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A Method of Contourlet Transform Image Denoising Based on Improved Contraction Function

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Abstract. *In the situation of traditional image details are lost in denoising process and poor effect, the paper puts forward an improved contraction function which combines with the advantages of soft threshold and hard threshold contraction function. Using the improved method, coefficient N_1, N_2 and contract order N can be chosed by self-adapting pattern. Experiments show that, for noise images, our method can eliminate the noise effectively. Image quality indexes, such as peak signal-to-noise ratio, mean square error and image enhancement factor have obvious enhancement. In addition, the denoising images have better visual quality.*
Keywords: Improved contraction function, Contourlet transform, Contract order, Threshold function, Boundary point.

1. **Introduction.** Image processing technology has played an important role in all aspects of human life and production. However, due to various factors, people get the image tends to be polluted by cloudy, foggy weather, the photography jitter, and other noise (noise contamination). In recent years, wavelet theory obtained significant development and has been widely used in image denoising. But because of the commonly used two-dimensional tensor product wavelet is isotropic and poor directional selectivity, only can depict image point singularity, and its difficult to depict images high dimensional geometric features such as edges and textures. Therefore, there have several super wavelet, such as wavelet[1-3], outline wave[4-6], etc., which use the super wavelet to image denoising and keep the linear structure of the image well at the same time.

As a new multi-scale transform, Contourlet transform can achieve any direction in any scale decomposition, which is good at describing image contour and directional texture information. Combined wavelet transform and Contourlet transform can maximize the realization of complementary advantages of both, so as to better realize image denoising. Currently, Contourlet transform has been widely used in image denoising and other fields [7-10]. However, Contourlet transform is lack of translation invariance and will cause pseudo gibbs in image edge distortion[11]. Ren[12] studied hard threshold based on wavelet Contourlet transform image denoising method, removed the pseudo gibbs phenomenon which generated by wavelet Contourlet transform translation variation and increased the denoising effect; Fang[13] used multi-scale multi-directional and translation invariance of unsampling Contourlet transform, obtained transform coefficient of noise image with unsampling Contourlet transform, proceeded the transform coefficient with best soft threshold, and realized image denoising with inverse transform. Hu[14] introduced fractional differential into the study of digital image processing, put forward digital image denoising method based on fractional differential calculus, while denoising can keep the details of image edge texture. Therefore, we combine the soft threshold and hard threshold processing in Contourlet transform, and construct new contraction function. In order to further verify the effectiveness of our method, we compare and analyze Contourlet transform soft threshold method, Contourlet transform hard threshold method and fractional method of image denoising methods. In addition, we analyze the experiment results from two aspects of qualitative and quantitative.

2. **Contourlet Transform.** Contourlet transform is a kind of "real" two-dimensional image representation method, which has not only inherited multi-resolution time-frequency analysis characteristics of wavelet transform, but also has good anisotropic characteristics. It can use less coefficient than the wavelet transform to express smooth curves, capture image edge and contour information better, and has the characteristics of multi-resolution and multiple direction. Contourlet transform realize multi-scale analysis and multiple direction analysis separately. Firstly, using laplacian pyramid (LP), the image is decomposed with multi-scale decomposition and the singularity is captured; Secondly, using direction filter bank (DFB)[15], singular points of distribution in the same direction are synthesized as a coefficient; Finally, the original image is approximated by the base structure which is similar to the contour segment. Fig.1 shows the Contourlet decomposition process.

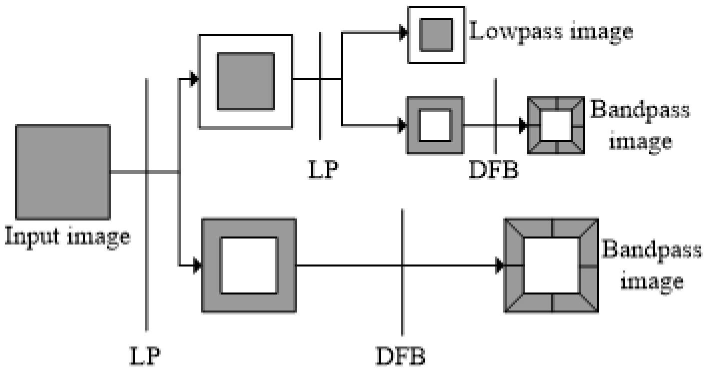

Figure 1. Contourlet decomposition process

3. **Contourlet Transform Based on Adaptive Contraction Function.**

3.1. **Soft Threshold and Hard Threshold Function.** High frequency information of image is mainly concentrated on normal, contour and texture edge, which reflect image detail information. Therefore, high frequency information of image is in the detail coefficient of these three directions and this part image also has the characteristics of larger shade changes. Noise is random, high frequency information is often contains noise and noise corresponding to the wavelet coefficient is generally small, only by the part of the details of the coefficient. After appropriate scale decompose to the noise and signal, it can be separated effectively.

The key of image denoising is to modify the detail coefficients, which determines the quality of final image effect. Soft threshold and hard threshold is the most commonly used methods.

(1) Mathematical expression of hard threshold contraction function

$$T_{hard} = \begin{cases} 0 & |\omega| < T \\ \omega & |\omega| \geq T \end{cases} \tag{1}$$

where, ω is wavelet coefficients of noise image, T is threshold, T_{hard} is contraction function. The purpose is to retain the coefficient which greater than or equal to the threshold value and remove the coefficient of small amplitude (wavelet coefficients of noise is small relatively). The estimation of wavelet coefficients is unbiased in hard threshold filtering. But the disadvantage is that it may produce abrupt changes, especially in image edge.

(2) Mathematical expression of soft threshold contraction function

$$T_{soft} = \begin{cases} 0 & |\omega| < T \\ sgn(\omega)(|\omega| - T) & |\omega| \geq T \end{cases} \tag{2}$$

where, sgn is symbolic function. In order to make the wavelet coefficient continuous in soft threshold domain, while the wavelet coefficient is greater than the threshold, and survived after operation on the contraction. In the whole process, a part of amplitude is cut off, and the estimation of coefficient is biased, so the disadvantage of the filtered image is too smooth.

Both hard threshold and soft threshold have their own advantages and disadvantages in denoising.

3.2. **Improved Contraction Function.** In order to take into account the advantages of two contraction, an improved contraction function is constructed as follows

$$T_{new} = \begin{cases} (T - |\omega|)/T * (\omega/N_1) & |\omega| < T \\ sgn(\omega)(|\omega| - T/N_2 * ((|\omega| - T)/|\omega|)^N) & |\omega| \geq T \end{cases} \tag{3}$$

where,N_1, N_2 take positive integer, N takes an integer greater than or equal to 0. In Eq.3, N_1 ,N_2 are decomposition coefficient, N is contract order.

Here, referred to N as the contract order, $(|\omega| - T/|\omega|)$ is contraction factor. After the decomposition, the image edge gain coefficient is generally larger, when order number N is fixed and $|\omega| \geq T$, transform Eq.3, we can obtain

$$T_{new} = sgn(\omega)(|\omega| - T/N_2 * (1 - T/|\omega|)^N) \quad |\omega| \geq T \tag{4}$$

For Eq.3, the selection of N_1, N_2 and N are selected as follows:

(1) Fixed N, when $|\omega|$ is greater, the operation retains original value in hard threshold. After modified coefficient with hard threshold method, $|\omega|$ is also great and image edge coefficient value is bigger generally. After wavelet inverse transformation, edge often have serious distortion. In Eq.4, if $|\omega|$ tends to $+\infty$, then $(1 - T/|\omega|)^N$ will tend to 1. When N_2 is equal to 1, Eq.4 becomes soft threshold, that is to say, edge mutation phenomenon of hard threshold will gradually ease with the increase of decomposition coefficient absolute value.

(2) For the smaller point $|\omega|$ in $|\omega| \geq T$, usually it is smooth area in the image. Smooth image is one of the characteristics of soft threshold. After dealing with soft threshold method and inverse transformation, a point tend to be too smooth. In Eq.4, if $|\omega|$ tends to T, then $(1-T/|\omega|)^N$ will tend to 0. When N_2 is equal to 1, it becomes the deformation of hard threshold. By analyzing, we can conclude that the decomposition coefficient is greater than or close to the threshold, using improved after contracting function, the more

close to the threshold, the more keep its own. Therefore, it is not going to happen too smooth phenomenon after the inverse transformation of image.

(3) While $|\omega| \geq T$, fixed $|\omega|$, if $|\omega|$ tends to $+\infty$, and N_2 is equal to 1, Eq.3 is the form of hard threshold. If N tend to 0 and N_2 is equal to 1, Eq.3 is the form of soft threshold. So when $N \in (0, +\infty)$, the state of Eq.3 is between hard threshold and soft threshold contraction function. As above (1) or (2) , it can take the advantages of both hard and soft contraction function.

(4) While $|\omega| < T$, for the smaller point $|\omega|$, the contraction function processing of hard threshold and soft threshold is consistent, so we set it to 0 directly. In Eq.3, if $|\omega|$ tends to T, then $T - |\omega|$ will tend to 0 and the formula is close to 0. When $|\omega|$ and T have a comparatively large difference, decomposition coefficient will get a degree of narrow, rather than we set it to 0 the contraction function processing of hard threshold and soft threshold is consistent that is to set to 0 directly. As you can see, for less than the threshold value and closer threshold decomposition coefficient, improved contraction function will make it tend to be 0; and for less than the threshold and large difference to the threshold decomposition coefficient, improved contraction function will be a certain degree of narrow, not 0. The decomposition coefficient which is less than the threshold is changed from the original without consideration processing to the reduction is taken into account in the final recovery operation, which can improve the soft and hard threshold contraction function.

Above analysis shows that, due to both factors of contraction factor and order, with the choice of decomposing coefficients and order, improved contraction function can be adjusted between soft threshold and hard threshold flexibly. By this way, it can improve adaptability of denoising process. Moreover, the contraction function, which combines soft threshold and hard threshold, can further improve the function effect.

3.3. **Algorithm steps.** Applying new contraction function to Contourlet threshold denoising, the algorithm steps are as following:

(1) Add noise to the original image;

(2) Multi-scale wavelet decompose (Contourlet) the noise image and establish detail component image sequence;

(3) Set up the threshold;

(4) According to Eq.3, deal with each layer details decomposition weight coefficient;

(5) Inversing the transformation to the processed coefficients with wavelet transform (Contourlet) and getting the reconstructed image.

4. **Experimental Results.** Using Eq.3, we take use of improved contraction function to realize image denoising. Due to the coefficient N_1, N_2 and contract order N are variable, so we choose different coefficients for image processing, and choose the best coefficients. In order to verify the superiority of our method, we take $\sigma = 25$ gaussian white noise image as research object, analyze and compare hard threshold denoising, soft threshold denoising method and fractional order method [14]($v = -0.8$).

From Fig.2, we find that the application of hard threshold and soft threshold Contourlet transform for image denoising, we can get a certain effect, but loss image detail information and image appears fuzzy phenomenon. Fractional method can get remarkable image denoising effect, image contrast and detail texture are improved significantly, but for big gray changes area occurred over sharpening of high frequency information and denoising result is not ideal. Applying our method, denoising effect is obvious, detail information is outstanding and the texture is relatively clear. Experimental results show that denoising effect of our method is better when the contraction function coefficient $N_1 = 6$, $N_2 = 6$

,$N = 1$. While we select other coefficient, parts of the image will appear fuzzy phenomenon. Thus in practical application, we can select proper parameters according to the degree of image denoising, so as to realize adaptive contraction function of Contourlet transform denoising.

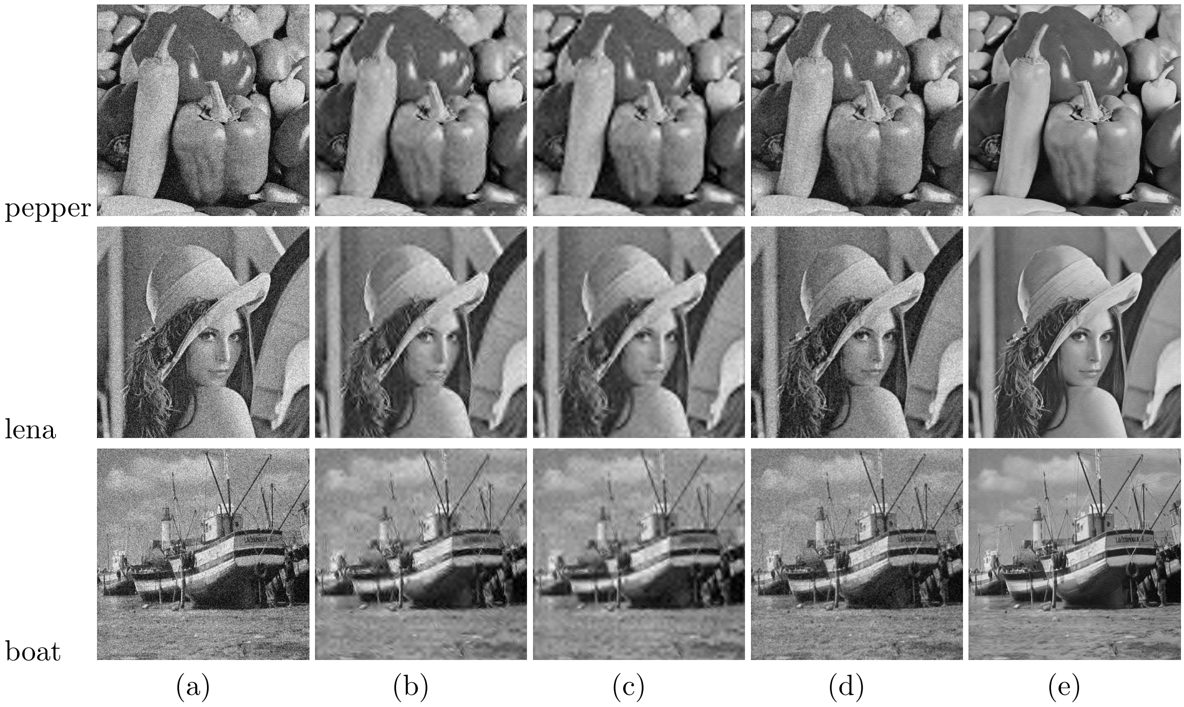

FIGURE 2. Processed image with different methods
(a) noising image ($\sigma = 25$);(b) hard threshold method;(c) soft threshold method;(d)fractional order method;(e) proposed method

5. **Image Denoising Quality Evaluation.** The quality of image denoising can be evaluated from two aspects: qualitative evaluation and quantitative evaluation. The qualitative evaluation depends on visual effect of image, as shown in Fig.2. But there is no unified standard to evaluate image denoising effect quantitatively. In experimental analysis, we take four index as comprehensive evaluation of denoising image, that is image grey value on direction projection,which can reflect image grey value changes, peak signal-to-noise ratio (PSNR), which can reflect the image detail information parameter, root mean square error (MSE) and image enhancement factor (IEF) [16].

5.1. **Image Gray Value Projection Analysis Method.** Comparison and analysis of denosing image grey value in longitudinal direction (x) projection of Fig.2, the projection results is shown in Fig.3.

From Fig.3, we can see that the image grayscale change drastically after adding gaussian white noise. With Contourlet hard threshold method, the denoising image grayscale becomes abate, the curve becomes smooth, some details information are lost, and the image appears fuzzy phenomenon; With Contourlet soft threshold method, the denoising image grayscale curve becomes more smooth, details information are lost seriously, fuzzy phenomenon aggravate; With fractional method, the image denoising effect is obvious, at the same time low frequency information of the image are increased, the image texture details keep is better, but the change of grey value is bigger, and local area appears sharpen phenomenon; With our method, it can take use of the advantages of soft and hard threshold and avoid disadvantages of other methods. After image denoising, the change of the gray level is basically the same as the original image, our method can preserve the original image texture detail information while denoising, and the denoising effect is obvious.

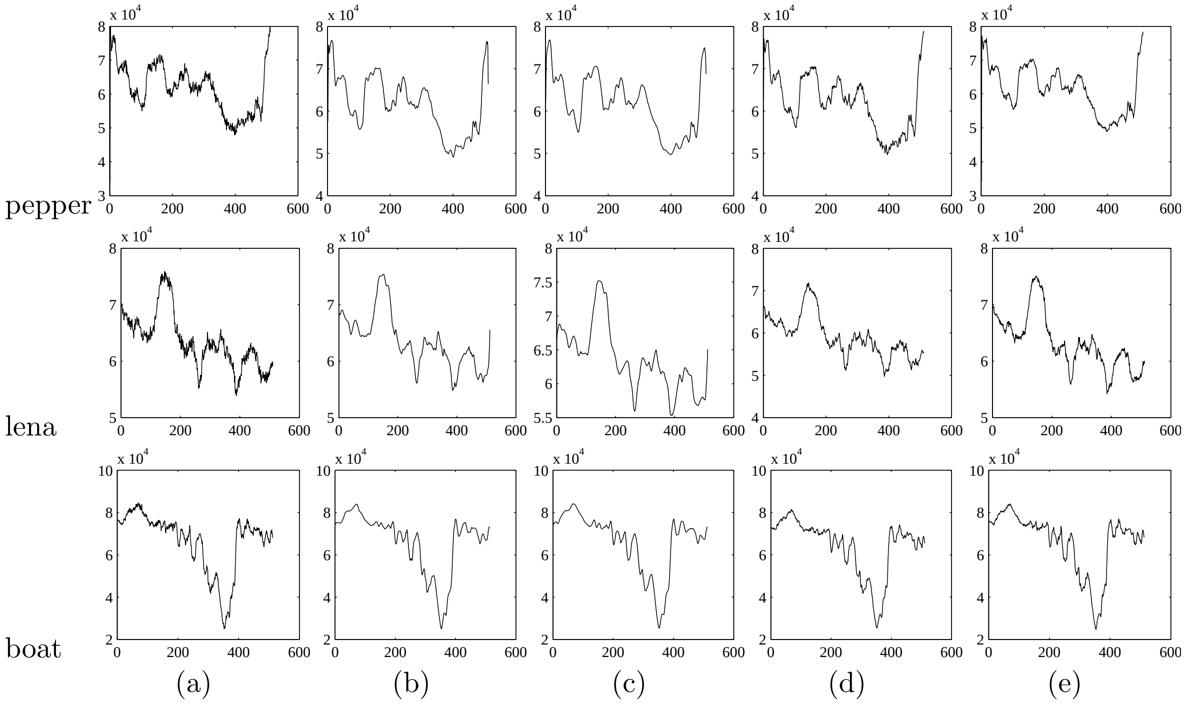

FIGURE 3. Gray value in the direction projection in Fig.2
(a) noising image ($\sigma = 25$);(b) hard threshold method;(c) soft threshold method;(d)fractional order method;(e) proposed method

5.2. **Image Gray Value Projection Analysis Method.** Suppose X_{in} is original image, X_{out} is the result of enhanced image, (M, N) is the size of the image, Y is the noise image, so:

(1) Root Mean Square Error

$$MSE = \frac{1}{M * N} \sum_{i=1}^{M} \sum_{j=1}^{N} (X_{in}(i, j) - X_{out}(i, j))^2 \tag{5}$$

MSE is the mean square error between original image and denosing image, the smaller value of MSE image denoising effect is better.

(2) Peak Signal to Noise Ratio

$$PSNR = 10log_{10}(\frac{(2^n - 1)^2}{MSE}) \tag{6}$$

The greater value of $PSNR$ image denoising effect is better.

(3) Image Enhancement Factor

$$IEF = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} (Y(i, j) - X_{in}(i, j))^2}{\sum_{i=1}^{M} \sum_{j=1}^{N} (X_{out}(i, j) - X_{in}(i, j))^2} \tag{7}$$

IEF is used to evaluate edge preserving ability of the method, the greater value of IEF the image edge to retain the more.

In order to compare the effect, hard threshold method, soft threshold method, fractional order method($v = -0.8$) and our method are applied to pepper, lena and boat images respectively. And we calculate every index of denoising image, such as root mean square error, peak signal-to-noise ratio and image enhancement factor, the calculation results is shown in Table 1.

TABLE 1. Comparision of various denoising in Fig.2

		noising image	hard threshold method	soft threshold method	fractional order method	proposed method
	$PSNR$	20.1829	26.8610	25.4889	24.3940	27.0627
pepper	MSE	623.4376	133.9613	183.7329	236.4170	127.8822
	IEF		4.6628	3.3942	0.0362	4.8751
	$PSNR$	20.1986	26.9309	25.7616	24.7190	26.9997
lena	MSE	621.1874	131.8238	172.5511	219.3737	129.7524
	IEF		4.7377	3.6220	0.0355	4.7875
	$PSNR$	20.1976	25.1121	24.3134	23.8359	25.6753
boat	MSE	621.3294	200.3876	240.8483	268.8353	176.0170
	IEF		3.1126	2.5947	0.0331	3.5299

From Table 1, with the fractional order method, peak signal-to-noise ratio and image enhancement factor are the minimum, denoising quality is poorer; With hard threshold method and soft threshold method, the root mean square error, peak signal-to-noise ratio and image enhancement factor indexes are relatively low, the visual effect of image denoising is also poorer relatively; With our method, peak signal to noise ratio and image enhancement factor are the highest, and the root mean square error is minimum, image denoising effect is best. In addition, contraction function coefficient of our method is variable, we can obtain ideal denoising image based on different contraction coefficient.

6. **Conclusion.** Contourlet transform is a new multi-scale transform, which has applied to the bottom layer of digital image processing as new research topic. In the paper, we study the principle of Contourlet transform image denoising, use the decomposition of arbitrary direction, which is good at describing the properties of the contour and directional texture information in the image. Then, we propose Contourlet transform method for adaptive contraction function, which can select the contraction function coefficient N_1, N_2 and the order N base on the reality, so as to achieve better denoising effect. The effectiveness of the method is verified by simulation experiments.

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