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An Improved Calculation Method of High-frequency Winding Losses for Gapped Inductors

Xiaosheng Huang[1] and Wei Chen[2]

[1]College of Information Science and Engineering, Fujian University of Technology,
No.3, Xueyuan Road, University Town, Minhou, Fuzhou, 350118, China
[2]College of Electrical Engineering and Automation, Fuzhou University,
No.1, Xueyuan Road, University Town, Minhou, Fuzhou, 350116, China
huangxiaosheng2010@qq.com;chw@fzu.edu.cn

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Abstract. *For the inductor with air gap, the magnetic field distribution in winding window is quite complex due to the air-gap fringing effects of the core. The Dowell model which is effectively used in the calculation of AC winding losses in transformers is no longer valid for gapped inductors. In this paper, the air-gap is equivalently replaced by surface current with certain geometry and current density distribution. And then, mirror-image method is utilized to deal with the boundary effect of magnetic core with high permeability. Most important, an novel analytic method is also proposed to improve the calculation speed and accuracy of high-frequency losses of round conductor within wider frequency range. Finally, the method proposed is verified by FEM (finite element method) simulation.It keeps very low error, which is less than* 8%*, when the ratio of wire diameter to skin depth reaches* 4.5*.*

Keywords: Calculation method; Gaped inductor; AC winding losses; Analytic model

1. **Introduction.** With the trend of power electronic converters towards high frequency, the calculation accuracy of magnetic component losses caused by high frequency eddy current affects the design and optimization of magnetic component directly. Normally, magnetic component losses consist of core losses and winding losses. The core losses are usually calculated by Steinmetz model [1]. And the numerical and analytic method are applied in the calculation of high frequency winding losses. For the winding losses of transformer, the magnetic field distribution in the winding window is quite regular and the fields are parallel with the winding layers. It can be simplified for the 1D model. And the Dowell theoretical model can be used for calculation [2–4]. However, for the inductors with high permeability core and air-gaps, such as PFC inductor, flyback transformer, etc., will have flux fringing effects near the air gaps resulting in complex magnetic field distribution in the winding window [5, 6]. The location, size, and the number of air gaps and even the shape of winding windows will affect the field distribution. Therefore, it is a great challenge for the theoretical calculation of winding losses in gapped inductor. Although the FEM simulation is a potential way to calculate the winding losses, it is not suitable for wide usages in small and medium-size companies because of its high cost and trained engineers. Besides, for the winding with a lot of turns especially for the strand Litz wires, there may be thousands of round conductor objects in the model. It needs very high cost of computer resources even with 2D simplification, not to mention the

optimization. On the other side, the analytic method contains clear physical concept and faster calculation speed, which is the key factor of the wire optimization [7–14].

Literature [15] presents line shape surface current, which replaces the actual air gap with same magnetic potential difference on the air gap, with evenly density distribution. Meanwhile, the mirror-image method is taken in to eliminate the effect of core boundary equivalently, and the current distribution in free space can be got. And then, the magnetic density of each conductor can be calculated through Biot-Savart Low. Furthermore, the winding losses can be calculated through the approximate equation of skin effect loss and proximity effect loss in the current-carrying round conductor in the uniform external magnetic field in free space [16]. However, the method in [15] keeps accurate only if the air gap is small or the conductors are far away from the air gap. On the other hand, the present equations for calculating the proximity eddy current losses in a round conductor is only accuracy with the condition that the eddy current effect will not influence the magnetic field distribution in the conductor, i.e. the magnetic field distribution is supposed to be uniform in the conductor. This condition is valid when the ratio of round conductor diameter to the skin-depth is less than 1.5, which will limit the applicative frequency range and conductor diameter.

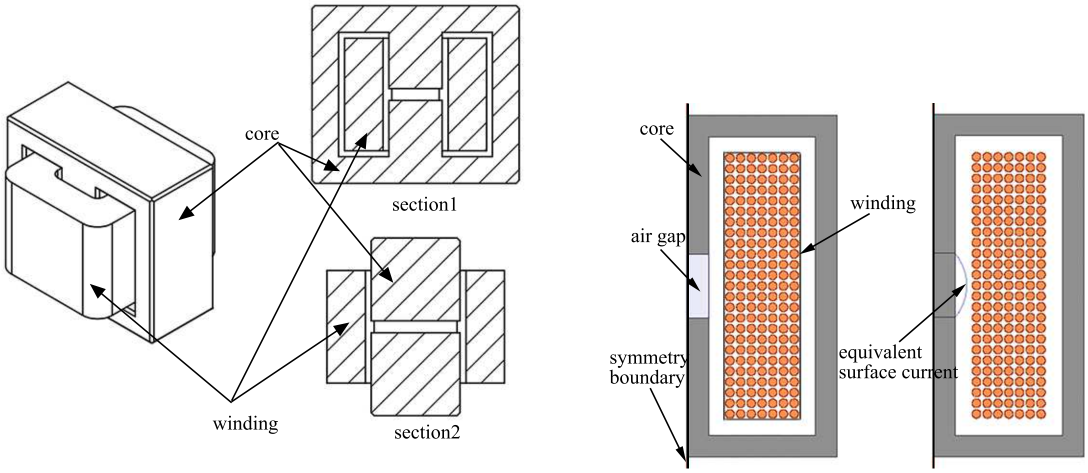
Literature [17] uses the nonlinear arc equivalent current source instead of line shape current source, and get a higher accuracy when doing equivalent treatment of the air gap. Unfortunate, the calculation accuracy of proximity effect losses in [17] is also limited as in [15].

Compared with the methods in literatures above, an improved analytic method, which has higher accuracy and keeps the calculation efficeincy due to the using of nonlinear equivalent current and improved analytic calculation of proximity effect, is proposed in this paper. The method keeps high accuracy even when the ratio of round conductor diameter to the skin-depth reaches 4.5. Presently, there is no effective methods to measure only the winding losses of inductor with magnetic core due to the difficulty in separating the winding and core losses. So, the FEM simulation is used to verify the accuracy of the proposed method [18, 19].

2. **Equivalent Treatment of The Air-gap.** Actual gapped inductors are always three dimensions. Normally, dual 2D method is applied to approximate the 3D model by two 2D models with acceptable accuracy of engineering application [20]. So the analysis in this paper is based on 2D model as in Fig.1

For gapped inductors, magnetic potential difference in the core can be neglected due to the ultda high permeability of the core. So, all of the MMF (Magnetic Motive Force) by winding current is balanced in the air-gap. [15] presents an equivalent surface current source to take the place of the air gap, and the magnetic field distribution in the winding window will not be changed.

It uses a linear surface current source which length is equal to that of the air gap in [15]. And the surface current density distribution is uniform. The total ampere-turns of the surface current is equal to the magnetic potential difference in the air gap. The linear surface current source can be equivalent to small air gap quite well. Otherwise, it will cause larger error if air gap is bigger or the winding get close to the air gap, and the magnetic field near the air gap should be considered. To solve this problem, FEM software is used in this paper to analyze the distribution characteristic of the magnetic field around the air gap as Fig.3 shows. According to the electromagnetic field theory, the direction of flux line stands for the direction of magnetic field and the distribution density means its strength. According to the uniqueness theorem of electromagnetic field, the function of any flux line to the surrounding areas can be replaced of the surface current



(a) Actual inductor with air-gap (b) Equivalent treatment of air-gap

FIGURE 1. Actual inductor model and its 2D half model

along it, and the surface current density is equal to the magnetic field intensity along the flux line. Therefore, this paper uses the method in [17], i.e. the arc shape equivalent surface current source is applied to eliminate the air gap. According to the simulation results of magnetic field distribution characteristic around the air gap, the applied arc section a-b, which angle is 80 degrees across the endpoints a and b, is shown in Fig.2. Its surface current density corresponds to the magnetic field intensity on the flux line which is shown on the right of Fig.2, and the total ampere-turns are equal to the magnetic potential difference of the air gap.

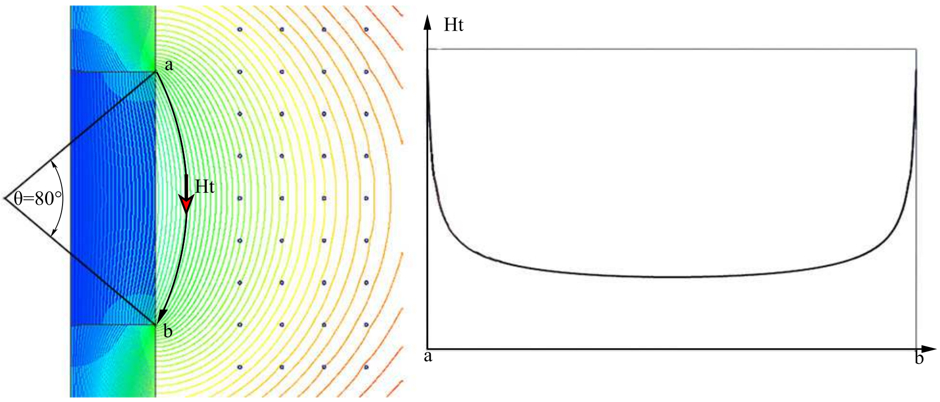
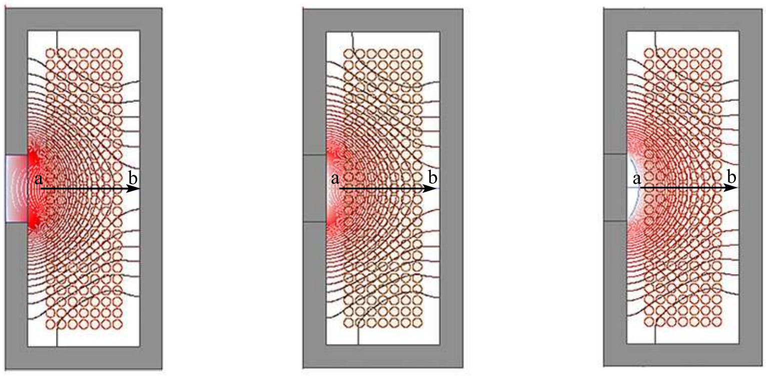


FIGURE 2. Flux lines around air gap (left) and field strength along line a-b (right)

Fig.3 illustrates magnetic field distribution of FEM model with the actual air gap, the linear line and the nonlinear arc equivalent current sources, respectively. The comparison of flux density distribution on line a-b in the 3 cases is in Fig.4. As B0 is the flux density at point 'a', we can see that the result of nonlinear arc current source is more close to that of the model with actual air gap, especially in the area near the air gap. Therefore, the nonlinear arc equivalent current source can get a higher accuracy when doing equivalent treatment of the air gap.

3. **Equivalent Treatment of the Winding Window Core Boundary.** In order to calculate the magnetic field in the winding window using analytic equations and get



(a) Air gap (b) Linear line (c) Nonlinear arc

FIGURE 3. Flux lines distribution in winding window with air gap and different surface currents

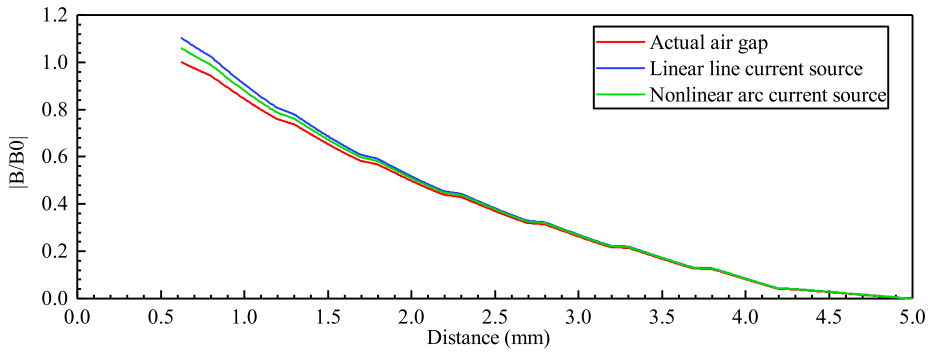


FIGURE 4. Comparison of Flux density distribution in line a-b

winding losses, it needs to do equivalent treatment for winding window boundary to get rid of the magnetic core. As in Fig.3, according to the boundary condition of magnetic field, the flux lines perpendicular to the core boundary if its permeability is high enough.

Therefore, the mirror-image method can be used in the equivalent treatment of core based on the uniqueness theorem of electromagnetic field just as showed in Fig.5. With the original winding window in the center of the whole images, the accuracy for engineering applications will be satisfied if there are 14 times images.

4. Improved Calculation of High-frequency Winding Losses in Round Conductor.

4.1. Limitation of Present Winding Losses Calculation Method.

After equivalent treatment of air-gap and ferrite core, there are only current-carrying conductors exist in the free space. In practice design, in order to reduce the eddy current effects, wire diameter was usually small enough. So it is reasonably assumed that the currents of each wire turn are concentrated in the center of wire. So field distribution in winding window could be simply got by Biot-Savart Low as

$$\vec{B}(x,y) = \frac{\mu_0}{2\pi} \cdot \sum_{k=all\_currents} \frac{\vec{I}_k \times \vec{r}_0}{r} \tag{1}$$

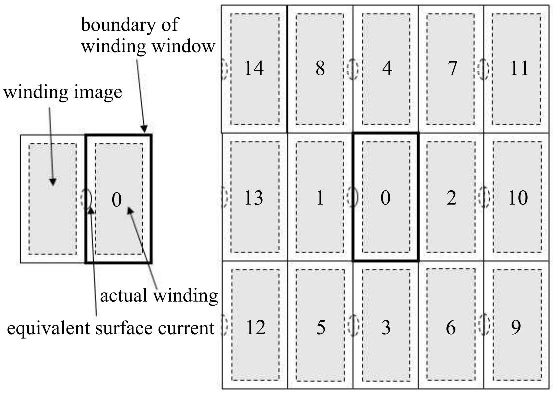


FIGURE 5. Images of winding window

where, r was the distance from the field point (x, y) to the current source $\vec{I_k}$, which should include all of the currents, i.e. winding, equivalent air-gap and all mirror-image current sources. $\vec{r_0}$ was unit vector from source to the field point.

With the known magnetic flux density in each wire conductor by (1), the winding losses of the wire, i.e. round conductor could be approximately calculated by

$$P_{total} = P_{skin} + P_{prox} \tag{2}$$

where,

$$P_{skin} = \frac{1}{2\sigma}\int_0^{r_0} 2\pi r \left|\frac{\alpha I}{2\pi r_0}\frac{I_0(\alpha r)}{I_1(\alpha r_0)}\right|^2 dr \tag{3}$$

$$\alpha = \sqrt{\omega\sigma\mu_0} \tag{4}$$

$$P_{prox} = \frac{\pi\sigma\omega^2 B_0^2 (2\cdot r_0)^4}{128}\cdot l \tag{5}$$

where, ω is angular frequency, σ is conductivity, r_0 is wire radius of round conductor, I means the peak current of wire, B_0 is the external magnetic density, and l is the length of wire [16].

Equation (5) is based on the assumption that magnetic field distribution in the conductor is uniform. It is reasonable if the eddy current effects are not strong enough with the ratios of conductor diameter to the skin depth, noted as $DS = \frac{d}{\delta}$, are smaller than 1.5. However, it is not true if the operation frequencies become higher when the eddy current effects are strong enough to make the flux distribution un-uniform as in Fig.6 [21]. Fig.7 shows the comparison of calculated losses results by present analytic method in [16] and FEM simulation, where $Loss0$ is the losses at $DS = 0.5$ by FEM simulation. It is shown the error is below 5% when $DS < 1.5$. If $DS = 4$, the error will reach to 219%.

4.2. **Improved Analytic Calculation Method.** According to Faraday's Law, EMF (Electromotive Force) produced by alternating fluxes tends to opposite the flux changing. In conductors, the EMF will produce eddy currents to offset the external magnetic field exciting on the conductor, resulting that the flux density in the conductor is lower than that of outside. Suppose the external flux density is $\boldsymbol{B}_o$, and the average flux density considering the eddy current effect is $\boldsymbol{B}_{sn}$. The calculation of $\boldsymbol{B}_{sn}$ is the key to calculate

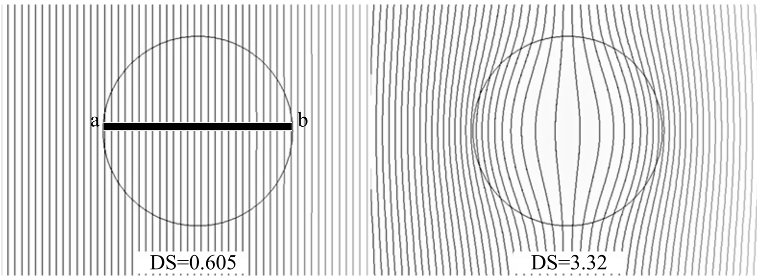


FIGURE 6. Flux line distribution in wire under different frequency

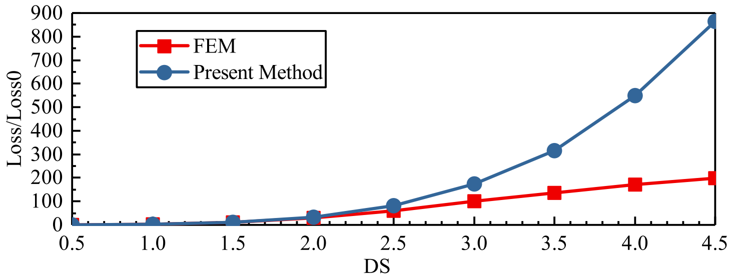


FIGURE 7. Comparison of winding loss by analytic method and FEM

the actual equivalent flux density in the conductor $\boldsymbol{B}_{eq}$ exactly. The deduction follows according to Fig.8.

$$\boldsymbol{E} = -C_1 \cdot f \cdot \boldsymbol{B}_o \cdot e^{j\frac{\pi}{2}} \tag{6}$$

$$\frac{j\omega L}{R + j\omega L} \cdot \boldsymbol{E} = C_1 \cdot f \cdot \boldsymbol{B}_{sn} \cdot e^{j\frac{\pi}{2}} \tag{7}$$

$$\boldsymbol{B}_{eq} = \boldsymbol{B}_o + \boldsymbol{B}_{sn} = \left(1 - j \cdot e^{j \cdot \theta} \cdot \sin\theta\right) \cdot \boldsymbol{B}_o \tag{8}$$

$$\theta = \arctan\left(\frac{\omega L}{R}\right) \tag{9}$$

$\boldsymbol{E}$ means induced electromotive force, C_1 is a constant, L is the equivalent inductance of conductor, R is the equivalent resistance of conductor, and ω means angular frequency.

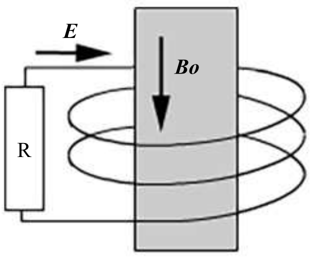


FIGURE 8. Equivalent Circuit of Proximity Effect

For the round conductor, it is approximately regarded as:

$$R \propto \frac{\rho}{d^2} \tag{10}$$

$$L \propto \mu \cdot d^{\alpha} \tag{11}$$

$$\theta = \arctan(\frac{C_z \cdot f \cdot \mu \cdot d^{\alpha+2}}{\rho}) \tag{12}$$

μ is the permeability of conductor, that is permeability of vacuum; C_1 and C_z are constants related to the section shape of conductor. According to the data analyzed by FEM, for the round conductor, α takes 0.25, and Cz takes 2.28. The equation (10) and (11) can be improved with a further work which has not been done in this paper. We can see that there is a scaling factor between the actual flux $\boldsymbol{B}_{eq}$ and external flux outside the conductor B0 which showed in (14). Equation (5) can be modified as follows after analyzing above.

$$C_e = \left|1 - j \cdot e^{j \cdot \theta} \cdot \sin\theta\right| \tag{13}$$

$$P_{\Pr ox} = \frac{\pi(2\pi f)^2 B_0^2 (2 \cdot r_0)^4}{128\rho} \cdot {C_e}^2 \cdot l \tag{14}$$

In order to verify the applicability of scale factor C_e proposed, both FEM simulation and equation (13) are applied to calculate the flux through the line,i.e. a to b, which is illustrated in Fig.8. As in Fig.9, the scale factor C_e can accurately evaluate the reduction of equivalent flux density at higher operation frequency,which is up to $DS = 4$. Therefore, the application frequency range and accuracy of the analytic calculation method for the winding losses are improved significantly.

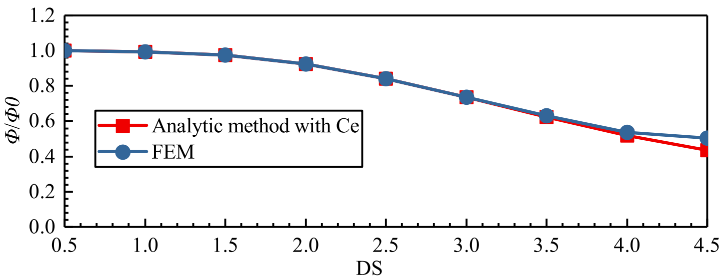


FIGURE 9. Comparison of the flux (a-b) calculated by analytic method proposed and FEM

5. **EXAMPLE AND VERIFICATION.** The inductor mentioned above is taken as example and verified by FEM simulation. In section 4.1, DS should lower than 1.5 in (5) for making sure the calculation accuracy of proximity effect loss. Most of the time, the operation current of gapped inductor is no-sinusoidal and there is high frequency component in the current wave. The calculation accuracy of high frequency harmonic losses should be ensured for evaluation of winding losses overall. It is very important for the design and optimization of gapped inductor.

The simulation parameters in Fig.10 are the same as that in Fig.7. The DS equal to 4.5 at the highest frequency point which is about 9 times that of $DS = 1.5$. It shows that the applicative frequency is extended significantly by (14). The calculation error is about 7.7% when $DS = 4.5$, while the error reaches to above 300% by (5). Therefore, the calculation accuracy of winding losses in gapped inductor will be improved by (14).

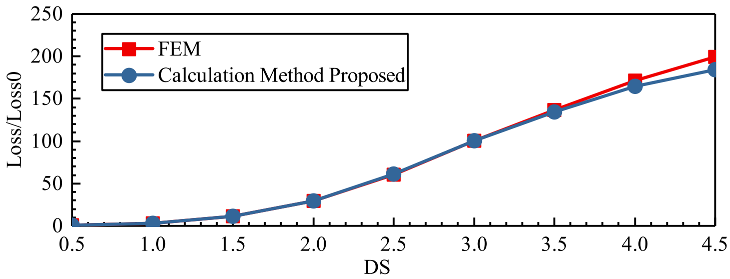


FIGURE 10. Comparison of Winding Losses by Analytic Method Improved and FEM

6. **Conclusion.** In this paper, equivalent treatment method, by using an arc-sharped surface current, is applied to eliminate air gap. The equivalent surface current with nonlinearly distributed current density is assigned along the flux line near the air gap. The equivalent treatment method has better accuracy than present method which replacing air gap with an straight-shaped surface current. An improved analytic calculation method of high-frequency losses in round conductor is proposed. The applicative frequency range and the wire diameter are extended significantly. And it keeps very low error, which is less than 8%, when the ratio of wire diameter to skin depth reaches 4.5. The accuracy of proposed method was verified through FEM simulation. The computational efficiency(speed) and accuracy are both improved significantly, contrast to FEM simulation. It completely meets the requirement of engineering design and optimization of gapped inductor.

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