Journal of Information Hiding and Multimedia Signal Processing ©2017 ISSN 2073-4212
Ubiquitous International Volume **8**, Number **5**, September **2017**

A New Method for Observing the Bifurcation of a Nonlinear System Based on CPLD

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Received February 2017; revised July 2017

Abstract. *In order to observe the stability and running states of a nonlinear system, a new method in this paper is proposed. It can directly make a dual trace oscilloscope observe the bifurcation of the nonlinear system changing with its internal parameters, which is called OOB method for short. Both the partial and the whole bifurcations of the nonlinear system with the change of its internal resistance parameter can be observed by this method. Firstly the complete definition of OOB method is given in the paper, and then it is explained and analyzed detailedly. Then based on CPLD the observing circuit system of this method has been designed, and its working principle is expounded in detail. Finally the direct observation experiments on a double trace oscilloscope are carried out using the designed system. The experimental results show the effectiveness and practicability of the proposed method.*
Keywords: Bifurcation, CPLD, Dual trace oscilloscope, OOB method

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1. **Introduction.** Since the bifurcation of a nonlinear system is the embodiment of the system being in a unstable state under certain conditions, it is very useful to use bifurcation phenomenon to research the stability of the nonlinear system changing with its parameters [1]-[3]. Therefore, bifurcation becomes one of the most important methods to research the stability and characteristics of a nonlinear system, and has been used in many research fields [4]-[9]. In practice, when some internal parameter of a system is changed, we often need to know whether the stability and running state of the system will be changed and whether it can work normally. If the bifurcation phenomenon of the tested system can be observed directly and quickly, its running states in a particular situation can be known timely, and the problems that may occur in the system can be timely corrected and processed. Thus, a lot of researches are carried out, and people hope there are more methods to observe the stability and the running states of a system. Reference [10] presents the analog electronic circuits of two finite difference equations with quadratic return maps, which can display the bifurcation diagrams of the systems on an inexpensive oscilloscope. Reference [11] implements the circuit systems of observing bifurcation diagrams with digital electronic circuits. However, our paper reports the observation of a bifurcation diagram on an oscilloscope by using CPLD (Complex Programmable Logic

Device), that is because CPLD has the advantages of processing data quickly, high reliability, good cost performance and conveniently control in real time, in addition, it can make the hardware circuit structure simply. Therefore, a new method that can directly make a dual trace oscilloscope observe the bifurcation of a nonlinear system based on CPLD is proposed in this paper, and it is called OOB for short. OOB method can directly observe the partial and the whole bifurcations of the system by using oscilloscope. In this paper, the second chapter gives the definition of OOB method and introduces the principle of showing bifurcation diagram. The third chapter designs a circuit system of observing bifurcation diagram according to OOB method, and discusses the working principle of every module in the system and their functions. Then, in order to prove the validity of OOB method, a typical Chua's circuit [12] is chosen as tested nonlinear system because of its abundant states. The observed bifurcation diagrams of Chua's circuit are given in the fourth chapter. At last, the conclusions are given in the fifth chapter.

2. **Definition of OOB Method.** Definition of OOB method: For any nonlinear definite system which contains resistance parameter (or indirect resistance parameter), the change of its internal parameter can be controlled by a periodic signal $V_C(t)$ produced by CPLD. The period of $V_C(t)$ is T. It synchronously controls to generate two signals, one is parameter signal $R(t) = R(t + nT)$ $(n = 1, 2, 3, ...)$, the other is parameter voltage signal $V_1(R)(V_1(R) \propto R(t))$ which changes with the parameter. Under the control of $V_C(t)$, the tested system generates a signal $V_a(t)$ which changes with $R(t)$. Choose a reference voltage V_F arbitrarily, which satisfys $V_{a\,min}(t) < V_F < V_{a\,max}(t)$ (Where $V_{a\,min}(t)$ and $V_{a\,max}(t)$ are the minimum and maximum of $V_a(t)$. Therefore, V_F must intersect with $V_a(t)$ for $\mathrm{d}V_a(t)/\mathrm{d}t > 0$(or $\mathrm{d}V_a(t)/\mathrm{d}t < 0$) and the intersections correspond to each time for $t_i(t_i = 1, 2, 3, ...)$. Then, a time increment $\Delta t(0 < \Delta t < 0.5T_{vamin})$ form $t_i(t_i = 1, 2, 3, ...)$ is chosen, where T_{vamin} is the minimum period of $V_a(t)$. $V_a(t_i + \Delta t)$ $(t_i = 1, 2, 3, ...)$ corresponding to $t_i + \Delta t(t_i = 1, 2, 3, ...)$ are chosen as the characteristic signals $V_2(R)$ of the tested system. After this, $V_1(R)$ and $V_2(R)$ are sent to the X-axis and Y-axis of a dual trace oscilloscope respectively, the bifurcation of the tested system can be displayed directly.

Unlike linear system, the running trajectory of a nonlinear system is not reversible, so if we want an oscilloscope show a stable bifurcation diagram, a proper scan signal which can reflect internal resistance changes of the tested system must be needed. In the meantime, after changing the internal resistance of the system from small to large, it cannot be changed from large to small again. Otherwise, the oscilloscope may show a disorder image which caused by the inconsistent states. In order not to affect the shown state of the system during the process of parameter change, we should control the internal resistance of the system from maximum back to initial value very quickly. But when the resistance changed from minimum to maximum, it should be controlled very slowly in order to observe the whole process of the system states changing with its resistance clearly. Therefore, the proper period T of the control signal $V_C(t)$ should be selected. Obviously, the smaller the period T is, the better the shown diagram on an oscilloscope is. But T can't be too small because it will make the information shown on the oscilloscope too little, which limits the system resistance parameter, and the range of measuring bifurcation will be narrowed. Since the human eye persistence of vision effect is 0.1s to 0.4s, the period T of $V_C(t)$ is chosen as 0.1s$< T <$0.4s.

Now, we explain the definition and work principle of OOB using Fig.1. The control signal $V_C(t)$ in Fig.1 (a) is generated by CPLD. $V_C(t)$ controls a tested system to generate a resistance signal $R(t)$ which increases and decreases orderly in one period T, as shown in Fig.1(a) and Fig.1(b). At the same time, CPLD also control to generate a parameter signal $V_1(R)$ of sending the X-axis on the oscilloscope, as shown in Fig.1(a) and Fig.1(c). According to OOB definition, we know:

$$V_1(R) \propto R(t) \tag{1}$$

$$R(t) = R(t + nT)(n = 1, 2, 3, ...) \tag{2}$$

Therefore, When $V_C(t)$ is high, it controls to generate a increasing resistance signal $R(t)$ and a increasing parameter voltage signal $V_1(R)$ simultaneously. But when $V_C(t)$ is low, it controls to generate a resistance decrease signal $R(t)$ and a decreasing parameter voltage signal $V_1(R)$ simultaneously. Under the control of $V_C(t)$, both signal $R(t)$ and $V_1(R)$ are generated periodically, which $R(t+nT)(n = 1, 2, 3, ...)$ and $V_1(R) \propto R(t)$. From Fig.1(a), it can be seen that the time is very short during $V_C(t)$ is low, which lets the resistance go back to initial value quickly and avoid influencing the display of bifurcation in the period of callback resistance.

In order to easily understand the principle of OOB, the characteristic signals $V_2(R)$ of the tested system is shown in Fig.1(e), which reflects characteristic states of the system. In the process that the resistance parameter R of the system is changed from R_1 to R_2, to R_3 in a cycle, the states of the system are changed with R from period one to period two and period four, as shown in Fig.1 (d) and Fig.1 (e). That is to say, when R is within $[R_1, R_2]$, the state of the system is period one; when R is within $[R_2, R_3]$, its state is period two; and when R is within $[R_3, R_1]$, its state is the period four. The bifurcation points in the system occur twice in each cycle. When the parameter signal $V_1(R)$ of Fig.1 (d) and characteristic signal $V_2(R)$ of Fig.1 (e) are sent the X-axis and the Y-axis of an oscilloscope respectively, the bifurcation in Fig.1 (f) can be displayed on the screen of the oscilloscope.

For a definite system, when its resistance parameter R is changed from one value to another orderly, the states of the system will be changed in accordance with some regulations. Combining with Fig.2, the principle of extracting the characteristic signal $V_2(R)$ which reflects the characteristic states of the system is described as following:

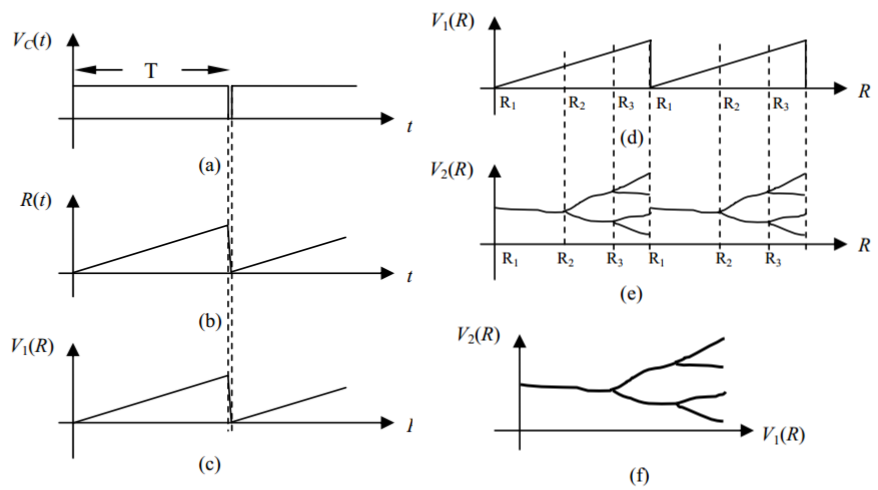


FIGURE 1. The principle of OOB: (a) control signal $V_c(t)$, (b) resistance signal $R(t)$, (c) voltage signal $V_1(R)$, (d) the Example of voltage signal $V_1(R)$, and (e) the Example of characteristic signal $V_2(R)$; (f) bifurcation example

The key of extracting the characteristic signal from the tested system is to decide which of signals to be gotten, that is where we can get the tested signal $V_a(t)$ which reflects the characteristic states of the system. First, we should determine the location of extracting the tested signal. According to OOB definition, we arbitrarily choose reference level V_F within the changing range of $V_a(t)$, which makes

$$V_{a\,min}(t) < V_F < V_{a\,max}(t) \tag{3}$$

where $V_{a\,min}(t)$ and $V_{a\,min}(t)$ are the minimum and maximum of $V_a(t)$ respectively, and V_F with range of (3) can be adjusted according to actual display. The time $t_i(t_i = 1, 2, 3, ...)$ correspond to intersection points of V_F and $V_a(t)$ for $\mathrm{d}V_a(t)/\mathrm{d}t > 0$ are time references. Then, we choose a time increment Δt from $t_i(t_i = 1, 2, 3, ...)$, as shown in Fig. 2(a). Finally, the voltage values of these dots on the tested signal $V_a(t)$ corresponding to $t_i + \Delta t(t_i = 1, 2, 3, ...)$ are the gotten characteristic signal of $V_2(R)$, as shown in Fig. 2(b), and their mathematical forms are

$$V_2(R) = \begin{cases} V_a(t_1 + \Delta) & t \in [t_1 + \Delta t, t_2 + \Delta t] \\ V_a(t_2 + \Delta) & t \in [t_2 + \Delta t, t_3 + \Delta t] \\ V_a(t_3 + \Delta) & t \in [t_3 + \Delta t, t_4 + \Delta t] \\ & \vdots \end{cases} \tag{4}$$

Where R satisfies (2). Considering not to lose the system states, we choose $\Delta t < 0.5T_{vamin}$.

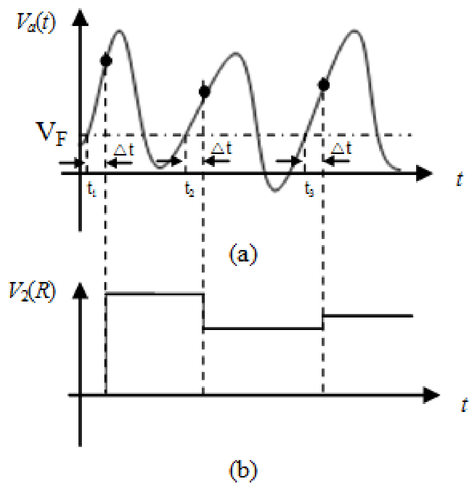


FIGURE 2. Extracting characteristic signals: (a) tested signal $V_a(t)$, and (b) characteristic signal $V_2(R)$

3. **The Design of OOB Observing System.** According to the OOB definition, we design a circuit system to observe the bifurcation of the tested system. The whole hardware module diagram is shown in Fig.3, which includes 4 modules: CPLD signal module, numerical control resistive array module, parameter voltage signal module and extracting characteristic signal module.

CPLD signal module generates three signals, one signal is one bit digital signal $V_C(t)$ which synchronously controls parameter voltage signal module and numerical control resistive array module, which works with another signal for adjusting the tested system parameter, as shown in Fig.1(a), Fig.1(b), Fig.1(c) and Fig.3. At the same time, CPLD signal module also generates one pulse signal $V_{INC}(t)$ for synchronous controls the numerical control resistive array for increasing or decreasing its value orderly, as shown in

Fig.3. The generated third signal is a set of digital signal V_t which is sent to the parameter voltage signal module for converting into the parameter signal $V_1(R)$ under the control of $V_C(t)$.

Numerical control resistive array module consists of a set of digital potentiometers. Its total resistance value is controlled by $V_C(t)$ and $V_{INC}(t)$ generated by CPLD. It must be paralleled with the internal resistance of the system or replace the resistance. The resistance values in numerical control resistive array module directly affect the internal resistance values of the tested system. Therefore, the changes of internal resistance in the system can be implemented. Under the control of $V_C(t)$ and $V_{INC}(t)$, the changes in increase or decrease and the speed of adjusting resistance values can be controlled, as shown in Fig.1, Fig.3 and Fig.4. Fig.4 shows that every time when one pulse of $V_{INC}(t)$ comes during $V_C(t)$ is high, $R(t)$ will increase one unit, and the increased total value of $R(t)$ is determined by the number of pulse $V_{INC}(t)$ and the high level of $V_C(t)$. Similarly, every time when one pulse of $V_{INC}(t)$ comes during $V_C(t)$ is low, $R(t)$ will decrease one unit, and the decreased total value of $R(t)$ is determined by the number of $V_{INC}(t)$ and the low level of $V_C(t)$. During the period of resistance value change from minimum to maximum and from the maximum back to the minimum, the number of $V_{INC}(t)$ appearing in the time of $V_C(t)$ being high must be equal to the number in the time of $V_C(t)$ being low, as shown Fig.4. In addition, numerical control resistive array module have one manual knob using for adjusting the resistance parameter, which can achieve the purpose of fine tuning the range of bifurcation diagram.

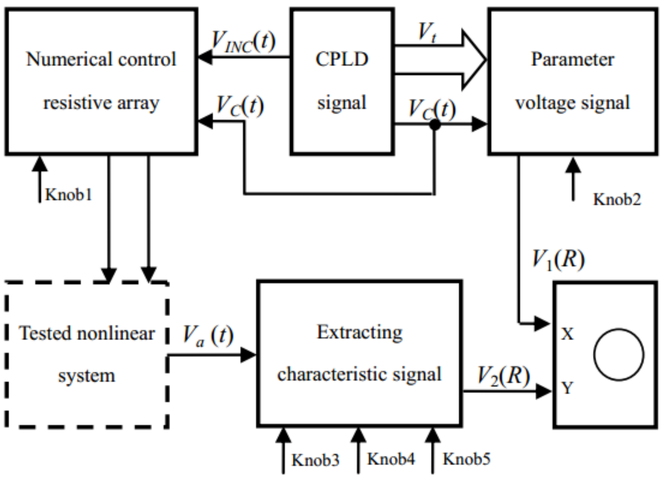


Figure 3. Hardware block diagram of OOB

The purpose of parameter voltage signal module is to generate resistance parameter signal $V_1(R)$ for sending to the X-axis of the oscilloscope, as shown in Fig.1 and Fig.3. Parameter voltage signal module has two sets of input terminals and one output terminal. One input terminal is used for receiving one bit digital control signal $V_C(t)$ generated by CPLD, the other is used for receiving a set of digital signal V_t generated by CPLD. Under the control of $V_C(t)$, a set of digital signal V_t is converted to parameter voltage signal $V_1(R)$ (analog voltage signal) and then $V_1(R)$ be sent to the X-axis of the oscilloscope. Because $V_C(t)$ also controls numerical control resistive array module at the same time, and must satisfy (1), it guarantees that the resistance value of numerical control resistive array is increased during $V_1(R)$ increasing, and the resistance value of numerical control resistive array is decreased during $V_1(R)$ decreasing, as shown in Fig.1(a), Fig.1(c) and

Fig.3. From Fig. 3 can also see that there is one knob 2. It is used for manual fine tuning $V_1(R)$ in the module.

The purpose of extracting characteristic signal module is to obtain the suitable characteristic signal from the tested system, and then to send it to Y-axis terminal on the oscilloscope. It has one input terminal, one output terminal and three manual knobs. The input terminal is used for receiving the output signal $V_a(t)$ of the tested system. In order to extract characteristic signal from $V_a(t)$, according to OOB definition, reference times must be chosen firstly. The knob 3 in the Fig.3 is used for getting the reference time locations, that is to adjust the location of V_F in the Fig.2(a), which let V_F satisfy (3) and $\mathrm{d}V_a(t)/\mathrm{d}t > 0$. The different value of V_F can get the different reference time $t_1, t_2, ...$ and we can get the $V_a(t_i)(i = 1, 2, 3, ...)$ corresponding to reference times $t_i(i = 1, 2, 3, ...)$, as shown in Fig.2. Then we make a delay $\Delta t(\Delta t < 0.5T_{vamin})$ at $t_i(i = 1, 2, 3, ...)$ respectively, and can get the value of those dots in Fig.2, that is characteristic signal $V_2(R) = V_a(t_i + \Delta t)(i = 1, 2, 3, ...)$. Since $V_a(t)$ is nonlinear signal, $V_a(t_1 + \Delta t) \neq V_a(t_2 + \Delta t) \neq ... \neq V_a(t_i + \Delta t)(i = 1, 2, 3, ...)$ as shown in Fig. 2, and the characteristic signal $V_2(R)$ satisfies (4). Then the output terminal outputs $V_2(R)$ to the Y-axis input terminal of the oscilloscope. The knob 4 is used for adjusting the width Δt, and the knob 5 is for adjusting the magnitude of the signal $V_2(R)$ in order to be convenient for observing bifurcation diagram.

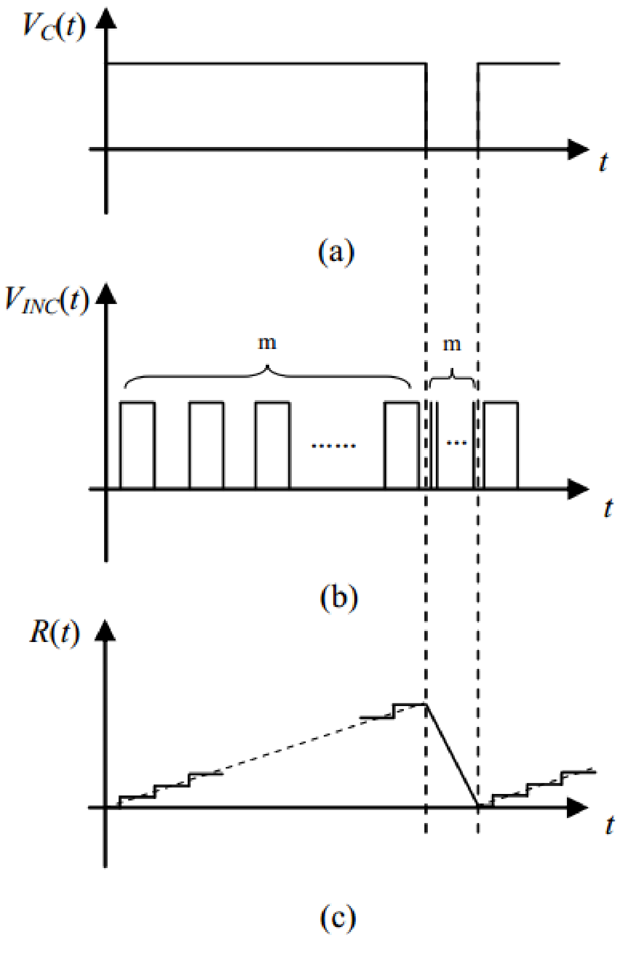


FIGURE 4. Changes of controlling resistance: (a) control signal $V_C(t)$, (b) pulse signal $V_{INC}(t)$, and (c) resistance signal $R(t)$

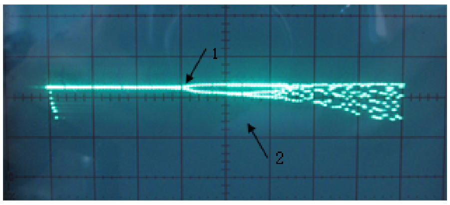
Since numerical control resistive array is connected to the tested system (in parallel with the internal resistance of the tested system or replaced the resistance), the changes of the array resistance are equivalent to the resistance parameter changes of the tested system. If we change the resistance parameter of the tested system, the stability of the system will be changed, and its running states will be influenced. Therefore, the running states of the system are changed with the resistance parameter. Similarly, its output signal $V_a(t)$ also changes with the resistance parameter. If we send the output signal $V_a(t)$ of the tested system to the extracting characteristic signal module, we can extract characteristic signal $V_2(R)$. When $V_1(R)$ and $V_2(R)$ are sent to oscilloscope simultaneously, we can online observe the bifurcation diagram, as shown in Fig.1 (f) and Fig.3.

4. **Observing Experiment.** The typical Chua's circuit system can generate rich states with the changes of its internal linear resistance, such as period one, period two, period 4,..., single-scroll chaos, period 3,..., double-scroll chaos, and so on [12]. And its circuit [12] state equations are:

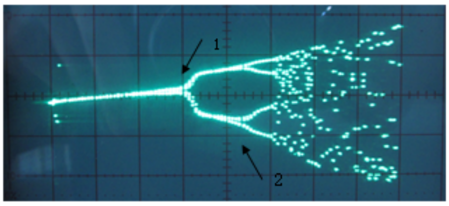
$$\begin{cases} \frac{du_{c1}}{d_t} = \frac{1}{c_1}G(u_{c2} - u_{c1}) - \frac{1}{c_1}g(u_{c1}) \\ \frac{du_{c2}}{d_t} = \frac{1}{c_2}G(u_{c1} - u_{c2}) - \frac{1}{c_2}i_L \\ \frac{di_L}{d_t} = -\frac{1}{L}u_{c2} \end{cases} \tag{5}$$

$$g(u_{C1}) = G_b u_{C1} + \frac{1}{2}(G_a - G_b)[|u_{C1} + E| - |u_{C1} - E|]$$

Let the resistance array module in the observing system designed according to OOB definition replace the linear resistor in the Chua's circuit (5). Under the X-Y model of an common analog dual trace oscilloscope, we can observe the bifurcation diagram of outputted by u_{C2}, as shown in Fig.5.



(a)



(b)

FIGURE 5. Online observing bifurcation diagram: (a) bifurcation diagram before manual adjustment, and (b) bifurcation diagram after manual adjustment

Since the horizontal axis of an oscilloscope represents resistance parameter of a tested system, $V_1(R)$ is the signal sent to X-axis of the oscilloscope. Also since the vertical axis of an oscilloscope represents the state characteristic of the tested system, $V_2(R)$ is the signal sent to X-axis of the oscilloscope. Therefore, that one line is shown on the oscilloscope means that the tested system is in the state of period one; two lines means that the system is in the period two; similarly, three lines means period three,..., and the countless chaotic points represents the non-period chaotic state. The Fig.5 is the bifurcation diagram observed by an analog dual-trace oscilloscope, which we can see the bifurcation point 1 and point 2. From Fig. 5(a) we also can see some different running states of the tested system in the range of different resistance values, such as period one, period two, chaos and so on. In order to observing clearly, we slightly adjust the manually knobs and get the Fig. 5(b). In the Fig. 5(b), we can see the clearer states of period one, period two, period four, chaos, period three, bifurcation points, and so on.

5. **Conclusions.** Whether a new designed nonlinear system can run stably is the concrete reflection of designed rationality and practicability. OOB is exactly suitable for this situation. Therefore, it can be used to test and analyze the stability of a designed new system changing with its parameter before use, and also can analyze the characteristic of an old system and predict faults after using a period of time. The system designed by OOB method is easy to implement. And it not only can predict the stability, running states and bifurcation detail of a nonlinear system changing with its resistance parameter, it but also can used for observing the whole process that a chaotic system changes with its resistance parameter from period into chaos, and it also can be applied to teaching demonstration of the bifurcation of a nonlinear system. The important thing is that it adds one new function of observing the bifurcation phenomenon of a nonlinear system for oscilloscope. Although OOB method in this paper applies to observing the tested system with its resistance change, if we modify slightly, it can be used for observing with the change of capacitors or inductors. Still, using CPLD can be convenient for adjusting the frequency of control signal on the software for adapting to the different kinds of the tested systems. In addition, based on CPLD the observing system designed by OOB method has the advantages of low cost, high measurement speed and strong practicability.

Acknowledgment. This work is supported by the National Natural Science Foundation of China (Grant No. 61471158).

REFERENCES

[1] G. T. Oumbe Tekam, C. A. Kwuimy, and P. Woato, Analysis of tristable energy harvesting system having fractional order viscoelastic material, *CHAOS*, vol.25, no.1, pp.013112- 1-10, 2015.

[2] A. Buscarino, C. Corradino, L. Fortuna, M. Frasca, and J. C. Sprott, Nonideal Behavior of Analog Multipliers for Chaos Generation, *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol.63. no.4, pp.396-400, 2016.

[3] J. Jahanpanah, M. Rezazadeh, and A. A. Rahdar, Formation mechanism of bifurcation in mode-locked class-B laser, *Chin. Phys. B*, vol.23, no.12, pp.124205-1-7, 2014.

[4] L. H. Yang and X. K. Ma, Analysis on Voltage Stability of Power System with Doubly Fed Induction Generator Wind Turbine Based on Bifurcation Theory, *Transactions of China Electrotechnical Society*, vol.27, no.9, pp.1-8, 2012.

[5] H. C. Liu, Y. Wang, and Z. X. Su, Bifurcation phenomena in single-phase three-level inverters, *Acta Phys. Sin.*, vol.62, no.24, pp.240506-1-8, 2013.

[6] Z. H. Zhang, J. C. Fu, G. N. Deng, Hopf Bifurcation Analysis and Bifurcation Control of a Lorenz-Like System, *Complex systems and complexity Science*, vol.12, no.1, pp.96-103, 2015.

[7] Y. L. Lu, G. P. Jiang, Y. R. Song, Stability and bifurcation of epidemic spreading on adaptive network,*Acta Phys. Sin.*, vol.62, no.13, pp. 130202-1-9, 2013.

[8] X. Wu, G. L. Wen, H. D. Xu, and L. P. He, Anti-controlling Neimark-Sacker Bifurcation of a three-degree-freedom vibration system with clearance,*Acta Phys. Sin.*, vol.64, no.20, pp.200504-1-7, 2015.

[9] R. Tchitnga, T. Nguazon, P. H. Louodop Fotso, and J. A. C. Gallas, Chaos in a Single Op-AmpCBased Jerk Circuit: Experiments and Simulations,*IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol.63. no.3, pp.239-243, 2016.

[10] E.Hellen, Real-time finite difference bifurcation diagrams from analog electronic circuits, *American Journal of Physics*, vol.72, no.4, pp. 499-502, 2004.

[11] Chi K. Tse, Experimental Techniques for Investigating Chaos in Electronics, in Chaos in Circuits and Systems, *http://www.eie.polyu.edu.hk/ cktse/pdf-paper/WS-chap18.pdf*, pp.367-384, 2002

[12] T. Matsumoto, L. O. Chua, and M. Komuro, The double scroll,*IEEE Trans. Circuits and Syst.*, vol.CAS-32, no.8, pp.797-818, 1985.