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Research on the Statistical Characteristics of PAPR in NC-OFDM

Jing-Bo Zhang, Er-Lei Li*, and Shu-Fang Zhang

College of Information Communication Engineering
Dalian Maritime University, Dalian, Liaoning, 116026, China
*Corresponding author: erleili@dlmu.edu.cn.

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Abstract. *The problem of the peak to average power ratio (PAPR) is more prominent in non-continuous orthogonal frequency division multiplexing (NC-OFDM) modulation than in orthogonal frequency division multiplexing (OFDM) modulation. To study the influence of subcarrier distribution on PAPR, we defined the autocorrelation function $R(\varepsilon)$ within a symbol period and the inter-symbol autocorrelation coefficient $\rho(\varepsilon)$, which was used to measure the relationship between the PAPR, the total number of subcarriers, and the number of cognitive users. Only when the total number of subcarriers was constant, the number of cognitive users was the main factor affecting PAPR; when the total number of subcarriers and the number of cognitive users were constant, the values of the autocorrelation coefficient varied with the distribution scheme where at this time, the statistical probability of PAPR increases monotonically with the increase of the autocorrelation coefficient. Using the appropriate subcarrier allocation scheme, which has a low autocorrelation coefficient, the PAPR of the NC-OFDM modulated signal could be effectively reduced by 5% to 6%.*
Keywords: NC-OFDM, PAPR, autocorrelation coefficient, subcarrier distribution

1. **Introduction.** As wireless communication technology rapidly develops, limited spectrum resources become a barrier to the development of wireless communication technology. A large number of studies by the Federal Communications Commission (FCC) have shown that the spectrum utilization is extremely unbalanced; some licensed bands always remain idle, while some unlicensed bands are crowded. To use the increasingly tense spectrum resources effectively, cognitive radio (CR) technology [1] has been proposed.

In a CR system, data transmission is carried out using the idle spectrum. orthogonal frequency division multiplexing (OFDM) technology has become the preferred transmission mode as it allows for the flexible use of the spectrum and suits the transmission requirements of a CR system. Since the data transmission uses the idle spectrum in a CR system [2], the idle spectrum may be distributed in a non-continuous frequency band where the OFDM system changes into an non-continuous orthogonal frequency division multiplexing (NC-OFDM) system [3].

Since the OFDM system uses multi-carrier modulation, this may cause it to inherit the high PAPR (peak-to-average power ratio) problem [4]. As cognitive users are non-continuous in NC-OFDM, the i.i.d (independent and identically distributed) condition is no longer valid [3], which along with a certain correlation between the subcarriers causes a more serious problem of PAPR. If the PAPR is too high, the power amplifier will work in non-linearity, therefore causing the nonlinear distortion of signals that results in a higher code error rate at the receiver of the NC-OFDM system; on the other hand, if the power amplifier has a large linear dynamic range, power efficiency will be at a very low level. Therefore, PAPR reduction is of particular importance.

Since the assumption that the i.i.d condition of subcarriers cannot be established in the NC-OFDM system [5], PAPR statistical methods are based on the assumption that the OFDM system with i.i.d cannot be applied to an NC-OFDM system directly. In this paper, we conducted an in-depth study of the PAPR statistics and systematically analyzed the influencing factors in an NC-OFDM system. To study the influence of subcarrier distribution on PAPR, we defined the autocorrelation function $R(\varepsilon)$ within a symbol period and the inter-symbol autocorrelation coefficient $\rho(\varepsilon)$, which was used to measure the relationship between the PAPR, the total number of subcarrier, and the number of cognitive users. The autocorrelation coefficient $\rho(\varepsilon)$ is also used to measure the relationship between the same cognitive users and different allocation schemes.

2. **The Definition of PAPR in NC-OFDM.** The NC-OFDM system allocates subcarriers by sensing external spectral information. The subcarriers that correspond to primary users is set to 0, making them become useless subcarriers. The modulation data is allocated on the subcarriers that correspond to cognitive users. Thus, the transmitter for the NC-OFDM system is formed. Assuming that the number of subcarriers for cognitive users in the NC-OFDM system are N_u, and the total number of subcarriers are N, the baseband signal after IFFT (inverse fast Fourier transform) is [6]:

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N} \tag{1}$$

where $1/\sqrt{N}$ represents the power normalization factor, and $0 \leq n \leq N-1$; X_k denotes the modulation signals allocated to the subcarrier k. When k is the primary subcarrier, $X_k = 0$; when k is the cognitive subcarrier, it can be defined by:

$$X_k = A_k e^{j\theta_k} \tag{2}$$

where A represents the amplitude, and θ represents initial phase. Then, the PAPR is defined as [7]:

$$PAPR(dB) = \frac{P_{peak}}{P_{average}} = 10\log_{10} \frac{\max\{|x(n)|^2\}}{E\{|x(n)|^2\}} \tag{3}$$

We defined γ as the ratio of cognitive users:

$$\gamma = N_u/N \tag{4}$$

3. **PAPR in an NC-OFDM System.** When analyzing the conventional OFDM signals, we assumed that the OFDM signal $x(n)$ had an i.i.d structure, and the total number of subcarriers was great enough. According to the central limit theorem (CLT), $x(n)$ after IFFT will be subject to Gaussian distribution [3]. The probability of N sampling values less than the threshold value λ is:

$$P(PAPR \leq \lambda) = (1 - e^{-\lambda})^N \tag{5}$$

When the OFDM signal oversamples, it can be seen as adding a certain number of independent sample values. Assuming that non-oversampling of αN subcarriers were used to approximate the oversampling of N subcarriers, the CCDF (complementary cumulative distribution function) [8] can be defined as:

$$P(PAPR > \lambda) = 1 - (1 - e^{-\lambda})^{\alpha N} \tag{6}$$

It was proven in Reference [8] that enough peak points could be captured when the sampling factor $J = 4$, and the PAPR obtained was very close to its theoretical value. At this time, we used $\alpha = 2.8$ in this paper as per Reference [9], where α was recommended as 2.8.

3.1. **The Three Types of Subcarrier Distribution in the NC-OFDM System.** In an NC-OFDM system, when cognitive users are considerably less than the total subcarriers, a large number of invalid subcarriers are allocated to primary users to avoid interference with them. At this point, the assumption of i.i.d will no longer exist, and the PAPR formulas applied in a conventional OFDM system no longer apply to the NC-OFDM system. As shown in Figure 1, cognitive users have three different types of distribution in NC-OFDM system, and the corresponding PAPR characteristics of the three different types of distribution are analyzed below.

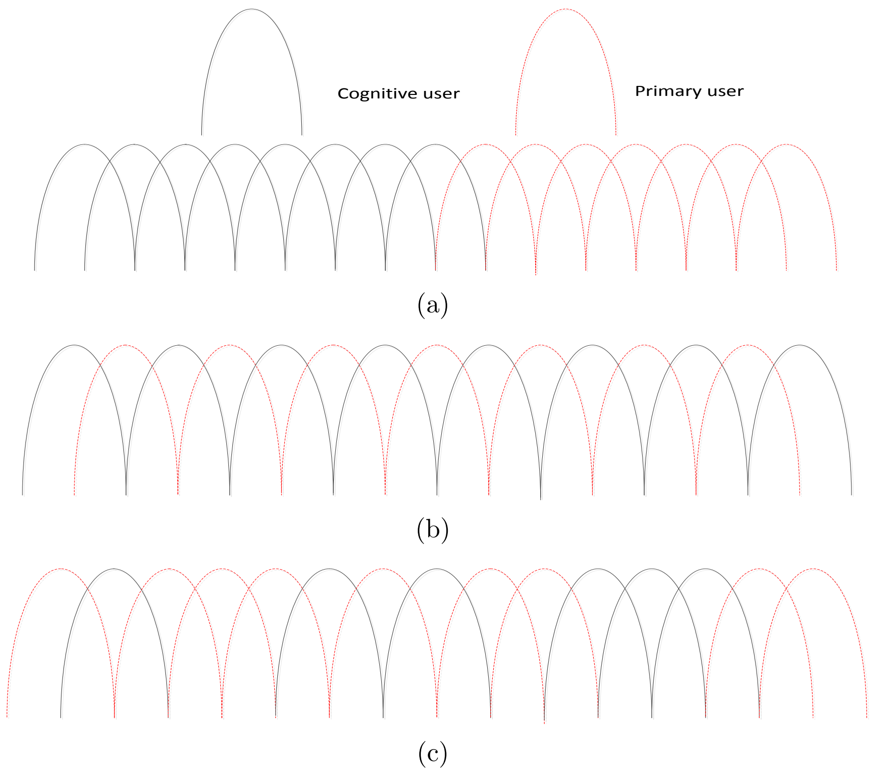


FIGURE 1. Three Types of Distribution of Cognitive Users in CR System. (a) The contiguous distribution; (b) distribution at regular intervals; and (c) random distribution.

When cognitive users are in continuous distribution (as shown in Figure 1a), the modulation data on the subcarriers is represented as [5]:

$$A_x = \begin{cases} 0, & k = 0, 1, ...n_1 - 1 \\ X_k, & k = n_1, n_1 + 1, ...n_2 - 1 \\ 0, & k = n_2, n_2 + 1, ...N - 1 \end{cases} \quad (7)$$

where X_k represents the data symbol; subcarriers between n_1 and $n_2 - 1$ correspond to cognitive users, and subcarriers beyond that range are set to 0. Since the empty subcarriers are continuous, which is equivalent to shifting the subcarriers of the OFDM signal in the frequency domain, or shifting the phase in the time domain. In fact, the PAPR characteristics of the NC-OFDM signals remain unchanged in this case.

When cognitive users are distributed at regular intervals, as shown in Figure 1b, assuming that an interval is $m-1$ primary subcarriers (N is divisible by m), the mapping data A_k in the NC-OFDM system is as follows:

$$A_k = \begin{cases} x_{im} & k = 1, m, ..., N \\ 0 & others \end{cases} \tag{8}$$

If $y_i = x_{im}$, $i = 0, 1, ..., N/m$ represents the mapping data pertaining to conventional OFDM signals, the relationship between the NC-OFDM signals and conventional OFDM signals is derived as:

$$\begin{aligned} x(n) &= \frac{1}{N}\sum_{k=0}^{N-1} A_k e^{j2\pi kn/N} = \frac{1}{N}\sum_{l=0}^{N/m-1} A_m e^{j2\pi nl/N} \\ &= \frac{1}{N}\sum_{l=0}^{N/m-1} x_{ml} e^{j2\pi nl/N} = \frac{m}{N/m}\sum_{l=0}^{N/m-1} y_l e^{\frac{j2\pi nl}{N/m}} \\ &= m \cdot IFFT[y_l]_{N/m} \end{aligned} \tag{9}$$

According to Equation (9), it can be obtained that the NC-OFDM time domain signals are m times the repetition of conventional OFDM signals. Evidently, $x(1) = x(m) = \cdots = x(N/m)$. In such cases, the PAPR is the same as that in conventional OFDM where only m sample values need to be evaluated.

When cognitive users are in random distribution, as shown in Figure 1c, if the cognitive users are noticeably less than the total subcarriers in the NC-OFDM system, the i.i.d characteristics no longer exist. The subcarriers that pertain to the primary users can significantly change the correlation of the subcarriers corresponding to cognitive users, which causes variations in the peaks of modulated symbols pertaining to cognitive users. Therefore, the PAPR in the case of random distribution in the NC-OFDM system is different from that in an OFDM system.

Figure 2 is the result of the Monte Carlo simulations (repeated 10,000 times) for the three types of distribution in Figure 1, where it was assumed that $N_u = 256$ and $N = 1024$.

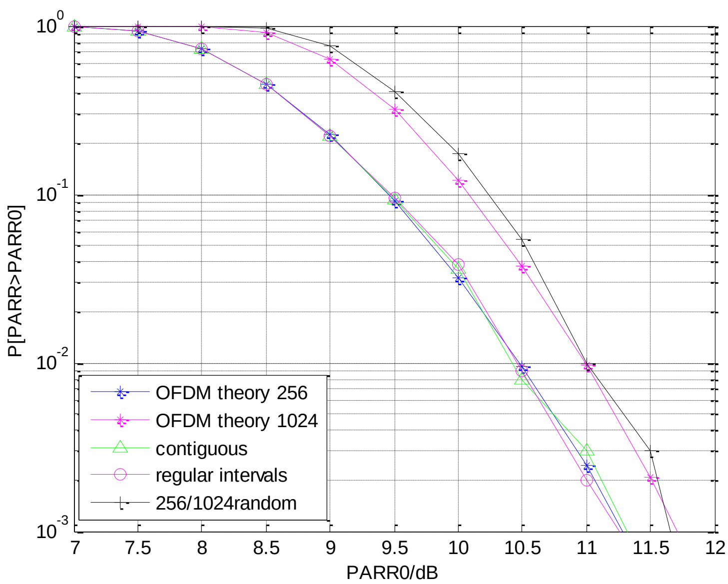


FIGURE 2. Comparison of three types of subcarrier distribution.

Figure 2 shows that in cases (a) and (b) of Figure 1, the statistical characteristics of PAPR in the NC-OFDM system is the same as the conventional OFDM. The statistical characteristics in the case of Figure 1c are different from those in the conventional OFDM, which has prominent PAPR problems. The case of Figure 1c is analyzed later in this paper.

3.2. **The Maximum PAPR in an NC-OFDM System.** PAPR reduction technology is aimed at reducing high PAPR values. As the linearity range of power devices is often subject to maximum PAPR in an NC-OFDM system, it is of great significance to calculate the maximum value of PAPR.

N indicates the total number of subcarriers in an NC-OFDM system, N_u represents the number of useful subcarriers pertaining to cognitive users, and the modulation data is represented by Equation (2).

According to Equation (3), the maximum value can be written as follows:

$$\begin{aligned}\max_{0\le n\le N-1}|x(n)|^2 &= \max_{0\le n\le N-1}|\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}X_k e^{j2\pi kn/N}|^2 = \frac{1}{N}\max_{0\le n\le N-1}|\sum_{k=0}^{N-1}X_k e^{j2\pi kn/N}|^2 \\ &\le \frac{1}{N}(\sum_{k=0}^{N-1}|X_k|)^2 \le \frac{N_u^2(A_{\max})^2}{N}\end{aligned} \tag{10}$$

According to Parseval's theorem, the average power can be written as follows:

$$E\{|x(n)|^2\} = E\{|\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}X_k e^{j2k\pi/N}|^2\} = \frac{1}{N}\sum_{k=0}^{N-1}E\{|X_k|^2\} = \frac{1}{N}\sum_{k=0}^{N-1}E\{|A_k|^2\} \tag{11}$$

and

$$PAPR(x(n))_{\max} = \frac{\max\limits_{0\le n\le N-1}|x(n)|^2}{E\{|x(n)|^2\}} \le \frac{N_u^2A_{\max}^2/N}{\frac{1}{N}\sum\limits_{k=0}^{N-1}E\{|A_k|^2\}} = \frac{N_u^2(A_{\max})^2}{\sum\limits_{k=0}^{N-1}E\{|A_k|^2\}} \tag{12}$$

If the probability of X_k is P_k, $k = 0, 1, ..., N-1$, $X_i, i \in (0, 1, .., N-1)$ is the modulation data that corresponds to $N_u(i), i \in (0, 1, ...N-1)$, the probability of $X_i$is $P_i, i \in (0, 1, ..., N-1)$, then Equation (12) can be rewritten as follows:

$$\begin{aligned}PAPR(x(n))_{\max} &= \frac{\max\limits_{0\le n\le N-1}|x(n)|^2}{E\{|x(n)|^2\}} \le \frac{N_u^2A_{\max}^2/N}{\frac{1}{N}\sum\limits_{k=0}^{N-1}E\{|A_k|^2\}} = \frac{N_u^2(A_{\max})^2}{\sum\limits_{k=0}^{N-1}E\{|A_k|^2\}} \\ &= \frac{N_u^2(A_{\max})^2}{\sum\limits_{k=0}^{N-1}P_kA_k^2} = \frac{N_u^2(A_{\max})^2}{\sum\limits_{i=0}^{N_u-1}P_iA_i^2}\end{aligned} \tag{13}$$

From Equation (13), it is known that factors influencing the maximum PAPR in an NC-OFDM system include the number of the cognitive users N_u, amplitude A_k, and the probability of modulation symbols P_k. When constant envelope modulation is used, the maximum PAPR can be expressed as $PAPR_{\max} = N_u$; when non-constant envelope modulation is used, the maximum of PAPR is related not only to the number of cognitive users N_u, but also to the amplitude A_k and the probability of the modulation symbol P_k.

3.3. **Analysis of the Autocorrelation between Symbols in an NC-OFDM System.** Unlike the conventional OFDM, the useful subcarriers pertaining to cognitive users in an NC-OFDM are less than the total subcarriers, and the assumption of i.i.d characteristics does not exist. PAPR cannot be calculated using the equations used in an OFDM system. Since PAPR in an NC-OFDM system depends on the aperiodic autocorrelation property of signals in a symbol period, PAPR can be analyzed by studying the autocorrelation property between symbols in the NC-OFDM.

The modulation data X_k satisfies the following conditions in an NC-OFDM system:
(1) $n \neq m, E(X_n X_m^*) = 0$, the cross correlation is 0;
(2) $X_n X_n^* = 1$, where subcarrier n belongs to the cognitive users;
(3) $X_n X_n^* = 0$, where subcarrier n belongs to the primary users.
Accordingly,

$$E(X_n X_n^*) = \frac{1}{N}(N_u \times 1 + (N - N_u) \times 0) = \frac{N_u}{N} \tag{14}$$

The autocorrelation function pertaining to $x(n)$ is:

$$\begin{aligned} R(m,n) &= E\{x(m)x^*(n)\} \\ &= \frac{1}{N}E\{[\sum_{k=0}^{N-1} X_k e^{j2\pi km/N}][\sum_{l=0}^{N-1} X_l e^{j2\pi kl/N}]*\} \\ &= \frac{1}{N}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} E(X_k X_l^*)e^{j2\pi(km-nl)/N} \end{aligned} \tag{15}$$

Accordingly,

$$\begin{aligned} R(m,n) &= \frac{1}{N}\sum_{k=0}^{N-1} E(X_k X_k^*)e^{j2\pi(m-n)k/N} \\ &= \frac{1}{N}\frac{N_u}{N}\sum_{k_1=0}^{N-1} e^{j2\pi(m-n)k/N} \end{aligned} \tag{16}$$

where k_1 represents the sequence number of subcarriers pertaining to the cognitive users; the autocorrelation function pertaining to the random process {x(n),0 ≤ n ≤ N-1} is only related to time difference, and is a wide stationary stochastic process [**?**]. For the sake of uniformity, the subcarrier sequence number k is used in Equation (16), and random variable a_k is introduced; when subcarrier k belongs to the primary users, $a_k = 0$. Thus a_k obeys the (0–1) distribution. Let $\varepsilon = m - n$, then the autocorrelation function is rewritten as:

$$R(m,n) = R(\varepsilon) = \frac{N_u}{N^2}\sum_{k=0}^{N-1} a_k e^{j2\pi\varepsilon k/N} \tag{17}$$

where a_k obeys the (0–1) distribution so that the function above has no analytical solution. The Monte-Carlo simulation was used to simulate the autocorrelation function. To compare the correlation of $x(n)$, the autocorrelation coefficient was adopted to measure the correlation, which was defined as follows:

$$\rho(\varepsilon) = \frac{R(\varepsilon)}{\sqrt{D(x(n))}\sqrt{D(x(m))}} \tag{18}$$

where

$$D(x(n)) = D(x(m)) = E\{x(n)x^*(n)\} = \frac{N_u^2}{N^2} \tag{19}$$

Accordingly,

$$\rho(\varepsilon) = \frac{N^2}{N_u^2} R(\varepsilon) = \frac{1}{N_u} \sum_{k=0}^{N-1} a_k e^{j2\pi\varepsilon k/N} \tag{20}$$

It can be seen from Equations (17) and (20) that when total number of subcarriers N was constant, N_u were the main factors influencing the autocorrelation coefficient ρ; when total number of subcarriers N and the number of cognitive users N_u were constant, different subcarrier allocation schemes also caused a change in autocorrelation coefficient ρ. Then, we conducted simulations for the above-mentioned influencing factors to further verify the relationship between the correlation coefficient ρ, total number of subcarriers N, the number of cognitive users N_u, and different allocation schemes.

4. **Simulations and Analysis of Results.**

4.1. **The Relationship between PAPR and the Modulation Methods in NC-OFDM.** Monte Carlo simulation (repeated 10,000 times) was carried out by using modulation methods of BPSK, QPSK, and 16QAM, respectively in the different modulation modes of Equation (12) with $N = 1024$ and $N_u = 16,\ 32,\ 64$. Results of the simulations are presented in Figure 3.

It can be seen from Figure 3 that when N_u is less than 64, the statistical probability of PAPR follows the order in the following: QPSK < BPSK < 16QAM. The reason for it is: QPSK and BPSK represent the constant envelope modulation, and carry no crest factor (CF) [11], while 16QAM represents the non-constant envelope modulation, and carries some CF by modulation itself; compared with BPSK, QPSK has more initial phases, therefore causing lower probability of amplitude overlay. When N_u equals or is greater than 64, the PAPR probability is approximately equal in the three modulation methods. The modulation methods can be selected according to the efficiency at the transmitter and the code error rate at the receiver.

4.2. **The Relationship between ρ, N_u and PAPR.** Monte Carlo simulation (repeated 10,000 times) was carried out using modulation methods of QPSK with $N = 1024$ and $N_u = 16,\ 32,\ 64$. Results of the simulations are presented in Figure 4.

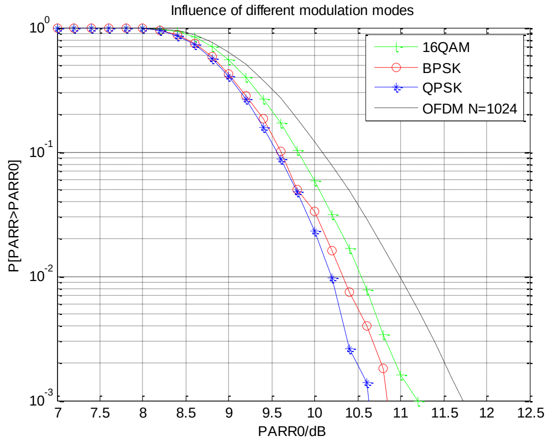
It can be seen from Figure 4 that an increase of cognitive users causes a higher statistical probability of PAPR; when N_u equals or is greater than 64, the PAPR probability is very close to the value of Complementary Cumulative Distribution Function (CCDF) pertaining to PAPR in an OFDM system with $N = 1024$.

According to Equation (20), Figure 5 uses N = 1024, and shows the simulations for autocorrelation coefficient ρ when $N_u = 32,\ 128,\ 512$.

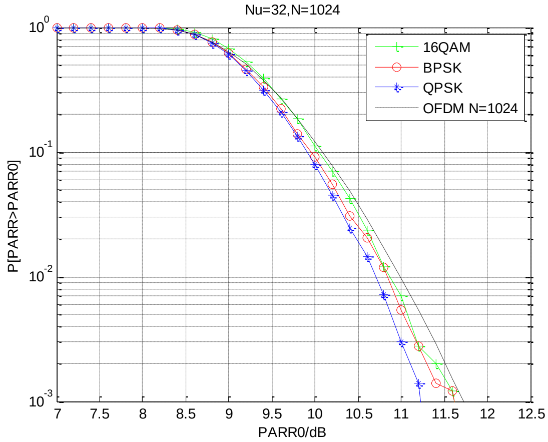
It can be seen from Figure 5 that the autocorrelation coefficient exhibits monotone decreasing with the increase of N_u, and greater N_u causes a smaller dynamic range of the autocorrelation coefficient. It can be seen that the main factor influencing the autocorrelation coefficient is the size of N_u.

4.3. **The Relationship between ρ and Different γ.** Figure 6 represents the mean values of the autocorrelation coefficient ρ obtained when the total number of subcarriers $N = 64,\ 128,\ 256,\ 512,\ 1024,\ 2048,\ 4096$, the ratio of cognitive users γ is between 0 and 1, and 10,000 times Monte Carlo simulation were performed.

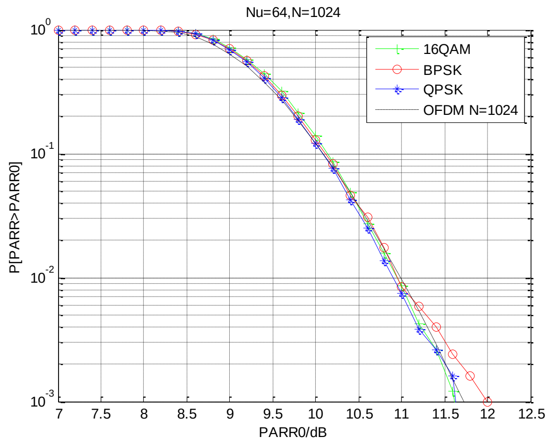
Figure 6 shows that when N is constant, greater γ leads to smaller autocorrelation coefficient; with the same γ, greater N causes a smaller autocorrelation coefficient.



(a)



(b)



(c)

Figure 3. The statistical characteristics of PAPR using different modulation methods. (a) PAPR with different modulation methods, when $N_u = 16$; (b) PAPR with different modulation methods; when $N_u = 32$; and (c) PAPR with different modulation methods, when $N_u = 64$.

4.4. **The Relationships between Different Allocation Schemes and ρ.** Figure 7 shows that the relationship between the autocorrelation coefficient ρ and different allocation schemes when cognitive users are allocated to different places with $N = 1024$, $N_u = 32$.

It can be seen from Figure 7 that when there is the same N_u, the mean values of the autocorrelation coefficients ρ are different with different allocation schemes. The following is an analysis of the relationship between ρ and PAPR while the mean values of the autocorrelation coefficients are different.

4.5. **The Relationship between ρ and PAPR.** As presented in Figure 8, a Monte Carlo simulation (repeated 100,000 times) was performed with $N = 1024$ and $N_u = 32$; and the simulation results showed that the statistical probability of PAPR exhibited monotonic increasing with the increase of the autocorrelation coefficient ρ. With cognitive users N_u and the total number of subcarriers as N, the allocation scheme where $\rho = 0.0739$ helped reduce the PAPR by 6%, rather than the allocation scheme with $\rho = 0.1239$.

5. **Conclusions.** The PAPR of an NC-OFDM system depends on the non-periodic autocorrelation characteristics for data within a symbol period. In this paper, we first analyzed

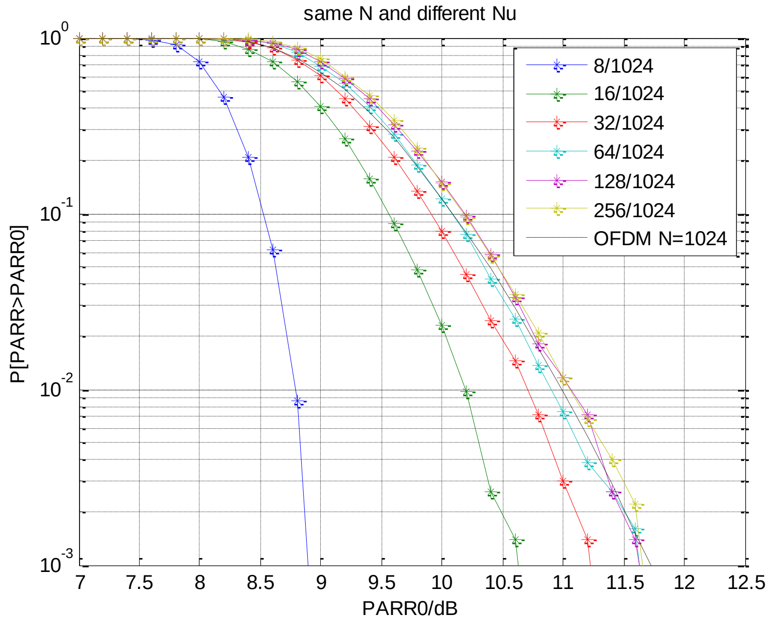


FIGURE 4. The statistical probability of PAPR with different N_u.

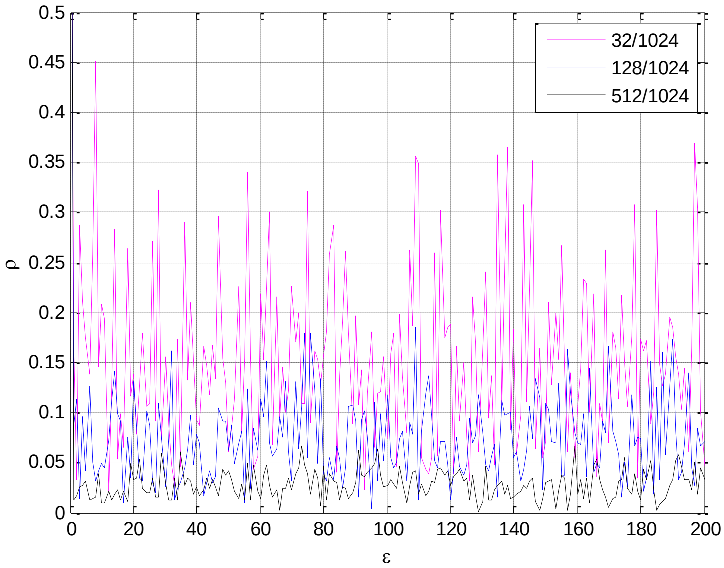


FIGURE 5. The relationship between autocorrelation coefficient and number of N_u.

the statistical probability of PAPR in three different distributions of cognitive users in a cognitive radio system. Second, for random distribution, we defined the autocorrelation coefficient to represent the schemes of different subcarriers distribution. When the number of subcarriers that corresponded to cognitive users N_u was constant, the PAPR exhibited a monotonically increasing relationship with the autocorrelation coefficient, the maximum value of PAPR was deduced in different modulation modes at the same time. Simulation results showed that non-constant envelope modulations had a higher statistical probability of PAPR when compared with constant envelope modulations; when N was constant, N_u was the major factor influencing PAPR; when N and N_u were constant, different allocation schemes for cognitive users led to varied autocorrelation coefficients ρ, and the increase of ρ came with the monotonic increasing of PAPR. Thus, choosing

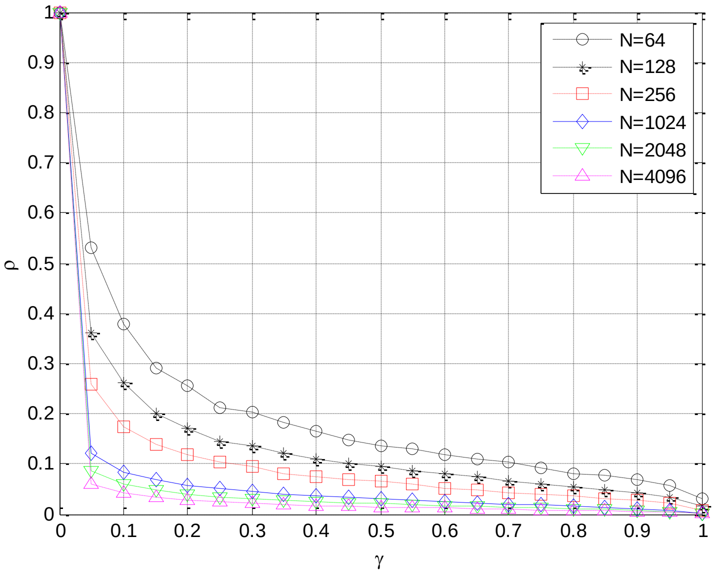


FIGURE 6. The Trend of Autocorrelation Coefficient with N and N_u.

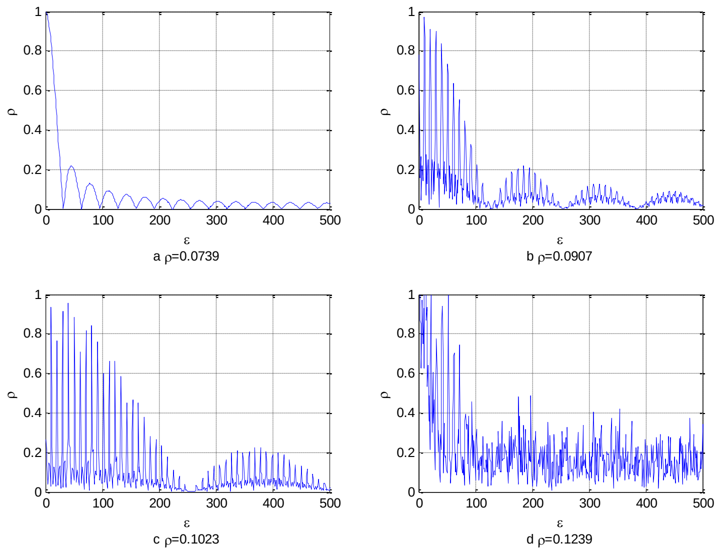


FIGURE 7. The autocorrelation coefficients in different allocation schemes.

an appropriate allocation schemes in accordance with the autocorrelation coefficient can help reduce PAPR by approximately 5–6%.

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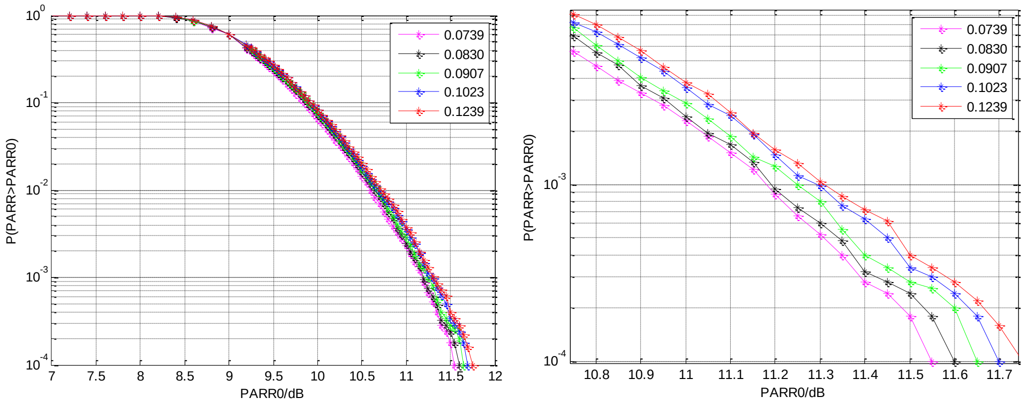

(a) (b)

FIGURE 8. The relationship between ρ and PAPR. (a) The relationship between ρ and PAPR; and (b) The amplified relationship between ρ and PAPR.

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