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A Novel Descending Dimension CKF Algorithm for A Class of Nonlinear System

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Abstract. *Nonlinear filter are applied to target tracking, signal processing, navigation, inertial navigation, attitude estimation and etc. For a class of nonlinear system, how to descend then sample dimension and sample points and then reduce filter calculation are studied. In this paper, a novel descending dimension CKF algorithm for a class of nonlinear system is proposed. And the simulations have performed to verify the effectiveness of the algorithm proposed. And the simulations have performed to verify the effectiveness of the algorithm proposed. The simulations results show the superiority to the traditional CKF algorithm.*
Keywords: Nonlinear system; CKF; Descending dimension

1. **Introduction.** Because of the essential characteristics of nonlinear system, the algorithms for nonlinear system are studied by many researchers. Nonlinear filter algorithms[1-5] are applied in most of real application systems, for example, target tracking, signal processing, navigation system, inertial navigation system, attitude estimation and etc. Therefore, filter algorithm to nonlinear system has important meaning for engineer applications. The aim of state estimation is to estimate the implied states of state system online by the observation with noise. The nonlinear filter algorithm became the most popular central issue recent sixty years. With the development of the computer technology, the nonlinear filter theory achieves the great development, for example, Unscented Kalman filter[6-7], particle filter[8-9] and the novel proposed Cubature Kalman filter[10-13]. In order to overcome the situation of numerical instability and accuracy reduced in UKF high system filter algorithm, Arasaratnam proposed the Cubature Kalman Filter (CKF) algorithm for nonlinear system recently. The proposed algorithm is easy to implement and has few adjustable parameters and then CKF has more rigorous mathematical theory supports and better convergence.

CKF can solve some problems in UKF and CDKF, so it is widespread concerned by most researchers when CKF is proposed. The CKF algorithm uses the Cubature transformation to probabilistic deduced. Firstly, selecting the Cubature point set to parameterize the mean and variance of probability distributions based on Cubature criterion. Secondly, apply the nonlinear transformation to all of the Cubature point. And then, calculating the deduced approximate Gaussian distribution parameters based on transformed Cubature. The filter processing of CKF and UKF is similar, calculating the transformed point sets to provide the next time system state prediction with the nonlinear system equation transformed by a class of point set with the weight. Therefore, the method avoids the linearization processing and does not depend on the system nonlinear equation. And then, compared with the derivation of the UKF algorithm, CKF algorithm is proposed by strict mathematical derivation based on Bayesian theory and Spherical-Radial Cubature rules.

Dealt with some special nonlinear system, how to descend then sample dimension and sample points and then reduce filter calculation are studied. In this paper, a novel descending dimension CKF algorithm for a class of nonlinear system is proposed.

2. **Problem Statement.** Consider the nonlinear Gaussian system as follows:

$$x_{k+1} = f(x_k) + \Gamma_k w_k \tag{1}$$

$$y_k = h(x_k) + v_k \tag{2}$$

Where, $x_k \in R^n$ is the system state, $y_k \in R^m$ is the system measure, $f(\cdot)$ and $h(\cdot)$ is the system nonlinear transfer function and measurement function respectively, Γ_k is $n \times p$ dimension noise input matrix, $w_k \in R^p$ is the system noise, $v_k \in R^m$ is the measure noise, $w_k \in R^p$ and $v_k \in R^m$ are all the Gaussian white noise and irrelevant, the statistical properties are as follows:

$E(w_k) = q_k, Cov(w_k, w_j) = Q_k \delta_{kj}$

$E(v_k) = r_k, Cov(v_k, v_j) = R_k \delta_{kj}$

Q_k is the non-negative definite symmetric matrix, R_k is the definite symmetric matrix, δ_{kj}.. is kronecker-δ function.

If the state posterior probability density function can be calculated, and then the statistical properties and different properties filter algorithms can be proposed by state probability density function and various estimation rules and approximation methods respectively. Therefore, calculating the state posterior probability density function is pivotal theoretically. However, the Bayesian estimation method in a recursive manner to calculate state posterior probability density function is accepted by many scholars and as the core idea of many filter algorithms. The idea of Bayesian recursive algorithm obtains $p(x_{k+1} \left| Y^{k+1} \right.)$ by prediction update and measure update processing when measure y_{k+1} at is known if state posterior probability density function $p(x_k \left| Y^k \right.)$ at time is known and $Y^{k+1} = \{y_1, y_2, \cdots, y_{k+1}\}$. Where, the prediction updates are state posterior probability density function $p(x_k \left| Y^k \right.)$ and one step state transition probabilities $p(x_{k+1} \left| x_k \right.)$. And the one step prediction probability density function can be obtained as follows:

$$p(x_{k+1} \left| Y^k \right.) = \int p(x_k \left| Y^k \right.) p(x_{k+1} \left| x_k \right.) dx_k \tag{3}$$

When one step prediction probability density function and the measure y_{k+1} are known, the measure update can be obtained. Therefore, the posterior probability density function $p(x_{k+1} \left| Y^{k+1} \right.)$ of x_{k+1} is:

$$p(x_{k+1} \left| Y^{k+1} \right.) = \frac{1}{c_{k+1}} p(x_{k+1} \left| Y^k \right.) p(y_{k+1} \left| x_{k+1} \right.) \tag{4}$$

Where,

$$c_{k+1} = \int p(x_{k+1} \left| Y^k \right.) p(y_{k+1} \left| x_{k+1} \right.) dx_{k+1} \tag{5}$$

The meaning of Bayesian recursive algorithm is to find a method that converting from the probability density function obtained to the recursive mode theoretically. Therefore, the Bayesian estimation aims to guide the nonlinear filter algorithm based on the recursive mode. Therefore, the CKF algorithm based on spherical-radial rule can be realized as follows:

Time Update: On the assumptions that the posterior density function $p(x_{k-1}) = N(\hat{x}_{k-1|k-1}, P_{k-1|k-1})$ at time is known, disintegration the error covariance $P_{k-1|k-1}$ by Cholesky:

$$P_{k-1|k-1} = S_{k-1|k-1} S_{k-1|k-1}^T \tag{6}$$

Calculate then Cubature points $(i = 1, 2, ...m, m = 2n)$:

$$X_{i,k-1|k-1} = S_{k-1|k-1} \xi_i + \hat{x}_{k-1|k-1} \tag{7}$$

Mapping the Cubature points by the state equation:

$$X_{i,k|k-1}^* = f(X_{i,k-1|k-1}) \tag{8}$$

Estimation the state prediction value at time :

$$\hat{x}_{k|k-1} = \frac{1}{m} \sum_{i=1}^{m} X_{i,k|k-1}^* \tag{9}$$

Estimation the state error covariance prediction value at time :

$$P_{k|k-1} = \frac{1}{m} \sum_{i=1}^{m} X_{i,k|k-1}^* X_{i,k|k-1}^{*T} - \hat{x}_{k|k-1} \hat{x}_{k|k-1}^T + Q_{k-1} \tag{10}$$

Measurement Update: Disintegration $P_{k|k-1}$ by Cholesky:

$$P_{k|k-1} = S_{k|k-1} S_{k|k-1}^T \tag{11}$$

Calculate the Cubature points ($i = 1, 2, ...m, m = 2n$):

$$X_{i,k|k-1} = S_{k|k-1}\xi_i + \hat{x}_{k|k-1} \quad (12)$$

Mapping the Cubature points by measurement equation:

$$Z_{i,k|k-1} = h(X_{i,k|k-1}) \quad (13)$$

Estimation the measurement prediction at time :

$$\hat{z}_{k|k-1} = \frac{1}{m}\sum_{i=1}^{m} Z_{i,k|k-1} \quad (14)$$

Estimation the autocorrelation covariance matrix:

$$P_{zz,k|k-1} = \frac{1}{m}\sum_{i=1}^{m} Z_{i,k|k-1}Z_{i,k|k-1}^T - \hat{z}_{k|k-1}\hat{z}_{k|k-1}^T + R_k \quad (15)$$

Estimation correlation covariance matrix:

$$P_{xz,k|k-1} = \frac{1}{m}\sum_{i=1}^{m} X_{i,k|k-1}Z_{i,k|k-1}^T - \hat{x}_{k|k-1}\hat{z}_{k|k-1}^T \quad (16)$$

Estimation the Kalman gain:

$$W_k = P_{xz,k|k-1}P_{zz,k|k-1}^{-1} \quad (17)$$

The state prediction value at time :

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + W_k(z_k - \hat{z}_{k|k-1}) \quad (18)$$

The state error covariance estimation at time :

$$P_{k|k} = P_{k|k-1} - W_k P_{zz,k|k-1}W_k^T \quad (19)$$

From the procedure of the CKF algorithm, we can know that CKF algorithm realizes based on spherical-radial rule of the nonlinear Gaussian filter.

3. **Descending dimension CKF algorithm.** A class of nonlinear system for system (1)-(2) is as follows:

$$x_{k+1} = F_k(\xi_k)x_k + g_k(\xi_k) + w_k \quad (20)$$

$$y_k = H_k x_k + v_k \quad (21)$$

Where, ξ_k is the previous l of state x_k, that is
$x_k = \left[\xi_k^T, \eta_k^T\right]^T$. $E[w_k] = 0$, $E[v_k] = 0$ $Cov[w_k, w_j] = Q_k\delta_{k,j}$, $Cov[v_k, v_j] = R_k\delta_{k,j}$ $Cov[w_k, v_j] = 0$

The system filter equation is as follows if $\hat{x}_{k/k-1}$ and $P_{k/k-1}^x$ can be obtained when the statistical properties $(\hat{x}_{k-1}, P_{k-1})$ of state x at time $k-1$ are known:

$$P_{k/k-1} = P_{k/k-1}^x + Q_{k-1} \quad (22)$$

$$K_k = P_{k/k-1}H_k(H_k P_{k/k-1}H_k^T + R_k)^{-1} \quad (23)$$

$$\hat{x}_k = \hat{x}_{k/k-1} + K_k(y_k - H_k\hat{x}_{k/k-1}) \quad (24)$$

$$P_k = (I - K_k H_k)P_{k/k-1} \quad (25)$$

That needs to obtain $2n$ samples for state x if by applying the regular CKF algorithm for solving $\hat{x}_{k/k-1}$ and $P_{k/k-1}^x$. $\hat{x}_{k/k-1}$ and $P_{k/k-1}^x$ are the Gaussian integral with respect to ξ_{k-1}:

$$\hat{x}_{k/k-1} = \int \Phi_{k-1}(\xi_{k-1})N(\xi_{k-1}; \hat{\xi}_{k-1}, P_{k-1}^{\xi})d\xi_{k-1} \quad (26)$$

$$P_{k/k-1}^x = \int \Psi_{k-1}(\xi_{k-1})N(\xi_{k-1}; \hat{\xi}_{k-1}, P_{k-1}^{\xi})d\xi_{k-1} - \hat{x}_{k/k-1}\hat{x}_{k/k-1}^T \quad (27)$$

Where,

$$\Phi_{k-1}(\xi_{k-1}) = F_{k-1}(\xi_{k-1})\left(\hat{x}_{k-1} + S_{k-1}\begin{bmatrix}(S_{k-1}^{\xi})^{-1}(\xi_{k-1} - \hat{\xi}_{k-1}) \\ 0\end{bmatrix}\right) + g_{k-1}(\xi_{k-1}) \quad (28)$$

$$\Psi_{k-1}(\xi_{k-1}) = \Phi_{k-1}(\xi_{k-1})\Phi_{k-1}^T(\xi_{k-1}) + F_{k-1}(\xi_{k-1})S_{k-1}\begin{bmatrix}0 & 0\\ 0 & I_{n-l}\end{bmatrix}S_{k-1}^T F_{k-1}^T(\xi_{k-1}) \tag{29}$$

It needs $2l$ sample points for ξ_{k-1} in order to approximate the formula (26)-(27) based on spherical-radial rule because ξ_{k-1} is l dimensions vector, therefore,

$$\begin{cases} \chi_{k-1}^i = \hat{\xi}_{k-1} - [\sqrt{lP_{k-1}^{\xi}}]_i & i = 1, \cdots, l \\ \chi_{k-1}^i = \hat{\xi}_{k-1} + [\sqrt{lP_{k-1}^{\xi}}]_i & i = l+1, \cdots, 2l \end{cases} \tag{30}$$

$$\hat{x}_{k/k-1} = \frac{1}{2l}\sum_{i=1}^{2l}\Phi_{k-1}(\chi_{k-1}^i) \tag{31}$$

$$P_{k/k-1}^x = \frac{1}{2l}\sum_{i=1}^{2l}\Psi_{k-1}(\chi_{k-1}^i) - \hat{x}_{k/k-1}\hat{x}_{k/k-1}^T \tag{32}$$

The formulas above (22)-(25) constitute the whole descending dimension CKF algorithm. Consider the measurement is nonlinear mode as follows:

$$x_{k+1} = F_k(\xi_k)x_k + g_k(\xi_k) + w_k \tag{33}$$

$$y_k = H_k(\varsigma_k)x_k + v_k \tag{34}$$

Where, ξ_k is as the system (20)-(21), ς_k is the s elements of state x_k not the previous s elements, therefore,

$$x_k = [x_{k,1}, \cdots, \varsigma_{k,1}, \cdots, \varsigma_{k,2}, \cdots, \varsigma_{k,s}, \cdots, x_{k,n}]^T \tag{35}$$

The descending filter algorithm of the system above is complicated; it can be preceded as two steps.

Step 1: State estimation. That is to obtain $\hat{x}_{k/k-1}$ and $P_{k/k-1}$. Step 2: State update. Regulating the elements order of x_k makes $x'_k = [\varsigma_k^T, \zeta_k^T]^T$ and regulating$P_{k/k-1}$, $H_k(\varsigma_k)$ as $P'_{k/k-1}$, $H'_k(\varsigma_k)$ corresponding. Therefore, $\hat{x}'_k$ and P'_k can be obtained as the following formulas if $\hat{y}_{k/k-1}$, $P_{k/k-1}^{xy}$ and $P_{k/k-1}^y$ are known:

$$P_k^y = P_{k/k-1}^y + R_k \tag{36}$$

$$K_k = P_{k/k-1}^{xy}(P_k^y)^{-1} \tag{37}$$

$$\hat{x}'_k = \hat{x}'_{k/k-1} + K_k(y_k - \hat{y}_{k/k-1}) \tag{38}$$

$$P'_k = P'_{k/k-1} - K_k(P_k^y)^{-1}K_k^T \tag{39}$$

And then,

$$\hat{y}_{k/k-1} = \int H'_k(\varsigma_k)\Theta_k(\varsigma_k)N(\varsigma_k; \hat{\varsigma}_{k/k-1}, P_{k/k-1}^{\varsigma})d\varsigma_k \tag{40}$$

$$P_{k/k-1}^y = \int H'_k(\varsigma_k)\Omega_k(\varsigma_k)(H'_k(\varsigma_k))^T N(\varsigma_k; \hat{\varsigma}_{k/k-1}, P_{k/k-1}^{\varsigma})d\varsigma_k - \hat{y}_{k/k-1}\hat{y}_{k/k-1}^T \tag{41}$$

$$P_{k/k-1}^{xy} = \int \Omega_k(\varsigma_k)(H'_k(\varsigma_k))^T N(\varsigma_k; \hat{\varsigma}_{k/k-1}, P_{k/k-1}^{\varsigma})d\varsigma_k - \hat{x}'_{k/k-1}\hat{y}_{k/k-1}^T \tag{42}$$

$$\Theta_k(\varsigma_k) = \hat{x}'_{k/k-1} + S_{k/k-1}\begin{bmatrix}(S_{k/k-1}^{\varsigma})^{-1}(\varsigma_k - \hat{\varsigma}_{k/k-1})\\ 0\end{bmatrix} \tag{43}$$

$$\Omega_k(\varsigma_k) = \Theta_k(\varsigma_k)\Theta_k^T(\varsigma_k) + S_{k/k-1}\begin{bmatrix}0 & 0\\ 0 & I_t\end{bmatrix}S_{k/k-1}^T \tag{44}$$

Approximation for formulas (40)-(42) based on spherical-radial rule:

$$\begin{cases} \varsigma_{k/k-1}^i = \hat{\varsigma}_{k/k-1} - [\sqrt{sP_{k/k-1}^{\varsigma}}]_i & i = 1, \cdots, s \\ \varsigma_{k/k-1}^i = \hat{\varsigma}_{k/k-1} + [\sqrt{sP_{k/k-1}^{\varsigma}}]_i & i = l+1, \cdots, 2s \end{cases} \tag{45}$$

$$\hat{y}_{k/k-1} = \frac{1}{2s}\sum_{i=1}^{2s} H'_k(\varsigma^i_{k/k-1})\Theta(\varsigma^i_{k/k-1}) \tag{46}$$

$$P^y_{k/k-1} = \frac{1}{2s}\sum_{i=1}^{2s} H_k(\varsigma^i_{k/k-1})\Omega_k(\varsigma^i_{k/k-1})\left(H'_k(\varsigma^i_{k/k-1})\right)^T - \hat{y}_{k/k-1}\hat{y}^T_{k/k-1} \tag{47}$$

$$P^{xy}_{k/k-1} = \frac{1}{2s}\sum_{i=1}^{2s} \Omega_k(\varsigma^i_{k/k-1})\left(H'_k(\varsigma^i_{k/k-1})\right)^T \tag{48}$$

The $\hat{x}'_k$ and P'_k can be solved by integrating (36)-(39) and then regulating them as $\hat{x}_k$ and P_k. Therefore, the whole descending dimension CKF algorithm proposed for a class of nonlinear system can be seen as the formula derived above.

Compared with the regular CKF algorithm, the sample vector dimension of descending dimension CKF algorithm reduce from n to l, the sample points reduce from $2n$ to $2l$ in state step prediction. The sample vector dimension reduces from n to s, sample points reduce from $2n$ to $2s$. Therefore, the algorithm computing load is improved. Besides, the estimation method applies for the spherical-radial rule to approximate the posterior mean and covariance, so the estimation accuracy can reach third-order Taylor series. Therefore, the algorithm proposed is more accuracy than the traditional algorithm.

4. **Simulations and analysis.** The simulation is performed on this section to illustrate the effective of the algorithm proposed. The nonlinear system is as follows:

$$\begin{cases} z_k = \frac{z_{k-1}}{2} + \frac{25z_{k-1}}{1+z^2_{k-1}} + 8\cos(1.2(k-1)) + r_{k-1} \\ x_k = Ax_{k-1} + Bz_k + w_k \\ y_k = Cx_k + v_k \end{cases} \tag{49}$$

Where, the means of r_k ,w_k, v_k are zero respectively and r_k ,w_k, v_k are Gaussian white noise with the variances Q_r, Q_w, R. z_k is the maneuvering target value. $x_k = [x, y, \dot{x}, \dot{y}]^T$ is the position, velocity and their differentials respectively at time k. The system above can be rewritten as follows:

$$\begin{cases} \begin{bmatrix} z_k \\ x_k \end{bmatrix} = F\begin{bmatrix} z_{k-1} \\ x_{k-1} \end{bmatrix} + g(z_{k-1}) + q_{k-1} \\ y_k = H\begin{bmatrix} z_k \\ x_k \end{bmatrix} + v_k \end{cases} \tag{50}$$

where, $F = \begin{bmatrix}\frac{1}{2} & 0\frac{1}{2}B & A\end{bmatrix}$; $g(z_{k-1}) = \begin{bmatrix}1 \\ B\end{bmatrix}\left(\frac{25z_{k-1}}{1+z^2_{k-1}} + 8\cos(1.2(k-1))\right)$; $H = \begin{bmatrix}0 & C\end{bmatrix}$; $q_{k-1} = \begin{bmatrix} r_{k-1} \\ Br_{k-1} + w_{k-1}\end{bmatrix}$.

The related parameters in simulations are as follows:

$$A = \begin{bmatrix} 1 & 0 & 0.1 & 0 \\ 0 & 1 & 0 & 0.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; B = \begin{bmatrix} 1.25 \\ -1.25 \\ 0.25 \\ -0.25 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}; Q_r = 2;$$

$$Q_w = 0.09\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; R = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Define as $x_0 = [20, 30, 1.2, 1]^T$, $z_0 = 0$ and initial variances are as follows respectively:

$$P_{x,0} = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, P_{z,0} = 10.$$

The simulation time is 200s, the simulation initial values are $x_{0/0} = x_0$, $z_{0/0} = z_0$, $P_{x/x,0} = P_{x,0}$, $P_{z/z,0} = P_{z,0}$. Compared the regular CKF algorithm with the descending dimension CKF algorithm under the 100 times Monte Carlo simulations, the compared results of state estimation mean are depicted in Fig.1-3.

It can be seen from Fig.1-Fig.3 that the averaged absolute value error of descending dimension CKF algorithm is smaller than the regular CKF. Therefore, the algorithm proposed is more effective and

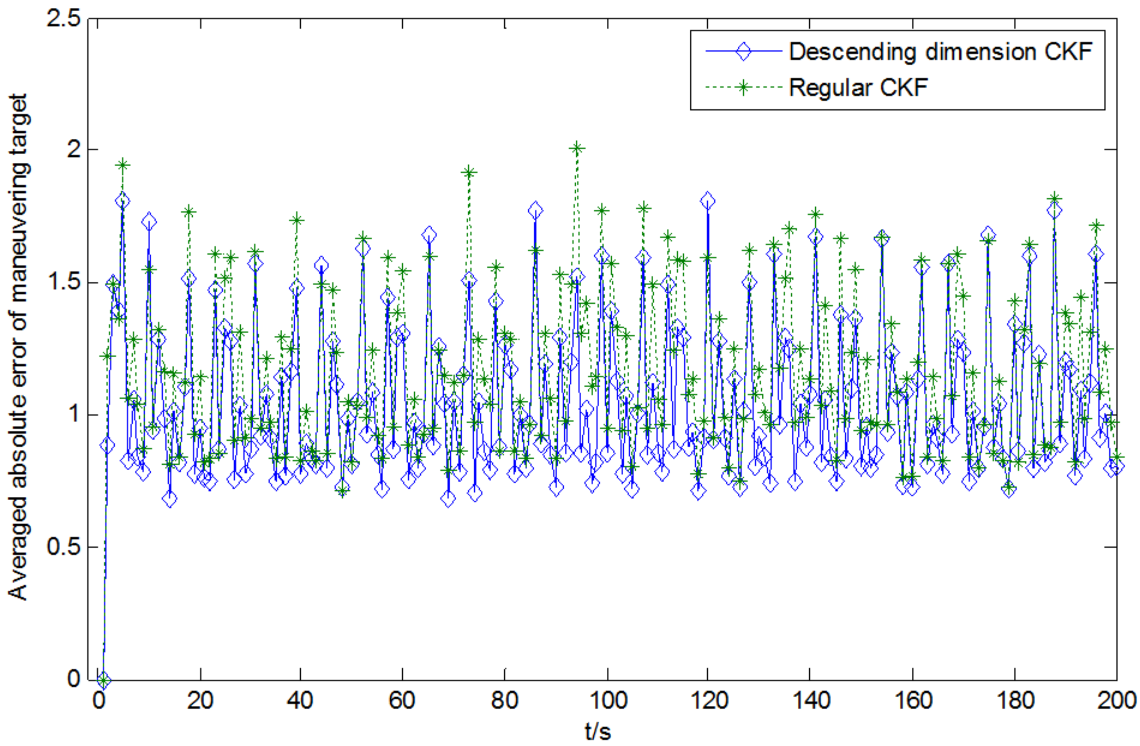

FIGURE 1. Averaged absolute error of maneuvering target

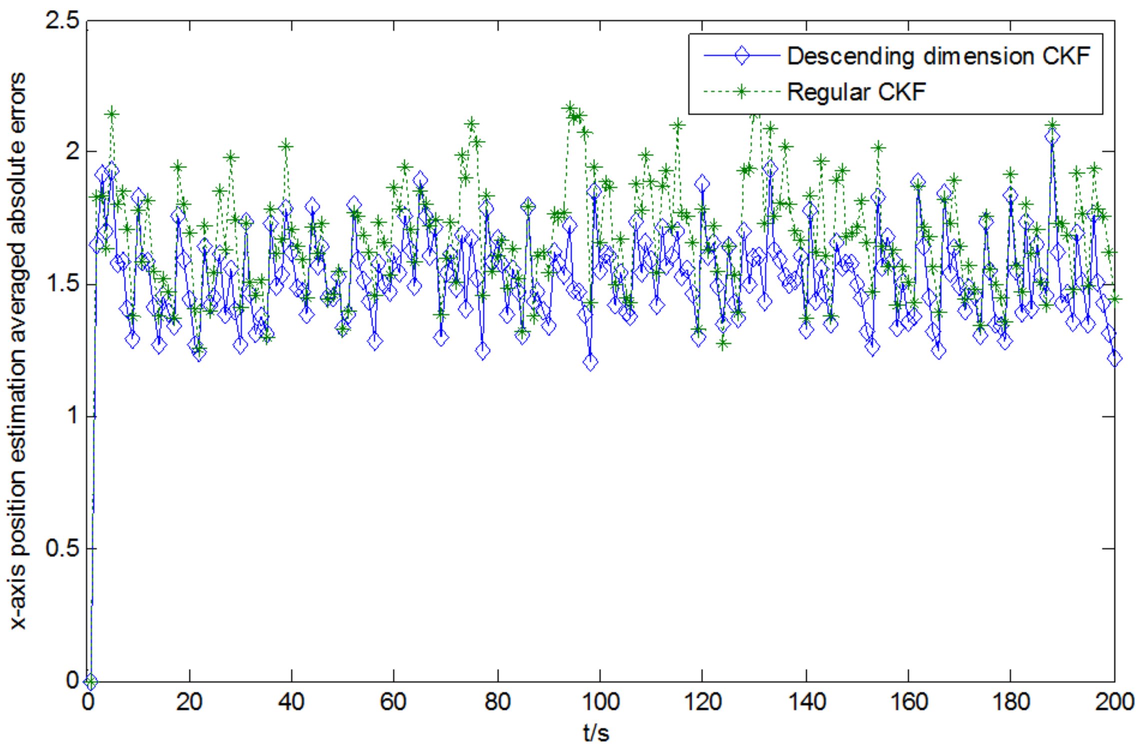

FIGURE 2. x-axis position estimation averaged absolute errors

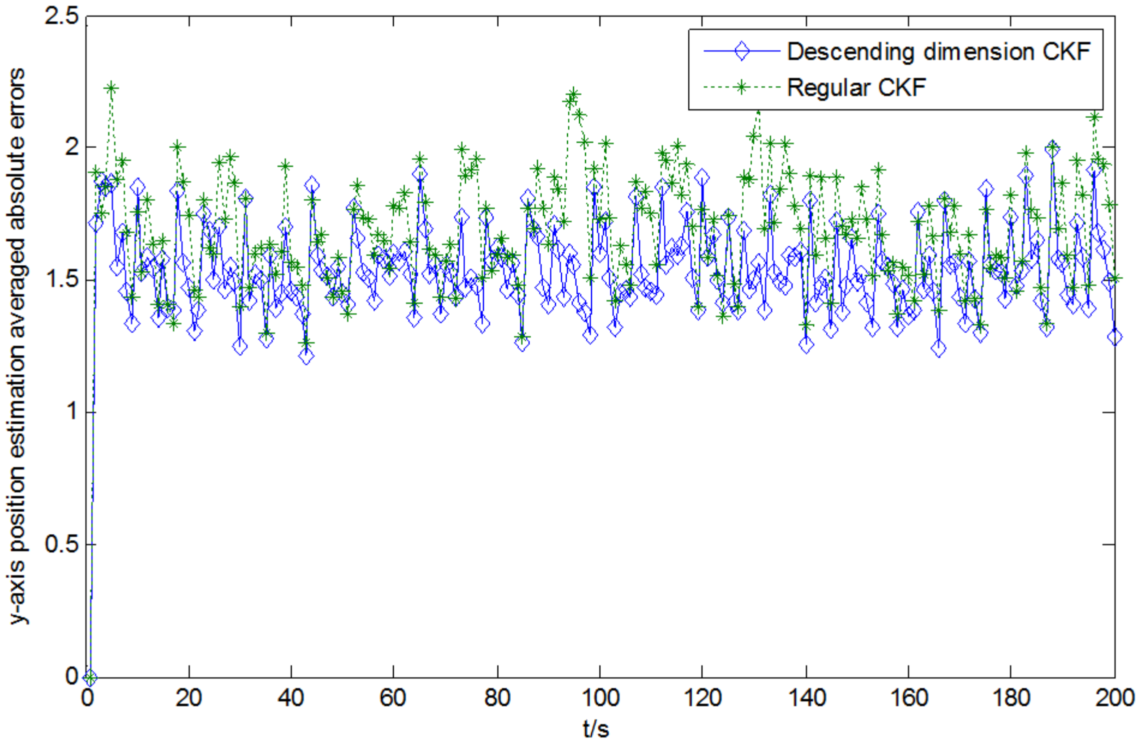

FIGURE 3. y-axis position estimation averaged absolute errors

TABLE 1. State estimation averaged absolute errors

Algorithm	**Regular CKF**		**Descending dimension CKF**	
	mean	**std**	**mean**	**std**
maneuvering target values	1.166	0.3143	1.045	0.2965
x-axis position estimation	1.674	0.2396	1.525	0.1967
y-axis position estimation	1.685	0.2363	1.573	0.1914

predominant than the traditional CKF algorithm. And then, the Table.1 also can verify the effective of the algorithm proposed.

5. **Conclusions.** In this paper, dealt with some special nonlinear system, how to descend then sample dimension and sample points and then reduce filter calculation are studied. A novel descending dimension CKF algorithm for a class of nonlinear system is proposed. And the simulations have performed to verify the effectiveness of the algorithm proposed. The CKF method can descend then sample dimension and sample points and reduce filter calculation for a class of nonlinear system.

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