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Upscale Gray Image using Mixing Transform Generation based on Tensor Product

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Abstract. *The increased size of grayscale images or upscale plays a central role in various fields such as medicine, satellite imagery, and photography. This paper presents a technique for improving upscaling gray images using a new mixing wavelet generation by tensor product. The proposed technique employs a multi-resolution analysis provided by a new mixing wavelet transform algorithm to decompose the input image into different frequency components. After processing, the low-resolution input image is effectively transformed into a higher-resolution representation by adding a zeroes matrix. Discrete wavelets transform (Daubechies wavelet Haar) as a 2D matrix is used but is mixed using tensor product with another wavelet matrix's size. MATLAB R2021b is used as the main program, and the performance metrics of upscale are evaluated in gray images. The results exhibit the supremacy of the proposed approach in terms of visual quality and quantitative metrics. The upscale by mix transform using wavelet transform provides efficient upscale quality metrics as compared with the technique that upscales image by mix transform using Slantlet transform, which has an upscaled blurred image. Also, wavelet and Slantlet upscaled Peak Signal-to-Noise Ratio (PSNRus)=79.4698 and 76.6695, respectively. This approach is often used in cryptography and computer vision.*
Keywords: upscale image; mixed transform; tensor product; orthogonal matrices; discrete w- avelet transform; quality metrics

1. Introduction.

Image upscaling (generally image interpolation) techniques are implemented in various computer tools [1]. It is a method of enlarging the number of pixels in an area inside the image and usually present in image processing applications such as multiple image descriptions, super-resolution, and facial reconstructions [2, 3, 4]. Discrete wavelet transform is one of the more effective tools in image processing in frequency and time domains, allowing for a better image localization feature analysis. Orthogonality is an important aspect of signal processing because it is a simple way to build control and data structures in languages. It always reflects a right angle, 90°. Orthogonal wavelets are relevant to wavelet transform and are orthogonal with each other [5]. In discrete wavelet transform, there are different accessible filters, but Daubechies and Haar are widely used [6].

A mixed transform is a composite transform that refers to the attempt to simultaneously apply multiple image processing operations or transformations to an image [3, 7]. The upscaling images process involves analysis and synthesis. The analysis decomposes the input low-resolution image "original" into different frequency sub-bands, LL, LH, HL, and HH, using the mixing wavelet transform. The high-frequency sub-bands are processed to extract fine details and edge information. In the synthesis, the enhanced high-frequency HH sub-bands are combined with the original low-frequency sub-band to reconstruct an improved, higher-resolution image [3, 4, 6, 8, 9]. A low-SNR and low-resolution images are low-quality images [10]. Grayscale image sharpening improves and enhances the image quality and detected edges. The sharpening enhances the clarity of the image by enhancing the features of all objects present in an image [11].

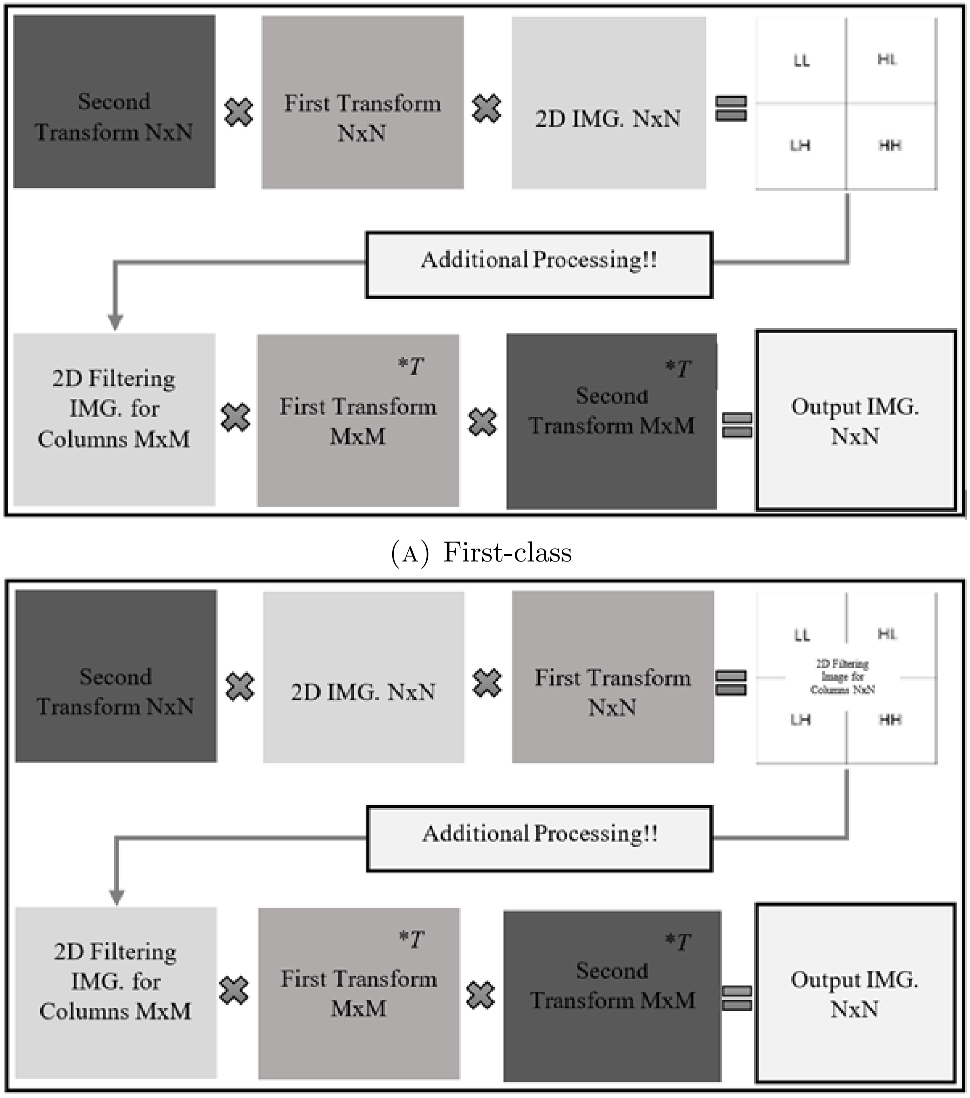
2. **Related work.** Andrea et al. proposed heuristics, statistical learning, and edge modeling then upscaling method based on two-step grid filling is used to enlarge image and enhance its quality [12]. Amanjot et al. presented a comparison of the various upscaling methods for images using image quality matrices. Upscaling gaussian filter based on denoising for images polluted with different noises has been considered [2]. A multi-dimensional upscaling to grow the image in depth and size is used via intermediate stages utilizing subscale pixel network [13]. Wanjie et al. proposed a differentiable up-scaling module which is employed to up-scale the low-resolution results from its underlying high-resolution counterpart [14]. Alptekin et al. suggested image upscaling involving cycle spinning consists of two stages, wavelet-domain zero padding and the application of cycle spinning operator [15].

Nathaniel et al. proposed an image upscaling using deep machine learning based on real-time image of super-resolution with a residue of neural network on prevailing resources at the receiver end [16]. Zainab et al. evaluated the operation of hybrid techniques consisting of wavelet (W), multi-wavelet (M), and mixed tensor product (T) transforms as 1-level, WT and MT as 2-level and WWT, WMT, MWT, and MMT as 3-level techniques. The results, which evaluate metrics such as peak signal-to-noise ratio and compression ratio indicated that the best technique is MMT [17]. This paper presents a method for upscaling grayscale image using a mixing transform generated by a tensor product. The method involves using discrete wavelet transform (Daubechies wavelet Haar) as a 2D matrix, which is then mixed with another additional matrix's size using tensor product. The proposed technique achieves significantly better upscale quality metrics than the compared technique, which results in an upscaled blurred image.

In this paper, Sec. 3 suggests the method and materials of the mixed transforms for the upscale image used. Sec. 4 introduces the results and discusses the technique's performance, and Sec. 5 provides the conclusion and future works.

3. **Materials and Methods.**

3.1. **Conventional mixed transform algorithms.** The conventional mixed transform algorithm has two classes. The first-class has the process of a 2D grayscale image using the conventional mixing methods. A multiplication between two transforms as matrices and then multiplied by a 2D grayscale image to generate a new filter [7, 18]. In the reconstruction step, gray image [11, 19] is multiplied by the result of multiplication the conjugate transpose of the two transforms (where *T is a conjugate Transpose). The second-class algorithm also uses two matrices but changes the order of multiplication. The two classes are shown in Figure 1 and both have not generated a new transform. The main benefits of using mixed transforms include enhanced feature extraction, robustness to variations, and improved generalization.



(A) First-class

(B) Second-class

FIGURE 1. Conventional mixed transform algorithms

3.2. **Mixed transforms for upscale.** Upscaling gray images using wavelet or wavelet mixed transform presents a promising avenue. However, finding mixed transform matrices that gain all the advantages of all transforms in classical methods may not improve performance as much as in [7], where the conventional mixed transform method for image processing is used. The mixed transform for image processing and upscale (where T is a Transform) is shown in Figure 2.

The new property in this paper can be functional for our investigation, and the new property is (all details can be seen in [7]):

Theorem: If $[R] : Z \to Z$ and $[T] : Z \to Z$ both are square matrices full orthogonal, then, on a matrix form, their tensor product will be given by $([R] \otimes [T]$ or $[T] \otimes [R] =$

$[K] : Z \rightarrow Z)$. The matrix $[K]$ will be square full orthogonal, where Z is a set of complex numbers or vector's numbers.

FIGURE 2. Mixed transform for image processing [2] and upscale

Based on the above theorem, which can be seen its proof in [7], a digital converter or transform using the tensor product is built and can be reconfigured, as seen in Figures 3 and 4. Where Figure 3 shows the proposed technique representation for upscale using a tensor multiplier, and Figure 4 shows a representation of how to add a zero matrix.

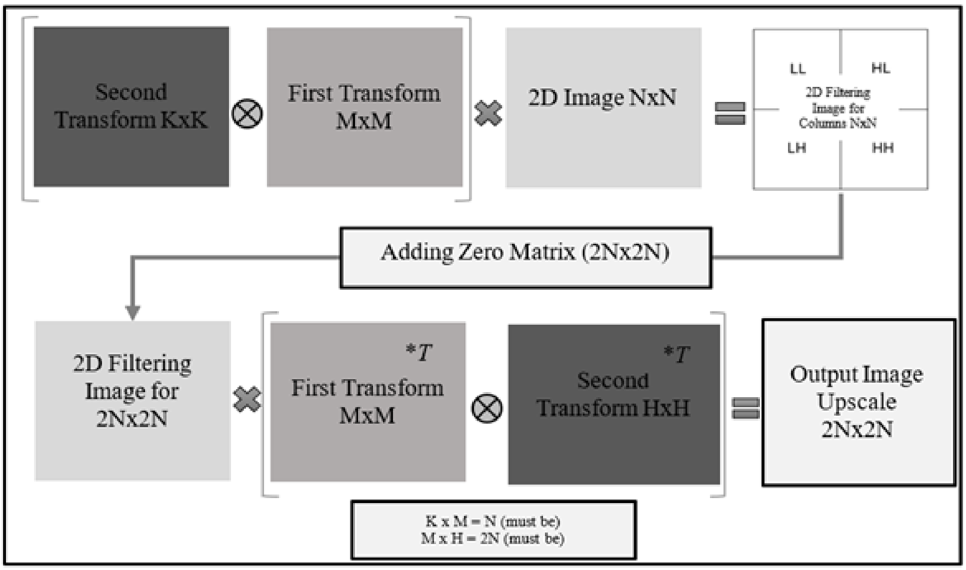


FIGURE 3. Proposed technique representation for upscale using tensor multiplier

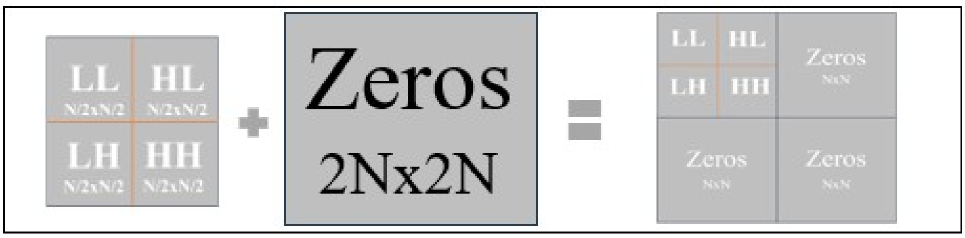


FIGURE 4. Representation of how adding zero matrix

3.3. **Image upscale using mix transform.** Image upscaling is a process of generating a high-resolution image from a low-resolution image that has a small size [20]. This paper utilizes a 256×256 or 28×28 original gray image and DWT-Db2 Haar. For image upscale shown in Figure 3, the generation complete orthogonal mixed transform using discrete wavelet is as follows:

(1) The transform construction equation will be:

$$[\text{Tensor.DWT}\#m]_{2^n \times 2^n} = [\text{Db2}]_{2^m \times 2^m} \otimes [\text{Db2}]_{2^{(n-m)} \times 2^{(n-m)}} \tag{1}$$

Then apply the tensor product, where n is the variation from 1 to $m-1$ as indicated below:

$$\begin{aligned}
&[\text{Tensor.DWT}\#1]_{2^8 \times 2^8} = [\text{Db2}]_{2^1 \times 2^1} \otimes [\text{Db2}]_{2^7 \times 2^7}, \text{ next round,} \\
&[\text{Tensor.DWT}\#2]_{2^8 \times 2^8} = [\text{Db2}]_{2^2 \times 2^2} \otimes [\text{Db2}]_{2^6 \times 2^6}, \text{ next round,} \\
&[\text{Tensor.DWT}\#3]_{2^8 \times 2^8} = [\text{Db2}]_{2^3 \times 2^3} \otimes [\text{Db2}]_{2^5 \times 2^5}, \text{ next round,} \\
&\vdots \\
&[\text{Tensor.DWT}\#7]_{2^8 \times 2^8} = [\text{Db2}]_{2^7 \times 2^7} \otimes [\text{Db2}]_{2^1 \times 2^1}, \text{ end round}
\end{aligned} \tag{2}$$

(2) The original image will be one part:

$$[\text{IMG}]_{2^8 \times 2^8} \tag{3}$$

(3) Find the result for the equations:

$$[\text{Pro.IMG}]_{2^8 \times 2^8} = [\text{Tensor.DWT}\#m]_{2^8 \times 2^8} [\text{IMG}]_{2^8 \times 2^8} [\text{Tensor.DWT}\#m]^T_{2^8 \times 2^8} \tag{4}$$

(4) Take all $[\text{Pro.IMG}]_{2^8 \times 2^8}$, then now can be upscale.

(5) For reconstruction of the image for upscale:

$$[\text{Up.Pro.IMG}]_{2^{n+1} \times 2^{n+1}} = [\text{Pro.IMG}]_{2^n \times 2^n} + [\text{Zeros}]_{2^{n+1} \times 2^{n+1}} \tag{5}$$

where $[\text{Zeros}]_{2^{n+1} \times 2^{n+1}}$ is the Zeros matrix added to filtered images.

(6) Then the final image will be:

$$[\text{UpImg}]_{2^{n+1} \times 2^{n+1}} = [\text{Tensor.DWT}\#m]^T_{2^{n+1} \times 2^{n+1}} [\text{Up.Pro.IMG}]_{2^{n+1} \times 2^{n+1}} [\text{Tensor.DWT}\#m]_{2^{n+1} \times 2^{n+1}} \tag{6}$$

To quantitatively evaluate the proposed technique, various upscaled metrics, Mean Square Error (MSE_{us}), (where the suffix "us" refers to upscale), Structural Similarity Index metric (SSIM_{us}), Upscale Ratio (USR), and Peak Signal-to-Noise Ratio (PSNR_{us}) are employed. The quality of upscale measurement is achieved by using the following metrics:

$$MSE_{us} = \frac{1}{M^2} \sum_{i=1}^{M} \sum_{j=1}^{M} (u(i,j) - o(i,j))^2 \tag{7}$$

where $u(i,j)$ and $o(i,j)$ are the upscaled and original images, respectively, while M^2 is the image dimensions [17, 21, 4].

$$PSNR_{us} = 20 \log_{10} \left(\frac{255^2}{MSE} \right)^{0.5} \tag{8}$$

$$SSIM_{us} = \frac{(2\mu_i \mu_j + c_1)(2\sigma_{ij} + c_2)}{(\mu_i^2 + \mu_j^2 + c_1)(\sigma_i^2 + \sigma_j^2 + c_2)} \tag{9}$$

where μ_j and μ_i are the mean intensity of j and i. $\mu_i, \mu_j, \sigma_i, \sigma_j$, and σ_{ij} are the mean intensity, variance, and co-variance of i and j, respectively. While c_1 and c_2 are the stability variables. Its value ranges from 0 to 1, and for maximum performance (upscaled and original images are similar), it should be nearly 1 [21].

$$USR = \frac{\text{Size of the upscaled image}}{\text{Size of the original image}} \tag{10}$$

4. **Results and Discussion.** After applying the theorem to an image of size $N \times N$, the result is an image with size $2N \times 2N$. Figure 5 shows the first mixed image (M1) of size $[128] \otimes [2]$ and the second mixed image (M2) for reconstruction, with its size $[256] \otimes [2]$. It shows the original image with a size of 256×256 pixels, then the image after converting it to gray and double class to be ready to make the mixing. After mixing, the figure shows that the reconstructed image appears similar to the original image. However, when adding the zero padding of size 512×512 to the mixed image, the reconstructed (upscaled) image appeared to be 512×512 pixels, which looks darker than the original image.

Figures 5 to 6 have the same simulation steps but with different matrices, as Figure 6 which has M1 of $[64] \otimes [4]$ and M2 of $[128] \otimes [4]$, and so on till Figure 11 which has M1 of $[2] \otimes [128]$ and M2 of $[4] \otimes [128]$. The best results are for Figure 5, but the reconstructed image is blurred in all other figures. This lack of clarity increased until Figure 11, which has the worst reconstructed image.

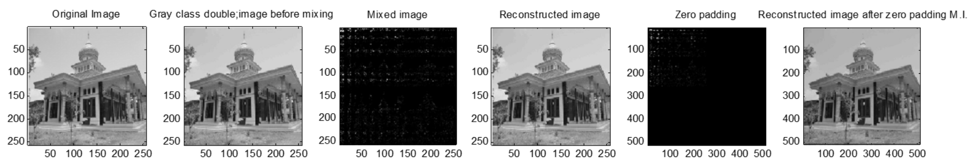


FIGURE 5. M1 of $[128] \otimes [2]$, M2 for reconstruction $[256] \otimes [2]$

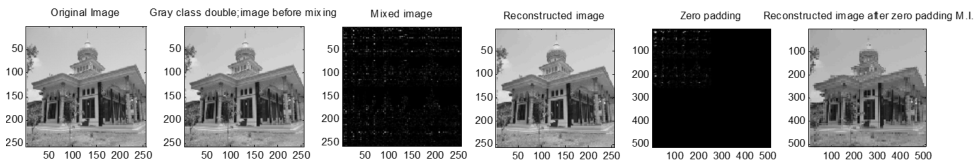


FIGURE 6. M1 of $[64] \otimes [4]$, M2 for reconstruction $[128] \otimes [4]$

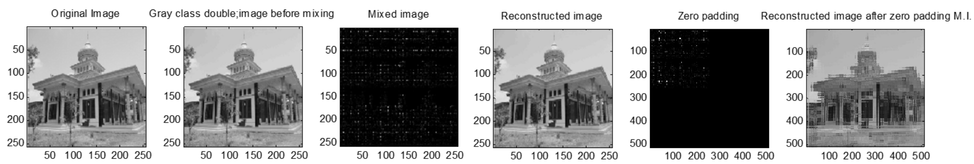


FIGURE 7. M1 of $[32] \otimes [8]$, M2 for reconstruction $[64] \otimes [8]$

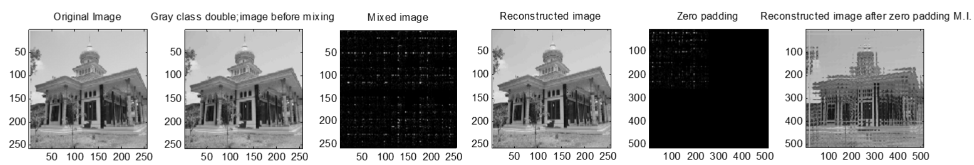


FIGURE 8. M1 of $[16] \otimes [16]$, M2 for reconstruction $[32] \otimes [16]$

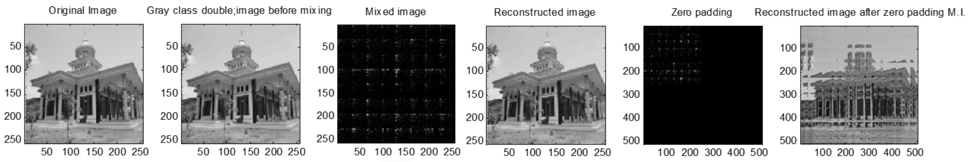


FIGURE 9. M1 of [8] $\otimes$ [32], M2 for reconstruction [16] $\otimes$ [32]

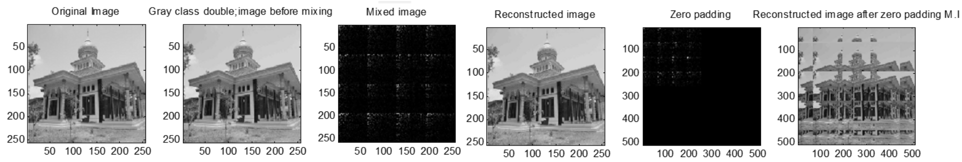


FIGURE 10. M1 of [4] $\otimes$ [64], M2 for reconstruction [8] $\otimes$ [64]

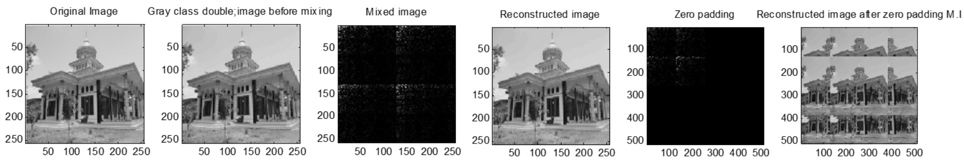


FIGURE 11. M1 of [2] $\otimes$ [128], M2 for reconstruction [4] $\otimes$ [128]

The high-quality [19] image has low MSE_{us}, high $PSNR_{us}$, and $SSIM_{us}$ near number 1. The proposed technique can perform the image upscaling four times [22] as shown in Table 1. This table shows the quality metrics values of the upscaled image for all values of the first mixed image size (M1) and second mixed image size (M2).

As shown in this table, in the case of using wavelet transform, the $MSE_{us} = 0.0007$, which is the lowest value, $PSNR_{us} = 79.4698$, which is the highest value, $SSIM_{us} = 0.9223$ the nearest value to 1, and $USR = 4$ for all values of M1 and M2 sizes. These values indicated that the size M1 of [128] $\otimes$ [2] and M2 of [256] $\otimes$ [2] have efficient quality metrics and upscaling.

Table 1 also compares the proposed technique with the same one, but instead of applying the wavelet transform, the slantlet transform is applied. This comparison is implemented according to upscale quality metrics. The size M1 of [128] $\otimes$ [2] and M2 of [256] $\otimes$ [2] have good quality metrics and upscaling.

While in a comparison with the proposed technique, the table indicated that the values of MSE_{us} in the proposed technique are less than that in the compared technique, which leads to higher $PSNR_{us}$ and $SSIM_{us}$. So, if the similarity between the upscaled and original images is high (near 1), then the technique is said to be improved upscaling, and the upscaling quality metrics justify the improvement and development in quantitative simulation upscaling results.

Another comparison is illustrated in Table 1, which shows a comparison between the proposed and compared techniques for M1 of [128] $\otimes$ [2] and M2 of [256] $\otimes$ [2]. It shows that the compared technique has upscaled blurred images, while the proposed technique has a clear image. So, the upscale by mix transform using wavelet transform is the best technique for upscaling.

5. **Conclusions.** This paper presents a technique with a new mixing wavelet transform algorithm to decompose the input image into different frequency components. After processing, the low-resolution input image is effectively transformed into a higher-resolution

Table 1. Quality metrics and comparison between proposed and compared techniques

1st Mixed image size	**2nd Mixed image size**	**Proposed technique (Wavelet Transform)**				**Compared technique (Slantlet Transform)**			
(M1)	(M2)	MSE_{us}	$PSNR_{us}$	$SSIM_{us}$	USR	MSE_{us}	$PSNR_{us}$	$SSIM_{us}$	USR
$[128] \otimes [2]$	$[256] \otimes [2]$	0.0007	79.4698	0.9223	4	0.0014	76.6695	0.9135	4
$[64] \otimes [4]$	$[128] \otimes [4]$	0.0174	65.7129	0.7144	4	0.0177	65.6511	0.6994	4
$[32] \otimes [8]$	$[64] \otimes [8]$	0.0177	65.6470	0.5039	4	0.0183	65.5063	0.4852	4
$[16] \otimes [16]$	$[32] \otimes [16]$	0.0912	58.5323	0.3980	4	0.0931	58.4413	0.3743	4
$[8] \otimes [32]$	$[16] \otimes [32]$	0.1701	55.8243	0.3447	4	0.1855	55.4474	0.3015	4
$[4] \otimes [64]$	$[8] \otimes [64]$	0.2687	53.8389	0.3081	4	0.3690	52.4605	0.2621	4
$[2] \otimes [128]$	$[4] \otimes [128]$	0.9240	48.4739	0.29	4	1.0037	48.1148	0.2240	4

Table 2. Comparison between the proposed and compared techniques

Comparison	**Original image**	**Upscaled image**
Upscale using Proposed Technique		
Upscale using Compared Technique		

representation by adding a zeroes matrix. This paper aims to improve the upscale images using this new mixed technique and the discrete wavelet transform (Daubechies wavelet Haar) as a 2D matrix and mixed with another wavelet's matrices' size using a tensor product. The results indicated that upscale by mix transform using wavelet transform has the values ($MSE_{us} = 0.0007$), ($PSNR_{us} = 79.4698$), and ($SSIM_{us} = 0.9223$), while using slantlet transform (compared technique) are ($MSE_{us} = 0.0014$), ($PSNR_{us} = 76.6695$), and ($SSIM_{us} = 0.9135$). Therefore, the proposed technique achieves significantly efficient upscale quality metrics and consistently produces visually appealing and detail-rich images as compared with slantlet transform-based upscaling, which has an upscaled blurred image. This approach can conduct long studies for all types of transforms and combine two or more transforms, such as wavelet and slantlet or Fourier transform. It can also be used in cryptography and image-processing tasks.

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