into physical layer security can improve transmission security through generating jamming signals with the assistance of external helpers. In this paper, we study robust beamforming design for multiple-input single-output (MISO) wiretap channels under CJ by exploiting imperfect channel state information (CSI), in which the source employs transmit beamforming for delivering information to the destination and the helper generates Gaussian noise to confuse the eavesdropper. We assume that all the wireless channels from the source/helper to the destination/eavesdropper are subject to quadratic channel uncertainty. Our objective is to minimize the total transmit power at both the source and the helper subject to the minimum signal-to-interference-plus-noise ratio (SINR) constraint at the destination and the maximum SINR constraint at the eavesdropper. Relying on the techniques of semi-definite relaxation (SDR) and S-procedure, we obtain the optimal robust beamforming solution. Simulation results show the effectiveness of our proposed scheme.*
Keywords: Physical layer security, cooperative jamming, robust beamforming, semi-definite relaxation (SDR), quadratic channel uncertainty.

---

1. **Introduction .** Wireless secure transmissions have recently drawn significant attention due to the broadcast nature of radio propagation and the inherent openness of the transmission medium [1]. Traditionally, encryption techniques become more complex and have high computation burden, which is very hard to implement in practical systems. Alternatively, physical layer security has been recognized as a prominent technique to realize secure communication by exploiting wireless channel fading [2, 3]. In particular, multi-antenna techniques are most promising to further improve the physical layer security by employing spatial degrees of freedom [4–7].

Recently, physical layer security has received an intensity of attention and various physical layer security techniques were proposed to realize secure communications. Artificial noise (AN) was an effective way by sending artificially generated noise at the source to improve the physical layer secrecy performance, several AN-aided works have been investigated in [5], [6], [8]. Also, it has been extended to the cooperative jamming (CJ) schemes recently for the secure transmission system with external helpers [9–11]. The basic idea of CJ is to generate jamming signals independent of the source message by employing friendly helpers (a.k.a. relay nodes or jammers) to interfere eavesdroppers. To explore the both benefits of multi-antenna and CJ techniques, multi-antenna CJ systems have been investigated in [11], [12]. Specifically, [11] investigated the secrecy capacity for Gaussian multiple-input multiple-output (MIMO) wiretap channels with an external helper. [12] considered a cooperative wireless networks with one or more eavesdroppers, in which the optimal transmit beamforming weights at the cooperating nodes were designed to maximize the achievable secrecy rate or minimize the total power. In [11], [12], the global channel state information (CSI) is perfectly known at both the source and the helper.

In practice, it is difficult to obtain perfect CSI at the source and the jammer due to channel estimation and quantization errors [13]. As a result, it is essential to investigate the robust transmission design under imperfect CSI [14], [15]. Robust transmit designs for MISO wiretap channels under both individual and global power constraints for the source and the helper were presented in [14], where transmit covariance matrices at the source and the helper were jointly optimized to maximize the secrecy rate. By extending [14], the authors of [15] investigated secret transmit design for a MISO system by jointly optimizing the beamforming vector at the source and the covariance matrix of jamming signals at the helper. The two aforementioned works only considered imperfect CSI of the source-eavesdropper and the helper-eavesdropper. In addition, robust beamforming design for the amplify-and-forward (AF) and decode-and-forward (DF) relaying secure communications were respectively investigated in [16, 17], where the imperfect CSI of relay-destination and relay-eavesdroppers links was modeled using a norm-bounded CSI error model.

In this paper, we focus on the robust beamforming design for secure MISO communication with the assistance of a jammer, in which the source employs transmit beamforming for delivering information to the destination and the jammer generates Gaussian noise to confuse the eavesdropper. Our objective is to minimize the total transmit power at both the source and the jammer subject to a minimum SINR constraint at the destination and a maximum SINR constraint at the eavesdropper. All the wireless channels from the source/jammer to the destination/eavesdropper are subject to quadratic channel uncertainty. The considered robust beamforming designs can be efficiently solved relying on some subtle transformations e.g., semi-definite relaxation (SDR) and S-procedure.

The rest of this paper is organized as follows. Section 2 introduces the system model and problem formulation. Section 3 investigates the robust beamforming design under channel quadratic uncertainty. Section 4 presents simulation results to validate the effectiveness of the proposed schemes. Finally, Section 5 concludes this paper.

Notations: Throughout this paper, boldface lowercase and uppercase letters denote vectors and matrices, respectively. The transpose, conjugate transpose, rank and trace of a matrix $\mathbf{A}$ are denoted as $\mathbf{A}^T$, $\mathbf{A}^H$, $\mathrm{rank}(\mathbf{A})$ and $\mathrm{tr}(\mathbf{A})$, respectively. $\mathbf{I}$ denotes an identity matrix. $\mathbb{E}[\cdot]$ denotes the expectation operator. $\otimes$ denotes the Kronecker product. $\mathrm{MAT}(\mathbf{A})$ returns an $N \times N$ square matrix from a vector of size N^2. $\mathrm{vec}(\mathbf{A})$ returns a vector by stacking the columns of a matrix $\mathbf{A}$. $\mathbf{A} \succeq 0$ means that $\mathbf{A}$ is a positive semi-definite matrix. $\mathbb{H}_+$ denotes the set of positive semi-definite Hermitian matrix.

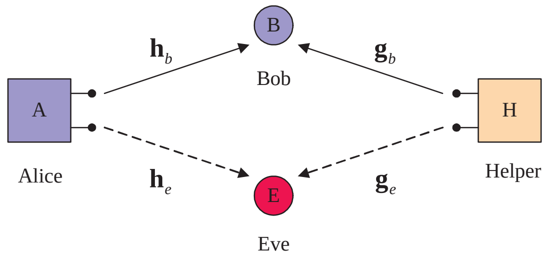

FIGURE 1. System model with friendly helper.

2. **System Model and Problem Statement .** We consider a MISO communication system with a source node (Alice), an external helper (Helper), a destination node (Bob), and an eavesdropper (Eve), as shown in Fig. 1. Alice and Helper are equipped with N_a and N_h transmit antennas, respectively, while both Bob and Eve are equipped with a single receive antenna. CJ is implemented in this system, in which Helper transmits jamming signals to confuse Eve for the purpose of improving the secrecy rate at Bob. It is assumed that Alice and Helper can (partially) obtain the CSI associated with Eve through e.g., estimating channels based on Eve's transmitted signals, while they do not know the location of Eve. As Eve's location changes, the proposed beamforming design is able to adapt dynamically to ensure the secure communications of Alice based on the associated channels. Note that the location-based secure robust beamforming design is beyond the scope of this paper.

Alice performs beamforming to deliver the desired signal to Bob, while keeping the signal private from Eve. Let $\mathbf{w} \in \mathbb{C}^{N_a}$ and s denote the beamforming vector sent by Alice and the desirable data signal for Bob, respectively. Without loss of generality, we assume $\mathbb{E}[ss^T] = 1$. In this paper, Bob is equipped with a single antenna and only a single data stream transmitted in the MISO system is considered. Then, the transmitted signal by Alice is given by

$$\mathbf{x} = \mathbf{w}s. \tag{1}$$

On the other hand, we denote $\mathbf{z}$ as the complex Gaussian jamming signal artificially generated by Helper to confuse Eve. It is also assumed that Helper does not know the confidential message and transmits only a Gaussian jamming signal which is not known at Bob nor Eve. The jamming signal is treated as noise at both Bob and Eve. Note that the jamming signals do not need to be decoded by Bob. As a result, in order to optimize the jamming performance and without loss of generality, $\mathbf{z}$ is modeled as a circularly symmetric complex Gaussian (CSCG) random vector with zero mean and covariance $\mathbf{W}_\mathbf{z} \succeq \mathbf{0}$, i.e., $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \mathbf{W}_z)$. Then, the received signals at Bob and Eve are denoted as follows

$$y_b = \mathbf{h}_b^H \mathbf{x} + \mathbf{g}_b^H \mathbf{z} + n_b, \tag{2}$$

and

$$y_e = \mathbf{h}_e^H \mathbf{x} + \mathbf{g}_e^H \mathbf{z} + n_e, \tag{3}$$

respectively, where $\mathbf{h}_b \in \mathbb{C}^{N_a \times 1}$ and $\mathbf{h}_e \in \mathbb{C}^{N_a \times 1}$ denote the channel vectors from Alice to Bob and Eve, and $\mathbf{g}_b \in \mathbb{C}^{N_h \times 1}$ and $\mathbf{g}_e \in \mathbb{C}^{N_h \times 1}$ denote the channel vectors from Alice to Bob and Eve, respectively. n_b and n_e are the complex Gaussian noises with zero mean and variances σ_b^2 and σ_e^2, respectively, i.e., $n_b \sim \mathcal{CN}(0, \sigma_b^2)$ and $n_e \sim \mathcal{CN}(0, \sigma_e^2)$. Note that the first term of the right-hand side of (2) is the signal intended for Bob, while the second term is the interference signal from Helper, and the remaining term is background noise.

Accordingly, the received SINR at Bob and Eve are, respectively, given by

$$\Gamma_b = \frac{\mathbb{E}[|\mathbf{h}_b^H \mathbf{w} s|^2]}{\mathbb{E}[|\mathbf{g}_b^H \mathbf{z}|^2] + \sigma_b^2} = \frac{\mathbf{h}_b^H \mathbf{w}\mathbf{w}^H \mathbf{h}_b}{\mathbf{g}_b^H \mathbf{W}_z \mathbf{g}_b + \sigma_b^2}, \tag{4}$$

$$\Gamma_e = \frac{\mathbb{E}[|\mathbf{h}_e^H \mathbf{w} s|^2]}{\mathbb{E}[|\mathbf{g}_e^H \mathbf{z}|^2] + \sigma_e^2} = \frac{\mathbf{h}_e^H \mathbf{w}\mathbf{w}^H \mathbf{h}_e}{\mathbf{g}_e^H \mathbf{W}_z \mathbf{g}_e + \sigma_e^2}. \tag{5}$$

Due to the fact that the information rate is monotonically increasing with respect to the SINR at Bob but decreasing in that at Eve, the minimum SINR target at Bob and maximum SINR target at Eve as the performance metrics will be chosen as the performance metrics alternatively [17]. By carefully optimizing the two SINR targets, the desirable secure rate and outage probability can be achieved. Let γ_b denotes the minimum SINR target at Bob and γ_e denotes the maximum SINR target at Eve, respectively. Our objective is to minimize the total transmit power at both Alice and Helper (i.e., $\mathbb{E}(\|\mathbf{x}\|^2) + \mathbb{E}(\|\mathbf{z}\|^2) = \mathrm{tr}(\mathbf{w}\mathbf{w}^H) + \mathrm{tr}(\mathbf{W}_z)$), subject to the minimum SINR constraint γ_b at Bob and the maximum SINR constraint γ_e at Eve, respectively. As a result, the optimization problem is formulated as

$$\begin{aligned} \min_{\mathbf{w}, \mathbf{W}_z} \quad & \mathrm{tr}(\mathbf{w}\mathbf{w}^H) + \mathrm{tr}(\mathbf{W}_z) \\ \text{s.t.} \quad & \Gamma_b \geq \gamma_b, \\ & \Gamma_e \leq \gamma_e, \\ & \mathbf{W}_z \succeq \mathbf{0}. \end{aligned} \tag{6}$$

The problem (6) is a joint optimization for the beamforming vector $\mathbf{w}$ at Alice and covariance matrix $\mathbf{W}_z$ of jamming signal at Helper. Similar to [18], when perfect CSI is available at both Alice and Helper, problem (6) can be efficiently handled by the SDR technique. Nevertheless, when the CSI is not perfectly known, how to solve problem (6) is a challenging task, which will be addressed in the following two sections by considering channel vector and channel quadratic uncertainties, respectively.

3. Robust Beamforming Design .

In practical communication systems, the perfect CSI is in general unavailable at the BS, due to imperfect channel estimation and quantization as well as feedback errors. In this section, the robust beamforming design with quadratic channel uncertainty is studied [21]. For example, in the fast-fading scenario, the second order statistics of the channel change significantly more slowly compared to the channel itself. As a result, estimating the channels in their quadratic forms (or its covariance) requires much less feedback and thus is generally more practical.

3.1. Quadratic Channel Uncertainty.

In this section, we consider the wireless channels are estimated by Alice and Helper in quadratic forms. In this case, we model the quadratic channel uncertainty as follows by using an ellipsoid model [21]

$$\mathbf{H}_b \triangleq \mathbf{h}_b \mathbf{h}_b^H = \tilde{\mathbf{H}}_b + \Delta\mathbf{H}_b \tag{7}$$

$$\mathbf{H}_e \triangleq \mathbf{h}_e \mathbf{h}_e^H = \tilde{\mathbf{H}}_e + \Delta\mathbf{H}_e \tag{8}$$

$$\mathbf{G}_b \triangleq \mathbf{g}_b \mathbf{g}_b^H = \tilde{\mathbf{G}}_b + \Delta\mathbf{G}_b \tag{9}$$

$$\mathbf{G}_e \triangleq \mathbf{g}_e \mathbf{g}_e^H = \tilde{\mathbf{G}}_e + \Delta\mathbf{G}_e \tag{10}$$

where $\tilde{\mathbf{H}}_b$, $\tilde{\mathbf{H}}_e$, $\tilde{\mathbf{G}}_b$, and $\tilde{\mathbf{G}}_e$ are the estimated channels in quadratic forms from Alice to Bob and from Alice to Eve, from Helper to Bob, from Helper to Bob, respectively. $\Delta\mathbf{H}_b$,

$\Delta\mathbf{H}_e$, $\Delta\mathbf{G}_b$, and $\Delta\mathbf{G}_e$ denote the CSI error which are bounded by the regions, denoted as $\tilde{\mathcal{H}}_b$, $\tilde{\mathcal{H}}_e$, $\tilde{\mathcal{G}}_b$, and $\tilde{\mathcal{G}}_e$, respectively, i.e.,

$$\tilde{\mathcal{H}}_b = \begin{cases} \tilde{\mathbf{H}}_b + \Delta\mathbf{H}_b \succeq 0,\ \Delta\mathbf{H}_b^H = \Delta\mathbf{H}_b \\ \mathrm{tr}(\Delta\mathbf{H}_b^H \mathbf{C}_1 \Delta\mathbf{H}_b) \leq 1 \end{cases} \tag{11}$$

$$\tilde{\mathcal{H}}_e = \begin{cases} \tilde{\mathbf{H}}_e + \Delta\mathbf{H}_e \succeq 0,\ \Delta\mathbf{H}_e^H = \Delta\mathbf{H}_e \\ \mathrm{tr}(\Delta\mathbf{H}_e^H \mathbf{C}_2 \Delta\mathbf{H}_e) \leq 1 \end{cases} \tag{12}$$

$$\tilde{\mathcal{G}}_b = \begin{cases} \tilde{\mathbf{G}}_b + \Delta\mathbf{G}_b \succeq 0,\ \Delta\mathbf{G}_b^H = \Delta\mathbf{G}_b \\ \mathrm{tr}(\Delta\mathbf{G}_b^H \mathbf{C}_3 \Delta\mathbf{G}_b) \leq 1 \end{cases} \tag{13}$$

$$\tilde{\mathcal{G}}_e = \begin{cases} \tilde{\mathbf{G}}_e + \Delta\mathbf{G}_e \succeq 0,\ \Delta\mathbf{G}_e^H = \Delta\mathbf{G}_e \\ \mathrm{tr}(\Delta\mathbf{G}_e^H \mathbf{C}_4 \Delta\mathbf{G}_e) \leq 1 \end{cases} \tag{14}$$

where $\mathbf{C}_i \succ 0, i \in \{1, 2, 3, 4\}$ specify the quality of CSI that are assumed to be known by Alice and Helper. $\mathbf{C}_i$'s can be decomposed as

$$\mathbf{C}_i = \mathbf{C}_i^{\frac{1}{2}} \mathbf{C}_i^{\frac{1}{2}},\ \forall i. \tag{15}$$

Under this model, the SINR Γ_b at Bob in (4) and Γ_e at Eve in (5) can be rewritten, respectively, as

$$\Gamma_b' = \frac{\mathrm{tr}((\tilde{\mathbf{H}}_b + \Delta\mathbf{H}_b)\mathbf{w}\mathbf{w}^H)}{\mathrm{tr}((\tilde{\mathbf{G}}_b + \Delta\mathbf{G}_b)\mathbf{W}_z) + \sigma_b^2}, \tag{16}$$

$$\Gamma_e' = \frac{\mathrm{tr}((\tilde{\mathbf{H}}_e + \Delta\mathbf{H}_e)\mathbf{w}\mathbf{w}^H)}{\mathrm{tr}((\tilde{\mathbf{G}}_e + \Delta\mathbf{G}_e)\mathbf{W}_z) + \sigma_e^2}. \tag{17}$$

In this paper, we jointly optimize the transmit beamforming vector $\mathbf{w}$ at Alice and the covariance matrix $\mathbf{W}_z$ at Helper under the worst-case SINR constraints at Bob and Eve. Accordingly, problem (6) is equivalently rewritten in terms of channel quadratic uncertainty as

$$\begin{aligned} \min_{\mathbf{w}, \mathbf{W}_z} \quad & \mathrm{tr}(\mathbf{w}\mathbf{w}^H) + \mathrm{tr}(\mathbf{W}_z) \\ \text{s.t.} \quad & \min_{\Delta\mathbf{H}_b \in \tilde{\mathcal{H}}_b, \Delta\mathbf{G}_b \in \tilde{\mathcal{G}}_b} \frac{\mathrm{tr}((\tilde{\mathbf{H}}_b + \Delta\mathbf{H}_b)\mathbf{w}\mathbf{w}^H)}{\mathrm{tr}((\tilde{\mathbf{G}}_b + \Delta\mathbf{G}_b)\mathbf{W}_z) + \sigma_b^2} \geq \gamma_b, \\ & \max_{\Delta\mathbf{H}_e \in \tilde{\mathcal{H}}_e, \Delta\mathbf{G}_e \in \tilde{\mathcal{G}}_e} \frac{\mathrm{tr}((\tilde{\mathbf{H}}_e + \Delta\mathbf{H}_e)\mathbf{w}\mathbf{w}^H)}{\mathrm{tr}((\tilde{\mathbf{G}}_e + \Delta\mathbf{G}_e)\mathbf{W}_z) + \sigma_e^2} \leq \gamma_e, \\ & \mathbf{W}_z \succeq 0. \end{aligned} \tag{18}$$

3.2. **Robust Beamforming Design.** In this subsection, we employ the SDR technique to solve problem (18). Define $\mathbf{W}_x = \mathbf{w}\mathbf{w}^H$ with $\mathbf{W}_x \succeq \mathbf{0}$ and $\mathrm{rank}(\mathbf{W}_x) \leq 1$. Then, by substituting $\mathbf{w}\mathbf{w}^H$ as $\mathbf{W}_x$ and introducing the auxiliary variables m, t, u, v, problem (18) can be equivalently reformulated as

$$\min_{\mathbf{W}_x,\mathbf{W}_z,m,t,u,v} \quad \mathrm{tr}(\mathbf{W}_x)+\mathrm{tr}(\mathbf{W}_z) \tag{19a}$$

$$\text{s.t.} \quad \min_{\Delta\mathbf{H}_b\in\tilde{\mathcal{H}}_b} \mathrm{tr}((\tilde{\mathbf{H}}_b+\Delta\mathbf{H}_b)\mathbf{W}_x)\geq m, \tag{19b}$$

$$\max_{\Delta\mathbf{G}_b\in\tilde{\mathcal{G}}_b} \mathrm{tr}((\tilde{\mathbf{G}}_b+\Delta\mathbf{G}_b)\mathbf{W}_z)+\sigma_b^2\leq t, \tag{19c}$$

$$\max_{\Delta\mathbf{H}_e\in\tilde{\mathcal{H}}_e} \mathrm{tr}((\tilde{\mathbf{H}}_e+\Delta\mathbf{H}_e)\mathbf{W}_x)\leq u, \tag{19d}$$

$$\min_{\Delta\mathbf{G}_e\in\tilde{\mathcal{G}}_e} \mathrm{tr}((\tilde{\mathbf{G}}_e+\Delta\mathbf{G}_e)\mathbf{W}_z)+\sigma_e^2\geq v, \tag{19e}$$

$$m\geq\gamma_b t,$$
$$u\leq\gamma_e v,$$
$$m\geq 0, t\geq 0, u\geq 0, v\geq 0,$$
$$\mathbf{W}_x\succeq\mathbf{0}, \mathbf{W}_z\succeq\mathbf{0},$$
$$\mathrm{rank}(\mathbf{W}_x)\leq 1. \tag{19f}$$

Problem (19) is challenging to be solved due to the semi-infinite constraints (19b)-(19e) and non-convex rank-one constraint (19f). To make problem (19) tractable, we first transform the constraints (19b)-(19e) into convex forms. Consider the constraint (19b), for which we have the following lemma.

Lemma 2: *By defining* $\tilde{\mathbf{C}}_1=\mathbf{I}\otimes\mathbf{C}_1+\mathbf{C}_1^H\otimes\mathbf{I}$*, we have*

$$\begin{aligned}&\min_{\Delta\mathbf{H}_b\in\tilde{\mathcal{H}}_b} \mathrm{tr}((\tilde{\mathbf{H}}_b+\Delta\mathbf{H}_b)\mathbf{W}_x)\\ &=\mathrm{tr}(\tilde{\mathbf{H}}_b\mathbf{W}_x)-2\|\mathbf{C}_1^{\frac{1}{2}}\mathrm{MAT}(\tilde{\mathbf{C}}_1^{-1}\mathrm{vec}(\mathbf{W}_x))\|\geq m.\end{aligned} \tag{20}$$

Proof: To prove (20), we first prove the following optimization problem

$$\min_{\Delta\mathbf{H}_b\in\tilde{\mathcal{H}}_b} \quad \mathrm{tr}(\Delta\mathbf{H}_b\mathbf{W}_x)$$
$$\text{s.t.} \quad \begin{cases}\Delta\mathbf{H}_b=\Delta\mathbf{H}_b^H\\ \mathrm{tr}(\Delta\mathbf{H}_b\mathbf{B}_b\Delta\mathbf{H}_b^H)\leq 1\end{cases} \tag{21}$$

The Lagrangian dual function of problem (21) is given by

$$\mathcal{L}=\mathrm{tr}(\Delta\mathbf{H}_b\mathbf{W}_x)-\lambda(\mathrm{tr}(\Delta\mathbf{H}_b\mathbf{B}_b\Delta\mathbf{H}_b^H)-1) \tag{22}$$

where $\lambda>0$ is the dual variable corresponding to the inequation constraint. By taking the first derivative of $\mathcal{L}$ with respect to $\Delta\mathbf{H}_b$ and setting the first derivative to be zero, we have

$$\frac{\partial\mathcal{L}}{\partial\Delta\mathbf{H}_b}=-\mathbf{W}_x+\lambda\Delta\mathbf{H}_b\mathbf{B}_b+\Delta\mathbf{B}_b\mathbf{H}_b=0 \tag{23}$$

Using Kronecker operation, the equation (21) can be transformed as

$$\lambda\Delta\mathbf{H}_b=\mathrm{MAT}(\tilde{\mathbf{B}}_b^{-1}\mathrm{vec}(\mathbf{W}_x) \tag{24}$$

Then, the dual objective of (23) is given by

$$\begin{aligned}\mathcal{L}&=2\lambda\mathrm{tr}(\Delta\mathbf{H}_b\mathbf{B}_b\Delta\mathbf{H}_b^H)-\lambda(\mathrm{tr}(\Delta\mathbf{H}_b\mathbf{B}_b\Delta\mathbf{H}_b^H)-1)\\ &=\lambda\mathrm{tr}(\Delta\mathbf{H}_b\mathbf{B}_b\Delta\mathbf{H}_b^H)+\lambda\\ &=2\lambda\end{aligned} \tag{25}$$

where $\mathrm{tr}(\Delta\mathbf{H}_b\mathbf{B}_b\Delta\mathbf{H}_b^H) = 1$ if it is in the optimum point, which in turn implied that $\|\mathbf{B}_e^{\frac{1}{2}}\Delta\mathbf{H}_b\| = 1$. Due to strong duality holds [20], so

$$\lambda = \lambda\|\mathbf{B}_e^{\frac{1}{2}}\Delta\mathbf{H}_b\| = \|\mathbf{B}_b^{\frac{1}{2}}\mathrm{MAT}(\tilde{\mathbf{B}}_b^{-1}\mathrm{vec}(\mathbf{W}_x))\| \tag{26}$$

Consequently, we have

$$\begin{aligned}&\min_{\Delta\mathbf{H}_b\in\tilde{\mathcal{H}}_b} \mathrm{tr}((\tilde{\mathbf{H}}_b+\Delta\mathbf{H}_b)\mathbf{W}_x)\\&= \mathrm{tr}(\tilde{\mathbf{H}}_b\mathbf{W}_x) - 2\|\mathbf{B}_b^{\frac{1}{2}}\mathrm{MAT}(\tilde{\mathbf{B}}_b^{-1}\mathrm{vec}(\mathbf{W}_x))\| \geq m\end{aligned} \tag{27}$$

which completes the proof.

Similarly, by defining $\tilde{\mathbf{C}}_2 = \mathbf{I}\otimes\mathbf{C}_2+\mathbf{C}_2^H\otimes\mathbf{I}$, $\tilde{\mathbf{C}}_3 = \mathbf{I}\otimes\mathbf{C}_3+\mathbf{C}_3^H\otimes\mathbf{I}$, $\tilde{\mathbf{C}}_4 = \mathbf{I}\otimes\mathbf{C}_4+\mathbf{C}_4^H\otimes\mathbf{I}$, respectively, we can obtain the equivalent forms of the constraints (19b)-(19e), i.e.,

$$\begin{aligned}&\max_{\Delta\mathbf{G}_b\in\tilde{\mathcal{G}}_b} \mathrm{tr}((\tilde{\mathbf{G}}_b+\Delta\mathbf{G}_b)\mathbf{W}_z)\\&= \mathrm{tr}(\tilde{\mathbf{G}}_b\mathbf{W}_z) + 2\|\mathbf{C}_3^{\frac{1}{2}}\mathrm{MAT}(\tilde{\mathbf{C}}_3^{-1}\mathrm{vec}(\mathbf{W}_z))\| \leq t\end{aligned} \tag{28}$$

$$\begin{aligned}&\max_{\Delta\mathbf{H}_e\in\tilde{\mathcal{H}}_e} \mathrm{tr}((\tilde{\mathbf{H}}_e+\Delta\mathbf{H}_e)\mathbf{W}_x)\\&= \mathrm{tr}(\tilde{\mathbf{H}}_e\mathbf{W}_x) + 2\|\mathbf{C}_2^{\frac{1}{2}}\mathrm{MAT}(\tilde{\mathbf{C}}_2^{-1}\mathrm{vec}(\mathbf{W}_x))\| \leq u\end{aligned} \tag{29}$$

$$\begin{aligned}&\min_{\Delta\mathbf{G}_e\in\tilde{\mathcal{G}}_e} \mathrm{tr}((\tilde{\mathbf{G}}_e+\Delta\mathbf{G}_e)\mathbf{W}_z)\\&= \mathrm{tr}(\tilde{\mathbf{G}}_e\mathbf{W}_z) - 2\|\mathbf{C}_4^{\frac{1}{2}}\mathrm{MAT}(\tilde{\mathbf{C}}_4^{-1}\mathrm{vec}(\mathbf{W}_z))\| \geq v.\end{aligned} \tag{30}$$

By replacing (19b)-(19e) as (20)-(30), problem (19) is equivalently rewritten as

$$\begin{aligned}\min_{\mathbf{W}_x,\mathbf{W}_z,m,t,u,v} \quad & \mathrm{tr}(\mathbf{W}_x)+\mathrm{tr}(\mathbf{W}_z)\\ \text{s.t.} \quad & (20),(28),(29),(30),\\ & m \geq \gamma_b t,\\ & u \leq \gamma_e v,\\ & m\geq 0, t\geq 0, u\geq 0, v\geq 0,\\ & \mathbf{W}_x \succeq 0, \mathbf{W}_z\succeq 0,\\ & \mathrm{rank}(\mathbf{W}_x)\leq 1.\end{aligned} \tag{31}$$

Problem (31) is still non-convex due to the rank-one constraint of $\mathbf{W}_x$. To meet this challenge, we here use the SDR technique. By dropping the rank-one constraint of $\mathbf{W}_x$, problem (31) becomes an SDP, which can be solved effectively via using the standard interior point method, i.e., CVX [20]. Let the optimal solution of $\mathbf{W}_x$ to the SDR of (31) be denoted by $\mathbf{W}_x^*$. If $\mathbf{W}_x^*$ is of rank-one, then the SDR of (31) is tight, for which the solution is indeed the optimal solution to the original problem (31). Alternatively, the Gaussian randomization technique can be applied to generate the approximate solution. Unfortunately, we cannot rigorously prove such tightness analytically. More surprisingly, in the simulations we find that the obtained optimal solution $\mathbf{W}_x^*$ to the SDR of (31) is always of rank-one. In this case, the optimal robust beamforming vector at Alice can be exactly obtained as $\mathbf{w}^*$ via employing the eigenvalue decomposition $\mathbf{W}_x^* = \mathbf{w}^*(\mathbf{w}^*)^H$.

4. **Simulation results .** In this section, we present simulation results to show the performance of the proposed robust beamforming design under channel quadratic uncertainties. We assume Alice and Helper both have four antennas, i.e., $N_a = N_h = 4$. All the channel coefficients are assumed to be independent and identically distributed (i.i.d.) CSCG random variables with zero mean and unit variance. The background noise power at Bob and Eve are set as $\sigma_b^2 = \sigma_e^2 = 0.01$. The channel uncertainty regions are assumed to be norm-bounded, i.e., $\mathbf{C}_i = \rho^{-1}\mathbf{I}$, $\forall i$, where ρ is the parameter of channel uncertainty and determines the qualities of the CSI. We average the transmit power via conducting 500 randomly generated channel realizations.

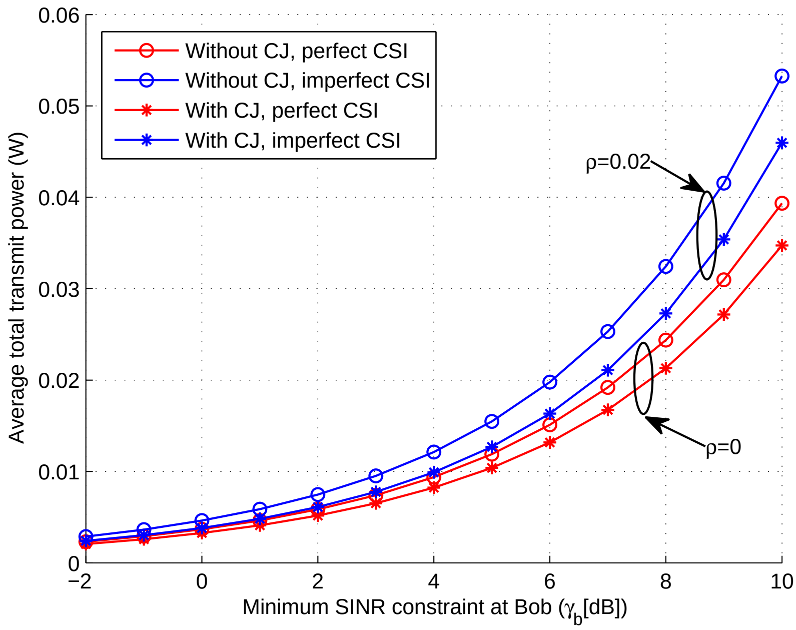

FIGURE 2. Average total transmit power versus the minimum SINR $\gamma_e = 1dB$.

Figs. 2 and 3 show the average total transmit power versus the SINR constraints at Bob and Eve, respectively. Intuitively, we use the scheme without CJ as the benchmark. It is observed that the average total transmit powers for imperfect CSI are higher than those required in the perfect CSI case. As expected, it is also observed from Figs. 2 and 3 that to meet the SINR requirements, the total transmit power with CJ is lower than that without CJ for both perfect and imperfect CSI cases, respectively. This is due to the generated jamming signals from friendly helper degrade the eavesdropper channel.

Fig. 4 depicts the average total transmit power versus the minimum SINR requirement γ_b at Bob for different ρ. It is observed that the total transmit power increases as γ_b increases. Furthermore, as the channel uncertainty increases, higher total transmit power is observed to be required to meet the SINR requirements for the proposed robust beamforming scheme.

Fig. 5 shows the average total transmit power versus the maximum SINR requirement γ_e at Bob for different ρ. It is observed that as γ_e increases, the total transmit power decreases, and the curves tends to become stable when γ_e is large. This is due to the fact that when γ_e is large enough, the SINR constraint at Eve becomes independent of the beamforming design and in such a case the system will be reduced to that without secrecy. By comparing the imperfect CSI case with the perfect CSI case, we can see that secrecy design has a greater impact on large channel uncertainty.

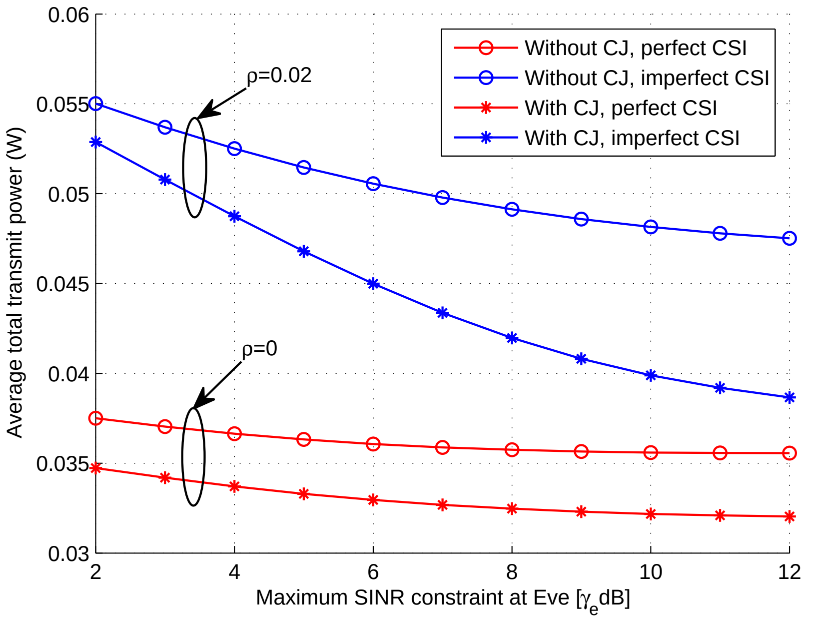

FIGURE 3. Average total transmit power versus the maximum SINR $\gamma_b = 10dB$.

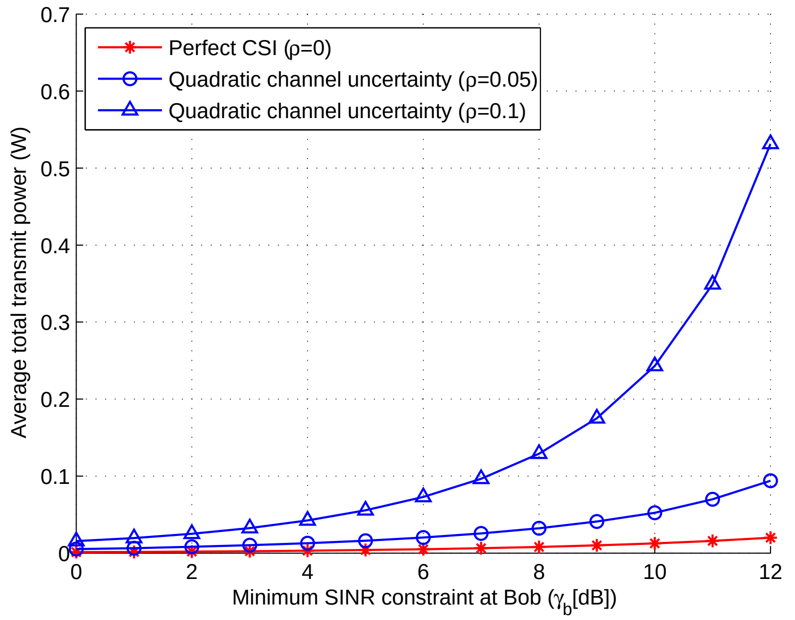

FIGURE 4. Average total transmit power versus the minimum SINR $\gamma_e = 1dB$.

5. **Conclusions .** In this paper, we investigated the robust beamforming design for secure MISO communications with cooperative jamming. We jointly optimized the beamforming vector at the source and the transmitted jamming signals at the helper to minimize their total transmit power while ensuring the worst-case SINR constraints at the destination and at the eavesdropper, in which we took into account the quadratic channel uncertainty with the CSI errors bounded by an ellipsoid model. Relying on the techniques of SDR and S-procedure, we have obtained efficient solution. Simulation results show the effectiveness

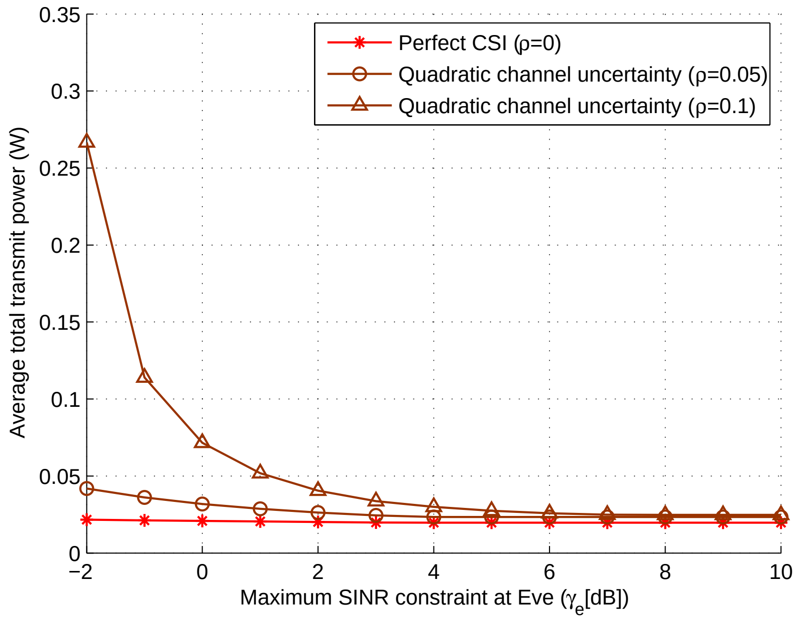

FIGURE 5. Average total transmit power versus the maximum SINR constraint at Eve with the minimum SINR constraint at Bob being $\gamma_b = 10dB$.

of our proposed scheme. In addition, our results provide useful guidelines on the robust beamforming design under quadratic channel uncertainty for multi-antenna secure wireless communications.

Acknowledgment. This work is partially supported by Natural Science Foundation of Hunan Province (2014FJ6095), Science and Technology Project of Hunan Province of China (2014FJ6095), and Science Research Foundation for Distinguished Young Scholars of Hunan Province Education Department (14B070). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

REFERENCES

[1] A. Mukherjee, S. A. A. Fakoorian, J. Huang, and A. L. Swindlehurst, Principles of physical layer security in multiuser wireless networks: A survey, *IEEE Commun. Surveys and Tutorials*, vol. 16, no. 3, pp. 1550–1573, 2014.
[2] Q. Shi, W. Xu, J. Wu, E. Song, and Y. Wang, Secure beamforming for MIMO broadcasting with wireless information and power transfer, *IEEE Trans. Wirel. Commun.*, vol. 4, no. 5, pp. 2841–2853, 2015.
[3] N. Yang and L. Wang and G. Geraci and M. Elkashlan and J. Yuan and M. Di Renzo, Safeguarding 5G wireless communication networks using physical layer security, *IEEE Commun. Mag.*, vol. 53, no. 4, pp. 20–27, 2015.
[4] Y. Zou, J. Zhu, X. Wang. V. C. M. Leung, Improving physical-layer security in wireless communications using diversity techniques, *IEEE Network*, vol. 29, no. 1, pp. 42–48, 2015.
[5] P. H. Lin, S. H. Lai, S. C. Lin, and H. J. Su, On secrecy rate of the generalized artificial-noise assisted secure beamforming for wiretap channels, *IEEE J. Sel. Areas Commun.*, vol. 31, no. 9, pp. 1728–1740, 2013.
[6] Q. Li and W. K. Ma, Spatially selective artificial-noise aided transmit optimization for MISO multi-eves secrecy rate maximization, *IEEE Trans. Signal Process.*, vol. 61, no. 10, pp. 2704–2717, 2013.
[7] M. L. Ammari and P. Fortier, Physical layer security of multiple-input-multiple-output systems with transmit beamforming in Rayleigh fading, *IET Commun.*, vol. 9, no. 8, pp. 1096–1103, 2015.

[8] N. Romero-Zurita, D. McLernon, and M. Ghogho, Physical layer security by robust masked beamforming and protected zone optimisation, *IET Commun.*, vol. 8, no. 8, pp. 1248–1257, 2014.
[9] R. Bassily, E. Ekrem, X. He, E. Tekin, J. Xie, M. Bloch, S. Ulukus, and A. Yener, Cooperative security at the physical layer: A summary of recent advances, *IEEE Signal Proc. Mag.*, vol. 30, no. 5, pp. 16–28, 2013.
[10] H. Long, W. Xiang, J. Wang, Y. Zhang, and W. Wang, Cooperative jamming and power allocation with untrusty two-way relay nodes, *IET Commun.*, vol. 8, no. 13, pp. 2290–2297, 2014.
[11] S. A. A. Fakoorian and A. L. Swindlehurst, Solutions for the MIMO Gaussian wiretap channel with a cooperative jammer, *IEEE Trans. Signal Process.*, vol. 59, no. 10, pp. 5013–5022, 2011.
[12] J. Li, A. P. Petropulu, and S. Weber, On cooperative relaying schemes for wireless physical layer security, *IEEE Trans. Signal Process.*, 2011, 59, (10), pp. 4985–4997
[13] A. Özcelikkale and T. M. Duman, Cooperative precoding and artificial noise design for security over interference channels, *IEEE Signal Process. Lett.*, vol. 22, no. 12, pp. 2234–2238, 2015.
[14] Z. Chu, H. Xing, M. Johnston, and S. L. Goff, Secrecy rate optimizations for a MISO secrecy channel with multiple multiantenna eavesdroppers, *IEEE Trans. Wireless Commun.*, vol. 15, no. 1, pp. 283–297, 2016.
[15] H. Ma and P. Ma, Robust QoS-based transmit beamforming design in MISO wiretap channels, *Proc. of IEEE Wireless Communications and Signal Processing (WCSP)*, 2012.
[16] S. Vishwakarma and A. Chockalingam, Amplify-and-forward relay beamforming for secrecy with cooperative jamming and imperfect CSI, *Proc. of IEEE International Conference on Communications (ICC)*, 2013.
[17] A. Sujana, S. Vishwakarma, and A. Chockalingam, Worst case SINR guarantees for secrecy using relay beamforming with imperfect CSI, *Proc. of IEEE Wireless Communications and Networking Conference (WCNC)*, 2014.
[18] J. Yang, I.-M. Kim, and D. I. Kim, Optimal cooperative jamming for multiuser broadcast channel with multiple eavesdroppers, *IEEE Trans. Wireless Commun.*, vol. 12, no. 6, pp. 2840–2852, 2013.
[19] A. Beck and Y. C. Eldar, Strong duality in nonconvex quadratic optimization with two quadratic constraints, *SIAM J. Optim.*, vol. 17, no. 3, pp. 844–860, 2006.
[20] S. Boyd and L. Vandenberghe, *Convex Optimization.* Cambridge, U. K.: Cambridge University Press, 2004.
[21] L. Zhang, L. Jin, W. Luo, Z. Zhong, and D. Yu, Robust artificial noise aided transmit method for multicast MISO wiretap channels with imperfect covariance-based CSI, *Proc. of 81st IEEE Vehicular Technology Conference (VTC-Spring)*, 2015, pp. 1–5.