Journal of Information Hiding and Multimedia Signal Processing ©2017 ISSN 2073-4212
Ubiquitous International Volume **8**, Number **5**, September **2017**

A High-Efficiency 3D-DOA Estimation Algorithm for Unknown Sources Number

Limin Zhao[1*], Qing Liu[1] and Haifeng Li[2]

[1]School of Electronic Information and Electrical Engineering
Tianshui Normal University
Tianshui 741001, China
*Corresponding author:melonlink@sina.com

[2]School of Software
Dalian University of Technology
Dalian 116620, China

Received February, 2017; revised July, 2017

Abstract. *Signal estimation is the basic function, applications and services of Internet of Things. Due to the problems of spike non-normal noise, multiple signal classification (MSC) algorithm will lose the tenacity for the 3D-DOA(3D-direction of arrival) estimation of multiple-sources and obtain inaccurate results. In this paper, therefore, we propose a new 3D-DOA estimation algorithm for amorphic multiple-sources based on nested array and FLOC-MSC method. This new scheme firstly quantifies to covariance matrix to extend the array aperture and it generates new covariance matrix. In addition, the new method utilizes row vector of covariance matrix to construct Toeplitz matrix and it uses the joint diagonal structure of Toeplitz matrix to create cost function. Afterwards, array output matrix is extended from second geometric moment to lower order moment. We can get spatial spectrum based on covariation matrix through analyzing covariation matrix. Eventually, after gradient operation for spatial spectrum, we conduct extremum searching through one-dimensional spectrum peak search and find DOA of each source. Our algorithm is compared to covariance fitting optimization technique and the fast approximated power iteration-total least square-estimation of signal parameters via rotational invariance technique (FAPI-TLS-ESPRIT) algorithm using the TLS-ESPRIT method and the subspace updating via FAPI-algorithm. At the end, experiments show that this new algorithm can accurately estimate 3D-DOA of mutiple-sources under the spike non-normal noise condition, especially at low signal-to-noise ratio (SNR) values with impulse. The new method has high estimation precision and resolution with unknown and underdetermined number of sources.*
Keywords: Multiple signal classification, 3D-direction of arrival, Nested array, FLOC-MSC method, Toeplitz matrix, Covariation matrix

1. **Introduction.** Currently, DOA estimation[1] is used widely in a variety of military and national economy areas, such as radar[1], sonar[2], communication[3], seismic exploration, radio astronomy, navigation, sound source tracking[4-5] etc. DOA of signal is one of the important parameters in electronic reconnaissance, it is also an important evidence for signal sorting, radiating source recognition and locating and tracking. People have raised more interest in DOA estimation. However, there is no prior information of signal in DOA of electronic reconnaissance compared with that in radar signal. Traditional direction-finding methods, such as maximum signal method, amplitude-comparing bearing method, are simple, easy to implement, but with low accuracy and low resolution.

Although the interferometer direction finding method is higher accuracy, there are many radiation sources with highly distribution density, at the same time, multi-signal problem seriously affects its performance in modern electronic countermeasure environment. To solve these problems, Liu J[6] proposed a low-complexity adaptive two-dimensional (2-D) frequency estimation algorithm to jointly track 2-D direction-of-arrival (DOA) of multiple moving targets with a uniform rectangular array (URA). The LOAFRI subspace tracking algorithm was applied to estimate the signal subspace recursively, then an adaptive eigenvector-based frequency estimation approach was used to resolve the 2-D DOA from the estimated signal subspace. Zheng Z[7] presented a fast algorithm for the central DOA tracking by using uniform linear arrays(ULA), which firstly updated the signal subspace in real-time by adopting the orthonormal projection approximation subspace tracking with deflation algorithm, and then a low-complexity DOA method was used to estimate the central DOA of distributed sources. Liu S[8] showed a new DOA method based on new array signal data model established by sequential sampling. After building a tracking equation, particle filter was improved with Reversible Jump Markov Chain Monte Carlo, and finally real time DOA and source number was estimated on time. Tao J[9] proposed novel quaternion data projection method, which had high robustness to the undulate phenomenon arisen from the initialization,and converged faster than the conventional data projection method. However, they suffer from a heavy computational load despite their accuracy. Some more accurate DOA algorithms have been proposed. Song D S etc[10] presented an improved likelihood function to improve the performance of traditional DOA estimate real-time dynamic tracking and the modified likelihood function was derived from multiple signal classification algorithm spectral function. Gao X[11] proposed a sequential Bayesian tracking approach based on the Maximum a posteriori principle to simultaneously update the arrival angles and the source signals in the Kalman filter step by converting the update process of the state vector into a joint optimization problem. The methods have been exhibited as above, if the DOA algorithm tracks detailed trajectory, it must own an identical angular distribution with a 2Dsearch.

Sonia[12] etc. proposed new tracking method based on a simple covariance fitting optimization technique exploiting the central and noncentral moments of the source angular power densities to estimate the central DOA. The current estimates were treated as measurements provided to the Kalman filter that model the dynamic property of directional changes for the moving sources. What's more, the covariance-fitting-based algorithm and the Kalman filtering theory were combined to formulate an adaptive tracking algorithm. And at the end of this paper, we conduct some experiments to compare our new method to S-method[12]. In this paper, we present a new source 3D-DOA estimation method based on rush non-normal noise covariant and 2-level nested array. Firstly, we make covariance matrix vectorization as a new array receiving data and construct a new covariance matrix. Then, we build up Toeplitz matrix for each row in the new covariance matrix and utilize joint diagonal structure to generate cost function. Finally, we solve DOA of each source through one-dimensional search. This new method is able to make high-precision DOA estimation with unknown number of source, and effectively expands the array aperture, saves the number of array elements. Because so far no papers present this method, so we study this method with many experiments and finally it is proved to be efficiency.

This paper is organized as follows. In section2, we give some preliminaries. Section3 detailed introduces the high-efficiency 3D-DOA estimation algorithm for amorphic multiple-sources. We give the experiments and analysis in section4. There is a conclusion in section5.

2. **Preliminaries.**

2.1. **Noise model.** Rush non-normal background noise pulse can be described by SAS(Alpha stable) distribution. It contains four parameters: feature parameter α, scale parameter σ, skewed parameter β and location parameter μ. Where α denotes trailing degree of distribution, the smaller α is, the heavier trailing is. σ represents dispersion degree of distribution. It is equal to the variance of second-order moments. $\beta = 0$ is symmetric degree of distribution relative to its distribution of center point. μ is the location, which is equivalent to mean value of second-order moments. p is the order of matrix. a and b are coefficient values. Autocorrelation matrix of array output cannot keep convergence under spike pulse noise environment. Therefore, we define 5-order covariant based on fractional lower order moments.

$$G_{xx} = Ex[x^{p-1}]^T =< x, x >_p, 1 < p < \alpha \leq 2. \tag{1}$$

If x is complex matrix, we define: $x^{<p>} = |x|^{p-1}x^*$. It is convergent with the following properties:

- for independence variable $< x_1, x_2 >_p = 0$.
- for aleatory variable $< ax_1 + bx_2, x >_p = a < x_1, x >_p + b < x_2, x >_p$.
- for autonomous variable $< x, ax_1 + bx_2 >_p = a^{p-1} < x, x_1 >_p + b^{p-1} < x, x_2 >_p$.

2.2. **Signal model.** There are far field, narrow band and stable source s[13]. M omni-directional undifferentiated array elements are arranged into circular array with evenly spaced. Radius is R. It is shown as figure1.

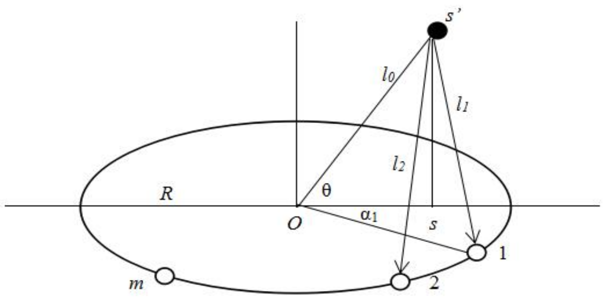

FIGURE 1. Circular array with evenly spaced

l_0 is the distance between circular array center O and source. Pedal point of signal source s' on circle flat is s. Pitch angle of signal source is θ. α_1 is horizontal angle of source, which is inclined angle between O_1 and O'_s. α_m is inclined angle between O_m and O'_s. l_m is the distance between array element m and s. For the $m-th$ array element,

$$\alpha_m = \alpha_1 + \frac{2\pi(m-1)}{M}, m = 1, 2, \cdots, M \tag{2}$$

$$l_m = \sqrt{R^2 + l_0^2 - 2Rl_0 cos\theta cos\alpha_m} \tag{3}$$

Time lag of information source arriving at array element is $\nabla t = 2\pi(l_m - l_0)/(f\lambda)$. f is source center angular frequency and λ is source wavelength. Phase difference can be expressed by:

$$\omega_m = \frac{2\pi(l_m - l_0)}{\lambda}. \tag{4}$$

So we can define direction matrix composed of p information source:

$$A_{M\times p}=\begin{pmatrix} e^{-\frac{j2\pi}{\lambda}(l_{11}-l_{01})} & e^{-\frac{j2\pi}{\lambda}(l_{12}-l_{02})} & \rightarrow & e^{-\frac{j2\pi}{\lambda}(l_{1p}-l_{0p})} \\ e^{-\frac{j2\pi}{\lambda}(l_{21}-l_{01})} & e^{-\frac{j2\pi}{\lambda}(l_{22}-l_{02})} & \rightarrow & e^{-\frac{j2\pi}{\lambda}(l_{2p}-l_{0p})} \\ \downarrow & \downarrow & & \downarrow \\ e^{-\frac{j2\pi}{\lambda}(l_{M1}-l_{01})} & e^{-\frac{j2\pi}{\lambda}(l_{M2}-l_{02})} & \rightarrow & e^{-\frac{j2\pi}{\lambda}(l_{Mp}-l_{0p})} \end{pmatrix}=[a(\omega_1),\cdots,a(\omega_p)] \quad (5)$$

2.3. **Array model.** 2-level nested array structure is as figure2. Number of array elements is N. Array element number of first level is N_1. The distance of array element is d_1. Array element number of second level is N_2. The distance of array element is d_2. And $d_2=(N_1+1)d_1$. The array element location of first level is $n_1,d_1(d_1=0,1,\cdots,N_1-1)$. The array element location of second level is $n_2,d_2-d_1(n_2=1,\cdots,N_1)$.

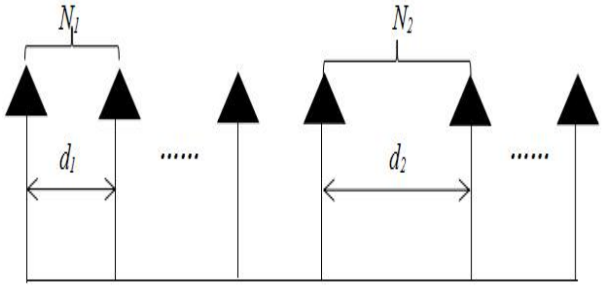

FIGURE 2. 2-level nested array structure

Assuming there are K uncorrelated narrow-band far-field signals which are incident on array a direction angle $\theta_k(k=1,2,\cdots,K)$ respectively. Also it satisfies the requirement $\theta_k\in(-90°,90°)$, $d_1\le\lambda_{min}/2$, λ_{min} is the minimum signal wavelength. Hence steering vector of $k-th$ source is represented by :

$$a(\theta_k)=[1,e^{-j2\pi d_1/\lambda sin\theta_k},\cdots,e^{-j2\pi(N_1-1)d_1/\lambda sin\theta_k},e^{-j2\pi(d_2-d_1)d_1/\lambda sin\theta_k},\cdots,e^{-j2\pi(N_2d_2-d_1)d_1/\lambda sin\theta_k}]^T. \quad (6)$$

Corresponding array flow is:

$$A=[a(\theta_1),a(\theta_2),\cdots,a(\theta_K)]. \quad (7)$$

So we can get the array receiving data $x(t)$ at t time:

$$x(t)=\sum_{k=1}^{K}a(\theta_k)s_k(t)+n(t)=As(t)+n(t). \quad (8)$$

Where $s(t)=[s_1(t),s_2(t),\cdots,s_K(t)]$ is $K\times1$-dimensional vector. A is $N\times K$-dimensional matrix. $n(t)$ is $N\times1$-dimensional white Gaussian noise.

3. High-efficiency 3D-DOA estimation algorithm for amorphic multiple-sources.

3.1. **Array aperture extension.** We get $T(T=L\times M)$ snapshot data from array model constructed by (8). And it is divided into M sections. Data length of each section is L. Assuming that each section of the source data satisfies smooth approximation conditions. Namely,

$$E|s_k(t)|^2=\sigma_{mk}^2,t\in[(m-1)L,mL-1],m=1,2,\cdots,M \quad (9)$$

Covariance matrix of $m-th$ section data is:

$$R_m=E[x(t)x^H(t)]=AD_mA^H+\sigma_n^2I,t\in[(m-1)L,mL-1] \quad (10)$$

Where $D_m = diag(\sigma_{m1}^2, \sigma_{m2}^2, \cdots, \sigma_{mK}^2)$ is signal covariance matrix of $m-th$ section data. $\sigma_{mk}^2 (k = 1, 2, \cdots, K)$ is signal power. σ_n^2 is white Gaussian noise power. We make vectorization for R_m and stack all columns into one column, get N-dimensional vector y_m:

$$y_m = vec(R_m) = (A^* \otimes A)p_m + \sigma_n^2 I_m. \tag{11}$$

Where $p_m = [\sigma_{m1}^2, \sigma_{m2}^2, \cdots, \sigma_{mK}^2]^T$, $I_m = [e_1^T, e_2^T, \cdots, e_N^T]^T$ and e_n^T is that all elements are zero vector except $n-th$ element (its value is 1). $\otimes$ denotes Khatri-Rao product. In fact, the function of $A^* \otimes A$ is equivalent to A in formula(8), which can be regarded as a new array manifold, it is recorded as $\overline{A} = A^*$. $\overline{A}$ has $2N_2(N_1+1)-1$ non-redundant rows. The reduplicative rows in $\overline{A}$ will result in non-uniqueness of some elements in y_m. Therefore, we select the new elements in y_m corresponding to non-redundant rows in $\overline{A}$. We use these elements to construct a new vector y_m', which can be expressed by:

$$y_m' = \overline{A}' p_m + \sigma_n^2 I_m'. \tag{12}$$

Where the new virtual array flow is $\overline{A}' = [a'(\theta_1), a'(\theta_2), \cdots, a'(\theta_K)]$, virtual array is uniform linear array with $2N_2(N_1+1)-1$ 1-array elements. Location of every array element is $nd_1, n = -N_i, \cdots, N_i (N_i = N_2(N_1+1)-1)$. The corresponding new direction vector:

$$a'(\theta_K) = [e^{-j2\pi(-N_i d_1)/\lambda sin\theta_k}, \cdots, 1, \cdots, e^{-j2\pi N_i/\lambda sin\theta_k}]^T. \tag{13}$$

So we can get new array receiving data:

$$Y = \overline{A}' P + \sigma_n^2 E. \tag{14}$$

Where $Y = [y_1', y_2', \cdots, y_M']$ is $2N_i \times M$-dimensional array receiving matrix. $P = [p_1, p_2, \cdots, P_M]$ is $K \times M$-dimensional signal matrix. $E = [I_1', I_2', \cdots, I_M']$ is $K \times M$-dimensional noisy matrix.

3.2. **3D-DOA estimation algorithm for amorphic multiple-sources.** By formula(14), we can get new array receiving data to make DOA estimation under the unknown number of source condition. First, we should calculate new covariance matrix of new array receiving data.

$$R = EYY^H. \tag{15}$$

We carry out Toeplitz transformation for the $n-th$ row of covariance matrix, and obtain Toeplitz matrix:

$$R_n = \begin{pmatrix} r(n,0) & r(n,1) & \cdots & r(n,N_i) \\ r(n,-1) & r(n,0) & \cdots & r(n,N_i-1) \\ \vdots & \vdots & & \vdots \\ r(n,-N_i) & r(n,-N_i+1) & \cdots & r(n,0) \end{pmatrix} = \tilde{A} S_n \tilde{A}^H + \sigma_n^2 \bar{I}_{N_{i+,n}}. \tag{16}$$

Where S_n is pseudo signal covariance matrix. $\tilde{A} = [\tilde{a}(\theta_1), \tilde{a}(\theta_2), \cdots, \tilde{a}(\theta_K)]$, its corresponding orientation vector is $\tilde{a}'(\theta_K) = [e^{-j2\pi d_1/\lambda sin\theta_k}, \cdots, e^{-j2\pi N_i d_1/\lambda sin\theta_k}]^T$. R_m has a total of $2N_i+1$ rows. In that $n-th$ row and $-n-th$ row are the conjugate symmetric. So we only need to make Toeplitz transformation from $-N_i$ to 0.

Under the circular array model, p sources select N snapshots to constitute source matrix $S_{p\times N}$ based on the classical MUSIC algorithm. Array additive noise matrix is $e_{M\times N}$. M array elements output $X_{M\times N}$ of N snapshots. Assuming signal phase difference of circle array is 0. s and e are independent of each other. Therefore, matrix form of the array output is $x = As + e$. The array output autocorrelation matrix is $R_{xx} = Exx^H$.

Under the rush non-normal noise background, R_{xx} cannot be convergent. The performance of classical MUSIC algorithm degrades seriously. However, covariation matrix G_{xx} is convergence. R_{xx} can be replaced by G_{xx}. The array output covariance matrix is:

$$G_{xx} = (As + e, As + e)_p = (As, As)_p + (As, e)_p + (e, As)_p + G_{ee}. \tag{17}$$

According to the above formula, we can obtain:

$$G_{xx} = (As, As) + G_{ee}. \tag{18}$$

Then

$$\begin{aligned} G_{xx} &= \sum_{i=1}^{p} (\alpha(w_i), As)_p + G_{ee} \\ &= \sum_{i=1}^{p} \alpha(w_i), (s_i, As)_p + G_{ee} \\ &= \sum_{i=1}^{p} \alpha(w_i)[(s_i, s_i)_p \sum_{i=1}^{p} \alpha^{p-1}(w_i)^T] + G_{ee} \\ &= A(s_i, s_i)_p[A^{p-1}]^T + G_{ee} \\ &= AG_{ss}A^H + G_{ee} \end{aligned}$$

For the reason that, $rank(G_{xx}) = M$, $rank(AG_{xx}A^H) = rank(G_{xx}) = p$, we execute eigenvalue decomposition for G_{xx}. It is easy to differentiate p larger eigenvalues(named signal eigenvalue) in G_{xx} and $M - p$ smaller eigenvalues (called noise eigenvalue). So characteristic matrix according to the column is divided into two matrices, that is: $S = [\mu_1, \cdots, \mu_p]$ and $D = [\mu_{p+1}, \cdots, \mu_M]$.

Finally,

$$G_{xx}D = AG_{xx}A^H D + G_{ee}D. \tag{19}$$

$$G_{xx}D = [S, D] \sum [S^H, D^H]^T D = [S, D] \sum [0, 1]^T = G_{xx}D. \tag{20}$$

And $D^H AG_{xx}A^H D = 0$, $A^H D = 0$, $a^H(w)D = 0$, $w = w_1, \cdots, w_p$. So the final spatial spectrum estimation $p(\theta_i)$ is :

$$p(\theta_i) = (N_i + 1 - maxeig \frac{\theta_i F^+ D(\theta_i)}{p(w)a^H(w)a(w)})^{-1}. \tag{21}$$

Where D is noise characteristic matrix of array output fractional low-order covariance matrix. $maxeig\cdot$ is the biggest characteristic value of matrix. $F = \sum_{n=-N_i}^{0} R_n^H R_n$, $\frac{1}{p(w)a^H(w)a(w)} = [R_{-N_i}^H \tilde{a}(\theta_i), \cdots, R_0^H \tilde{a}(\theta_i)$.

4. **Experiments and analysis.** In this subsection, we present experiments to illustrate the effectiveness of our proposed algorithm in different scenarios and compare it to the method derived in [11].

Experiment 1.

Space spectrum is three-dimensional matrix. We make angle and distance estimation for 3D-DOA estimation. The pitch angle, azimuth angle and distance of information source is $\alpha_1 = 20°$, $\beta_1 = 60°$, $d_1 = 30m$; $\alpha_2 = 40°$, $\beta_2 = 40°$, $d_2 = 80m$; $\alpha_3 = 60°$, $\beta_3 = 10°$, $d_3 = 120m$ respectively. SNR(Signal to Noise Ratio) is 5dB. Background noise has peak pulse characteristics. Angle and distance estimation are as figure3,4,5

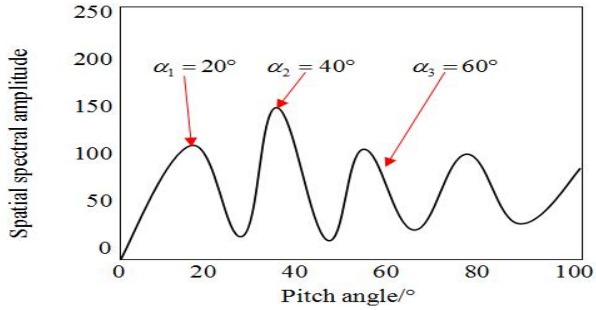

Figure 3. Pitch angle estimation

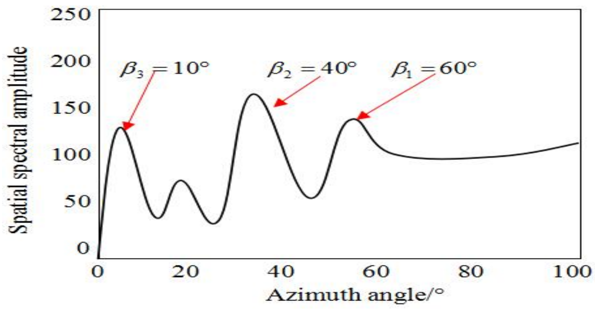

Figure 4. Azimuth estimation

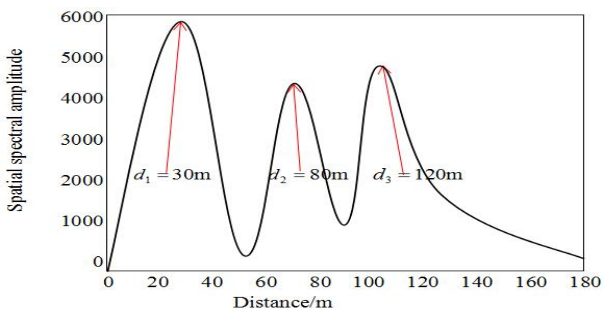

Figure 5. Distance estimation

In order to analyze the estimation performance difference between our new method(called N-FLOC-MSC) and the method in reference[11](named FAPI-TLS-ESPRIT) under different signal-to-noise ratios and the characteristic parameters, we take standard deviation as the evaluation criterion. The standard deviation is smaller, the better estimation performance is. When $\alpha = 1.2$, $p = 1.1$, signal-to-noise ratio ranges from 0dB to 10dB. Estimated performance curve is shown as figure 6. With the increase of signal-to-noise ratio, the estimation results of both methods improves gradually. And the standard deviation of FAPI-TLS-ESPRIT is greater than N-FLOC-MSC when signal-to-noise ratio is high. When $p = 1.1$, signal-to-noise ratio is 5dB, characteristic parameter ranges from 1.1 to 2. Estimated performance curve is shown as figure 7. The characteristic parameter is bigger, the weaker background noise impulse is. Second order matrix approaches to convergence, and arrives to normal distribution, estimation performance of the two algorithms is getting better.

Experiment 2.

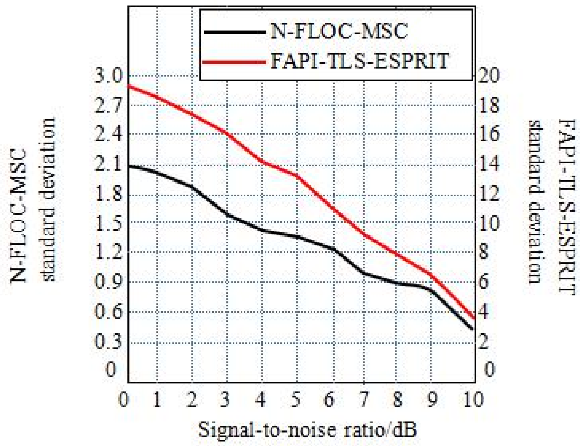

FIGURE 6. Estimation performance curve

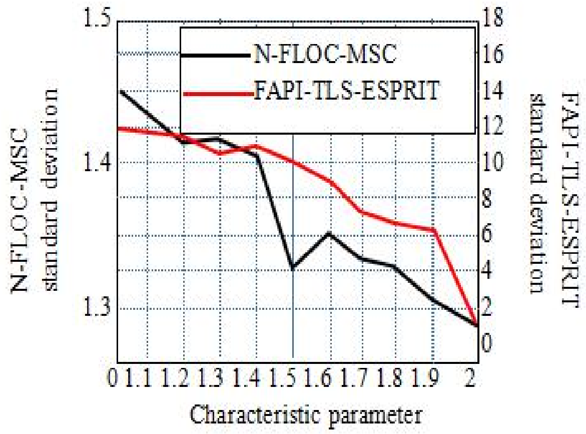

FIGURE 7. Estimation performance curve

We choose root mean square error (RMSE) to describe the estimated performance of algorithm, it is defined as:

$$RMSE = K^{-1}\sum_{k=1}^{K}(J^{-1}\sum_{j=1}^{J}(\hat{\theta}_k - \theta_k)^2)^{0.5}. \tag{22}$$

Where J is experiment time. K is source number. $\hat{\theta}_k$ is the DOA estimation value of $k-th$ source. θ_k is the DOA true value of $k-th$ source. Assuming array number N=5. And it uses 2-level nested structure, $N_1 = 2$, $N_2 = 3$. The number of snapshots $T = 8192$. The data is divided into 16 segments, $M = 16$, Length of each segment is $L = 512$. Four sources arrive to the array with angle 15°, 5°, 30° and 45° respectively. $SNR = 15dB$.

When $K = 4$, we get the DOA estimation result as figure8 compared to[11]. Due to the array aperture expansion through our algorithm, it increases the degree of freedom, which can, therefore, make DOA estimation accurately when number of sources is greater than the actual array number. The root mean square error(RMSE) with different signal-to-noise ratios is as figure9.

In order to further verify high efficiency of our new algorithm, we select eight sources with angle −55°, −35°, −20°, −5°, 5°, 20°, 30° and 45° respectively. When K=8, we

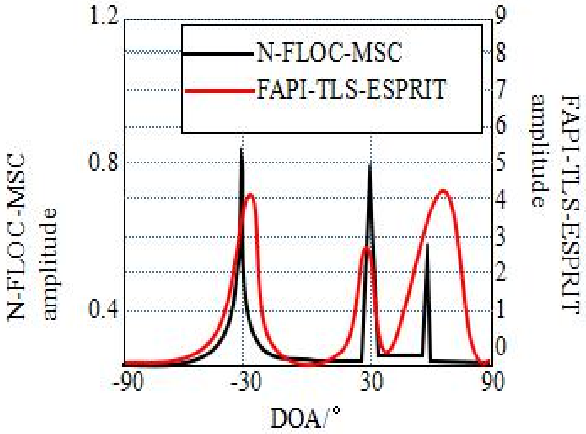

Figure 8. K=4, DOA estimation value

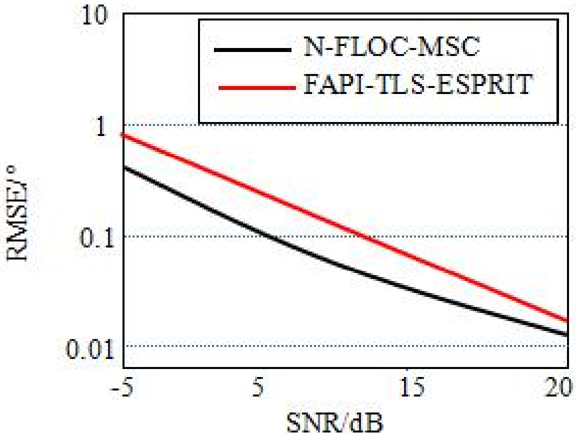

Figure 9. RMSE with two algorithms when K=4

get the DOA estimation result as figure10 compared to[11]. The root mean square error(RMSE) with different signal-to-noise ratios is as figure11. In engineering applications, the spectral peak coordinates of spatial spectrum need to be obtained directly, namely 3-D information. Spatial spectral peak is a bunch of larger values, if maximum value of the spatial spectral is computed. After many operations, it will remain at the highest peak. And no other spectral peaks will be obtained. In order to improve the computing speed, we use our new 3D-DOA estimation algorithm to execute gradient calculation for spatial spectral. The results are better than other methods.

5. **Conclusions.** In this paper, we develop a new method for tracking the DOA assuming multiple incoherently distributed sources, which is based on 2-level nested array. Also it makes eigenvalue decomposition for covariant matrix, and finds out the noise subspace. Then it constructs spatial spectrum, and carries out gradient calculation for any dimension in spatial spectrum. Finally, we take extreme value in turn and get the 3D-DOA estimation. Theoretical analysis and simulation experiments show that this method extends the array aperture, it can use less number of array elements at the same time to estimate DOA of multiple source, and it has high estimation precision and resolution, the signal-to-noise ratio requirement is not high also. In the future, we will study more signal estimation methods for improving the practical value, although there are still more

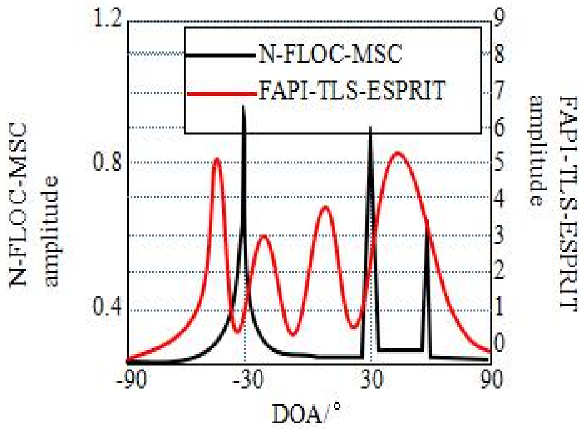

Figure 10. K=8, DOA estimation value

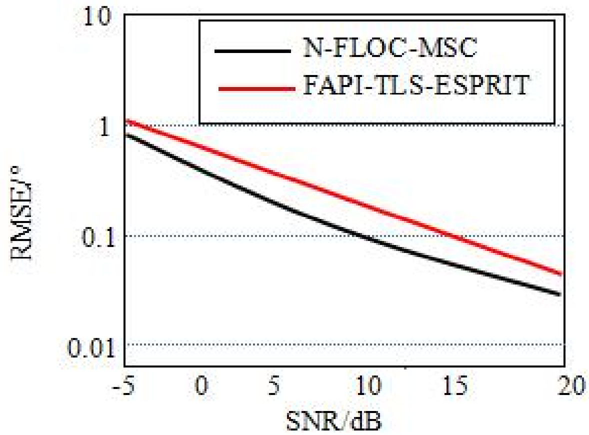

Figure 11. RMSE with two algorithms when K=8

challenges with information awareness and information interaction, large-scale network applications and multimedia sensor networks areas.

Acknowledgment. This work is supported by National Natural Science Foundation of China under Grant No. 61461046.

REFERENCES

[1] Z. D. Zheng, J. Y. Zhang, P. L. Xiong Joint DOD and DOA estimation for bistatic MIMO radar, *Journal of Systems Engineering & Electronics*, vol. 32, no. 2, pp. 2268-2272, 2010.
[2] J. S. Thompson , P. M. Grant , B. Mulgrew , et al., Generalised algorithm for DOA estimation in a passive sonar, *Journal of Iee Proceedings F*, vol. 140, no. 1, pp. 339-340, 1993.
[3] M. Jin ,G. Liao , J. Li Fast communication: Joint DOD and DOA estimation for bistatic MIMO radar , *Journal of Signal Processing,* vol. 89, no. 2, pp. 244-251, 2009.
[4] T. Wiese , H. Claussen, J. Rosca Particle filter based DOA estimation for multiple source tracking (MUST), *Conf. of Circuits, Systems and Computers, 1977. Conference Record.* 1977 11th Asilomar Conference on. pp. 624 - 628, 2011.
[5] T. H. Liu, S. L. Yin, An improved neural network adaptive sliding mode control used in robot trajectory tracking control, Journal of International Journal of Innovative Computing, Information and Control, vol.11, no. 5, pp. 1655-1666, 2015.

[6] J. Liu, X. Liu, Joint 2-D DOA Tracking for Multiple Moving Targets using Adaptive Frequency Estimation, *Conf. of Acoustics, Speech and Signal Processing, 2007. ICASSP 2007. IEEE International Conference on. IEEE,* pp. II-1113 - II-1116, 2007.

[7] Z. Zheng, L. I. Guang, A Fast Algorithm for the Central DOA Tracking of Coherently Distributed Sources, , *Journal of Signal Processing*, vol. 27, no. 4, pp. 529-533, 2011.

[8] S. Liu , W. Ying , L I. Dong-Hai, Joint Source Number Detection and DOA Tracking Based on Single Channel Array*, Journal of Electronic Information Warfare Technology*, 2013.

[9] J. Tao ,F. Yu,Z. Lin DOA Tracking Algorithm Based on Quaternion Data Projection Method, Journal of Journal of Data Acquisition & Processing, vol. 28, no. 2, pp. 244-249, 2013.

[10] D. S. Song ,G. L. Liang , Y. Wang Particle Filter Algorithm for DOA Tracking of Maneuvering Targets, *, Journal of Journal of Signal Processing*, 2014.

[11] X. Gao ,L.I. Xiang , Filos J, et al. A Sequential Bayesian Algorithm for DOA Tracking in Time-Varying Environments, *, Journal of Chinese Journal of Electronics,* vol. 24, no. CJE-1, pp. 140-145, 2015.

[12] S. B. Hassen, A. Samet, An efficient central DOA tracking algorithm for multiple incoherently distributed sources, *, Journal of EURASIP Journal on Advances in Signal Processing*, 2015.

[13] J. Liu,S. L. Yin, Improved Square Root Unscented Kalman Bilateral Filtering Used for Indoor Positioning System, *, Journal of ICIC Express Letters Part B: Applications*, vol.6, no. 12, pp. 3102-3107, 2015.