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An Adaptive Decomposition-based Multi-objective Evolutionary Algorithm for Fuzzy Portfolio Selection

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Abstract. *In this paper, a recently proposed model which is called Mean-variance-skewness for fuzzy portfolio selection is studied. Both the multidimensional nature of the portfolio selection problems and the requirements of the investor are considered in this model. The weighted possibilistic moments are used to approximate the quantification of the fuzzy variables. Then, an adaptive decomposition-based multi-objective evolutionary algorithm (AMOEA/D) is designed to solve this model which is considered as a constrained three-objective optimization problem. In this algorithm, this problem is firstly decomposed into a number of sub-problems through a set of weight vectors with good uniformly and aggregate functions, and these sub-problems are simultaneously optimized in a run; secondly, according to the distances of obtained non-dominated solutions, an adaptive weight vector adjustment strategy is proposed to redistribute the weight vectors of sub-objective spaces; thirdly, a crossover based uniform design is specially designed for portfolio selection problems; fourthly, an external elite population is introduced to help maintaining the diversity of obtained non-dominated solutions. Moreover, comparing with some efficient state-of-the-art algorithms NSGAII and MOEA/D on the Shanghai Stock Exchange, the results indicate the efficiency and effectiveness of the proposed algorithm.*
Keywords: Portfolio selection; Fuzzy variable; Possibilistic moments; Multi-objective evolutionary algorithm; Decomposition; Uniform design

1. **Introduction.** Portfolio selection theory is derived from the mean-variance (MV) model proposed by Markowitz [1]. MV model considers trade-off between return and risk. Due to the non-linear programming problem contained in this approach, portfolio selection problem has become a classical optimization problem. Since from Markowitz, several researchers have done some studies by using various approximation schemes. However, in recent years, many empirical studies think that the distributions of asset returns usually tend to be of asymmetric leptokurtic and heavy-tailed features, and are not normally distributed [2]. This indicates that an effective model should consider the higher order moments. The three or four moment's framework has been considered in the portfolio problems to solve this problem. Campbell [3] proposed the mean-variance-skewness framework with the skew normal distribution to incorporate higher order moments in portfolio selection; Adcock [4] studied the performance of the mean-variance- skewness portfolio model under the multivariate extended skew-Student distribution.

Besides the higher order moments, uncertainty is another important factor in portfolio model because investors may face uncertain, imprecise and vague data. If there is not enough historical data, the model is difficulty described by the statistical variable. This problem can be solved by using fuzzy variables. Moreover, many researchers [5-6] have studied the portfolio selection models with fuzzy variables. In this paper, a recently proposed model which is called Mean-varianceVskewness (MVS) for fuzzy portfolio selection is studied. Both the multidimensional nature of the portfolio selection problems and the requirements of the investor are considered in this model. The weighted possibilistic moments are used to approximate the quantification of the fuzzy variables.

This Mean-variance-skewness model is a constrained multi-objective problem (MOP), and it cannot be found efficient portfolios by using traditional optimization methods. To solve this problem, multi-objective evolutionary algorithms (MOEAs) are an effective method to solve this model because they can handle a set of solutions in parallel. Many MOEAs have been successfully used to solve many portfolio selection models [7]. Among these MOEAs, multi-objective evolutionary algorithms based on decomposition [8] have a good performance on searching a diversity of non-dominated solutions for various kinds of MOPs [9-10]. They make use of traditional aggregation methods and weight vectors to transform the task of approximating the Pareto front (PF) into a number of single objective optimization sub-problems which are simultaneously optimized in a run. Because the Pareto optimal fronts of MVS are unknown, multi-objective evolutionary algorithms based on decomposition should have adaptive weight adjustment strategy to set the weight vectors.

In this paper, a decomposition-based multi-objective evolutionary algorithm with adaptive weight vector adjustment (MOEA/DA) is especially designed to solve MVS. The main contributions of this work are that an adaptive weight vector adjustment strategy which some weight vectors are adaptively deleted or added according to the distances of obtained non-dominated solutions is proposed to solve this MVS with complex PF, a crossover operator based on uniform design is designed for portfolio selection problems, and a selection strategy is used to help crossover operators to improve the search efficiency.

The rest of this paper is organized as follows. Section 2 introduces the main concepts of the multi-objective optimization and the weighted possibilistic MVS portfolio selection problem. Section 3 presents a detailed description of our designed multi-objective evolution algorithm. Section 4 shows the experiment results of the proposed algorithm and the related analysis on the data of the Shanghai Stock Exchange Market. Finally, conclusion and future directions are drawn in Section 5.

2. **Preliminaries.** This section introduces some concepts and preliminary.

2.1. **Multi-objective optimization.** A continuous optimization problem is a mathematical programming problem with a vector-valued objective function, which can be formulated as follows [11]:

$$\begin{cases} \min F(x) = (f_1(x), f_2(x), ..., f_m(x)) \\ s.t. g_i(x) \le 0, i = 1, 2, ...q \\ h_j(x) = 0, j = 1, 2, ..., p \end{cases} \tag{1}$$

where $x = (x_1, ..., x_n) \in X \subset R^n$ is a n-dimensional decision variable bounded in the decision space X, m is the number of objective functions. $f_i(x)(i = 1, ..., m)$ is the i-th objective function to be minimized, $g_i(x)(i = 1, 2...q)$ defines the i-th inequality constraint and $h_j(x)(j = 1, 2...p)$ defines the j-th equality constraint. Moreover, all the inequality and equality constraints determine a set of feasible solutions which is denoted by Ω and $Y = \{F(x)|x \in \Omega\} \subset R^m$ is denoted as the objective space. Because the objectives often contradict each other, the improvement of one objective may cause to the deterioration of other objectives. So, MOPs have many optimal solutions which can be called non-dominated solutions [12]. A few important definitions are introduced as follows. Let $x, z \in \Omega$, x is said to be better than z, if $F(x) \neq F(z)$ and $f_i(x) \leq f_i(z)$ for $i = 1, 2...m$. If there is no other x such that x is better than x^*, x^* is called Pareto optimal. The set of all the Pareto optimal solutions is defined as the Pareto set (PS). The image of the PS $(PF = \{F(x)|x \in PS\})$, is called the Pareto optimal front (PF) [12].

2.2. **Mean-variance-skewness model.** In this section, some defines of mean-variance-skewness model are introduced and more details can refer to the literature [13]. Firstly, the concept of skewness for fuzzy variables is defined as follow:

$$s[\xi] = E[(\xi - E[\xi])^3] \tag{2}$$

where ξ is a fuzzy variable with finite expected value, and $E[\xi]$ is the expected value of the fuzzy variable ξ.

Let ξ_i be a fuzzy variable representing the return of the ith security, and let x_i be the proportion of the total capital invested in security i. The Mean-variance-skewness model which maximizes the expected return and the skewness, minimizes the risk is defined as follow:

$$\begin{cases} \max S\left[\sum\limits_{i=1}^{n} \xi_i x_i\right] \\ \max E\left[\sum\limits_{i=1}^{n} \xi_i x_i\right] \\ \max V\left[\sum\limits_{i=1}^{n} \xi_i x_i\right] \\ s.t., \sum\limits_{i=1}^{n} x_i = 1, x_i \geq 0, i = 1, 2, ...n \end{cases} \tag{3}$$

where $V[\xi] = (\xi - E[\xi])^2$ is the variance of the fuzzy variable ξ.

In this paper, the weighted possibilistic moments are used to approximate the quantification of the fuzzy variables. The possibilistic skewness [14] of $\sum_{i=1}^{n} \xi_i x_i$ is

$$\begin{aligned} S\left[\sum_{i=1}^{n} \xi_i x_i\right] &= \tfrac{19}{1080}\left[\left(\sum_{i=1}^{n} x_i\theta_i\right)^3 - \left(\sum_{i=1}^{n} x_i\delta_i\right)^3\right] \\ &+\tfrac{1}{24}\left[\sum_{i=1}^{n} x_i(d_i - c_i)\right]\left[\left(\sum_{i=1}^{n} x_i\theta_i\right)^2 - \left(\sum_{i=2}^{n} x_i\delta_i\right)^2\right] \\ &+\tfrac{1}{72}\left[\left(\sum_{i=1}^{n} x_i\delta_i\right)\left(\sum_{i=1}^{n} x_i\theta_i\right)^2 - \left(\sum_{i=1}^{n} x_i\theta_i\right)\left(\sum_{i=1}^{n} x_i\delta_i\right)^2\right] \end{aligned} \tag{4}$$

where $[c_i, d_i]$ is the core of the fuzzy variable ξ_i, $\delta_i > 0$ is the left width, $\theta_i > 0$ is the right width. The weighted possibilistic variance of $\sum_{i=1}^{n} \xi_i x_i$ is

$$V\left[\sum_{i=1}^{n} \xi_i x_i\right] = \frac{\left[\sum_{i=1}^{n} x_i(\theta_i + \delta_i)\right]^2 + \left[\sum_{i=1}^{n} x_i(\theta_i - \delta_i)\right]^2}{72} + \left[\sum_{i=1}^{n} x_i\left(\frac{d_i - c_i}{2} + \frac{\theta_i + \delta_i}{6}\right)\right]^2 \tag{5}$$

The weighted possibilistic expected value of $\sum_{i=1}^{n} \xi_i x_i$ is $\sum_{i=1}^{n} x_i\left(\frac{d_i - c_i}{2} + \frac{\theta_i + \delta_i}{6}\right)$

3. **A New Evolutionary Algorithm for the MVS.** For the MVS with unknown Pareto optimal fronts, a multi-objective evolutionary algorithms based on decomposition with adaptive weight adjustment strategy is designed to solve this problem. This proposed algorithm mainly consists of three parts: adaptive weight vector adjustment strategy,

a crossover operator based on uniform design and a selection strategy, which will be introduced in this section.

3.1. **Adaptive Weight Vector Adjustment.** In the subsection, an adaptive weight vector adjustment is presented. This adjustment strategy uses the distances of obtained non-dominated solutions to delete or add some weight vectors solve the problems with unknown PF and maintain relative stability of weight vectors. The details of the adjustment are as follows.

For the current weight vectors $W = (W_1, W_2, ..., W_H)$ and current population $POP = (x^1, x^2, ..., x^H)$, where H is the number of solutions or weight vectors and $x^i (i = 1 \sim H)$ is the current optimal solution of the corresponding sub-problem of the weigh vector W_i. The non-dominated solutions of POP are firstly found. For convenience, we suggest that $(x^1, x^2, ..., x^K)(K \leq H)$ are the non-dominated solutions of POP and denote $WW = (W_{1+K}, W_{2+K}, ..., W_H)$. The distances ND_i of obtained non-dominated solutions of $W_i (i = 1 \sim H)$ is calculated as $ND_i = \max\{|f_j(x^{j_1}) - f_j(x^i)|, |f_j(x^i) - f_j(x^{j_2})|, j = 1 \sim m\}$, where $W_{j_1,j}$ and $W_{j_2,j}$ are the supremum and infimum of $W_{i,j}$ among $\{W_{1,j}, W_{2,j}, ..., W_{K,j}\}$. The values of ND_i are mainly used to delete some weight vectors. In addition, all $|f_j(x^{j_1}) - f_j(x^i)|$ and $|f_j(x^i) - f_j(x^{j_2})|$ are sorted to add the weight vectors. For convenience, we use $PD_{i,u} = |f_j(x^{u_i}) - f_j(x^i)|(j = 1 \sim m, 1 \leq u \leq 2 * K * m)$ and W_{u_i} to denote the distance of obtained non-dominated solutions of W_{u_i} and W_i and the corresponding weight vector, respectively, where $W_{u_i,j}$ is the supremum or infimum of $W_{i,j}$ among $\{W_{1,j}, W_{2,j}, ..., W_{K,j}\}$.

The deleting strategy is as follows. If $K > N$ (where N is the size of the initial population), $N - K$ weight vectors with the minimum ND_i are deleted from W. Then, if $\max\{ND_i, i = 1 \sim N\}/\min\{ND_i, i = 1 \sim N\} > 2$, the corresponding weight vector with the minimum ND_i is deleted from W. After some weight vectors are deleted from W, the adding strategy is that, if the size of the current W is smaller than $H - K + N$, $H - K + N - |W|$ new weight vectors are generated as follows:

$$W_{new} = \begin{cases} (0.25 * W_{u_i} + 0.75 * W_i)/yy & if \exists W_k \in WW, W_i * tt' < W_k * tt') \\ tt & else \end{cases} \quad (6)$$

Where $yy = ||0.25 * W_{u_i} + 0.75 * W_i||_2, tt = (0.5 * W_{u_i} + 0.5 * W_i)/||0.5 * W_{u_i} + 0.5 * W_i||_2$, and the distances $PD_{i,u}$ of obtained non-dominated solutions of W_{u_i} and W_i are the $H - K + N - |W|$ maximum, where $|W|$ is the size of W. The condition $\exists W_k \in WW, W_i * tt' < W_k * tt'$ makes the optimal solution of the new sub-problem generated by the weigh vector W_{new} to be non-dominated solution. In other word, we don't want that the generate weight vectors locate these space which have no nod-dominated solution. The role of the deleting strategy and the adding strategy are to delete the sub-problems from the crowded regions and add the sub-problems into the sparse regions. The adaptive weight vector adjustment is summarized in Algorithm 1.

In the step 4, some weight vectors of WW are kept, which is to record these regions with no non-dominated solution and make these sub-problems to quickly find non-dominated solutions (if have).

3.2. **Crossover operator based on uniform design.** For the MVS problem, its optimal solutions should subject to $\sum_{i=1}^{n} x_i = 1, x_i \geq 0, i = 1, 2, ...n$. In this paper, a crossover operator based on uniform design is designed to satisfy this constraint condition. The main idea of this crossover operator is that, some vectors are firstly generated by the uniform design which can sample a small set of points from a given closed and bounded set such that the sampled points are uniformly scattered on the set, a special method is used to transfer those vectors into offspring which satisfy the constraint condition. The details of this crossover operator are as follows.

Algorithm 1 Adaptive Weight Vector Adjustment

Require: the size of the initial population N, the current weight vectors $W = (W_1, W_2, ..., W_H)$ and current population $POP = (x^1, x^2, ..., x^H)$
Output: the weight vectors W
Step 1: Find the non-dominated solutions $(x^1, x^2, ..., x^K)$ of POP and denote $WW = (W_{1+K}, W_{2+K}, ..., W_H)$. Calculate the ND_i and $W_{u_i,j}$.
Step 2: Deleting weight vectors:
If $K > N$, then $N - K$ weight vectors with the minimum ND_i are deleted from W.
While $\max\{ND_i, i = 1 \sim N\}/\min\{ND_i, i = 1 \sim N\} > 2$ **do**
The corresponding weight vector with the minimum ND_i is deleted from W and recalculate the ND_i and $W_{u_i,j}$.
Step 3: Adding weight vectors:
If $H - K + N > |W|$ **then**
Find the $H - K + N - |W|$ maximum distances $PD_{i,u}$ of obtained non-dominated solutions of W_{u_i} and W_i, and use Eq.(5) to generate the new weight vectors.
Step 4: Deleting some weight vectors of WW from W
If $|WW| > 0.5N$ **then**
Use the crowding distance to delete $|WW| - 0.5N$ weight vectors of WW from W.

The uniform design method is briefly shown. For a given bounded and closed set $G \subset R^M$ (where M is the dimension of the set G), the uniform design was developed to sample some points which have a small number and are uniformly scattered on G. The Good-Lattice-Point method (GLP) [15] is a simple and efficient method. And it can generate a set of uniformly scattered points on a given set $C = \{(\theta_1, \theta_2, ..., \theta_M)|0 \le \theta_i \le 1, i = 0, ...M\}$. The details of GLP are as follows. For given integer q, M and μ, it generates uniform array which is a $q \times M$ integer matrix $G(q, M)$ by the following expression:

$$G(q, M) = [G_{ij}]_{q\times M}, where\ G_{ij} = (\text{mod}(i\mu^{j-1}, q)) + 1, i = 1 \sim q, j = 1 \sim M \tag{7}$$

where, $2 \le \mu \le q$, $\text{mod}(i\mu^{j-1}, q)$ is the remainder of $i\mu^{j-1}/q$. Thus, these all μ can generate $q - 1$ different integer matrices. So, for given q and M, a number δ [16] is determined to determine an integer matrix with the smallest discrepancy among these $q - 1$ different integer matrices. Each row of matrix $G(q, M)$ determiners a point $C_i = (C_{i1}, C_{i2}, ..., C_{iM})$ of $C(q, M)$ by

$$c_{ij} = \frac{2G_{ij} - 1}{2q}, , i = 1 \sim q, j = 1 \sim M \tag{8}$$

$C(q, M)$ is given by $C(q, M) = \{C_i|i = 1 \sim q\}$.

For given two parents $y = (y_1, ..., y_n)$ and $t = (t_1, ..., t_n)$, two vectors are defined as follow:

$$l = (l_1, ..., l_{n-1})\ and\ u = (u_1, ..., u_{n-1}) \tag{9}$$

where $l_i = \min\{t_i, y_t\}$ and $u_i = \max\{t_i, y_i\}$ for $i = 1 \sim n - 1$. These two vectors define a hyper-rectangle,

$$[l, u] = \{(x_1, x_2, \cdots, x_{n-1})|l_i \le x_i \le u_i, i = 1, \cdots n - 1\} \tag{10}$$

Choose two proper integer q and M. Note, in general case, the parameters M and q of the uniform design should satisfy $M \leq q-1$ [32]. Thus, for given M and q, the q offspring which are denoted by $\mathrm{O}(q, n-1) = \{O_i = (o_{i1}, o_{i2}, \cdots, o_{in-1}) | i = 1 \sim q\}$ should be generated according to two cases: $M \leq q-1$ and $M > q-1$. The detail is introduced as the following algorithm 2.

Algorithm 2 Crossover operator based on uniform design

Require: a proper prime number q, two parents y and t

Output: q feasible solutions of the MVS problem

Step 1: Two vectors l and u are defined by Eq.(9)

Step 2: If $n-1 \leq q-1$, set $M = n-1$. Then, the j-th component of the i-th offspring $O_i = (o_{i1}, o_{i2}, ..., o_{in-1})$ is set to $o_{ij} = l_j + c_{ij}(u_j - l_j), i = 1 \sim q$, $j = 1 \sim n-1$, where c_{ij} is generated by according to Eq.(8).
If $n-1 > q-1$, we set $M = q_1 - 1$ and randomly divide l and u into M blocks of sub-vectors, respectively, in the following way:

$$l = (A^1, A^2, ..., A^{q-1})\ and\ u = (B^1, B^2, ..., B^{q-1}) \tag{11}$$

where A^j and B^j are sub-vectors of l and u with the same dimension.
Then the i-th offspring $O_i = (o_{i1}, o_{i2}, ..., o_{iq-1})$ can be generated by

$$o_{ij} = A^j + \frac{2G_{ij} - 1}{2q}\left(B^j - A^j\right), i = 1 \sim q, j = 1 \sim q-1 \tag{12}$$

where $G(q, q-1) = [G_{ij}]_{q \times q-1}$ is defined by (7) with $M = q-1$

Step 3: For the MVS problem, the offspring is converted as following expression:

$$oo_i = \begin{cases} 1 - o_{i1}, & if\ j = 1 \\ o_{i1} * o_{i2} * ... * o_{ij-1} * (1 - o_{ij}), & if\ 2 \leq j \leq n-1 \\ o_{i1} * o_{i2} * ... * o_{ij}, & if\ j = n-1 \end{cases} \tag{13}$$

Where $i = 1 \sim q$ and $OO(q, n-1) = \{OO_i = (oo_{i1}, oo_{i2}, ..., oo_{in}) | i = 1 \sim q\}$ is the feasible solutions of the MVS problem.

3.3. **Selection Strategy.** A good selection strategy can help crossover operators to carry out the local search and global search, thus an appropriate selection strategy can improve the search efficiency of an algorithm. In this paper, a selection strategy based on the decomposition is designed to improve the performance of the proposed algorithm. Firstly, the Euclidean distances of any two weight vectors are computed and then the T closet weight vectors of each weight vector are worked out. For each $i = 1, ...N$, set $B(i) = i_1, ..., i_T$ where $\lambda^{i_1}, \cdots \lambda^{i_T}$ are the T closet weight vectors to λ^i. Then set

$$P = \begin{cases} B(i), if rand1 < J \\ \{1, \cdots, N\}, otherwise \end{cases} \tag{14}$$

where $rand1$ and $rand2$ are two random number and its scope is [0,1], J and $p1$ are two parameters. J is set to 0.9 as the same in [15]. For the weight vector λ^i, when P is set, randomly select two indexes $r2$ and $r3$ from P. For $rand1 < J$ and $rand2 < p1$, Algorithm 2 is used to generate some offspring from x^{r2} and x^i , otherwise, a solution is generated f by the following formula:

$$x_j^{new} = \begin{cases} x_j^i + L\left(x_j^{r2} - x_j^{r3}\right), & if\ rand(0,1) < CR \\ x_j^i & otherwise \end{cases} \tag{15}$$

where $L \in [0, 2]$ is a scale factor which controls the length of the exploration vector $(x^{r2} - x^{r3})$; CR is a constant value namely crossover rate; $j = 1, ..., n$ and x_j^{r2} indicates the j-th component of x^{r2}.

If P is set to $B(i)$, the proposed crossover operator based on uniform design and the formula (15) can carry out the local search. Otherwise, the global search is implemented.

3.4. **Steps of the Proposed Algorithm.** Based on all above, an adaptive decomposition-based multi-objective evolutionary algorithm (AMOEA/D) is designed and the pseudo code of the algorithm AMOEA/D is as follows:

Algorithm 3 The pseudo code of the algorithm AMOEA/D

Require:
MOP (1)
A stopping criterion
N: the number of direction vectors
T the number of weight vectors in the neighborhood of each weight vector, $0 < T < N$
$\lambda^1, \lambda^2, \cdots, \lambda^N$: a set of N uniformly distributed weight vectors

Output: Approximation to the PF: $\{F(x^1), F(x^2), \cdots, F(x^N)\}$

Step 1: Generate an initial population $x^1, x^2, ..., x^N$ randomly or by a problem-specific method; determine $Z = (z_1, ..., z_m)$ by a problem-specific method; determine $B(i) = \{i_1, \cdots, i_T\}, (i = 1, \cdots, N)$, where $\lambda^{i_1}, \cdots, \lambda^{i_T}$ are the T closest weight vectors to λ^i.

Step 2: Generate offspring and updated
For $i = 1, ..., N$, **do**
Generate offspring $x_{new} = (x_{new,1}, \cdots, x_{new,n})$
Two indexes $r2$ and $r3$ are randomly selected from P.
If $rand1 < J$ and $rand2 < p1$
Offspring is generated by x_i, and x_{r1} according to Algorithm 2.
else
Offspring is generated by x_i, x_{r1} and x_{r2} according to the formula (15).
end if
For each offspring x_{new}
Update of Z: For $k = 1, ..., m$, if $z_k < f_k(x_{new})$, then set $z_k = f_k(x_{new})$
Update the population by the updated strategy of the literature [27].
end for
end for

Step 3: **If** gen is a multiple of 50, then, use Algorithm 1 to modify the weight vectors W, re-determine $B(i) = i_1, ..., i_T, (i = 1, ..., H)$ (where H is the size of W), and randomly select solutions from the current population to allocate the new sub-problem as their current solution.

Step 4: **If** the conditions are satisfied, output the $\{F(x^1), F(x^2), ..., F(x^N)\}$, or, change to **Step 2**.

4. **Numerical Examples and Analysis.** In this section, to demonstrate the effectiveness of the proposed algorithm for the MVS, the proposed algorithm compares with two other classical algorithms which are multiobjective genetic algorithm based on pareto dominance (NSGAII [17]) and multiobjective evolutionary algorithm based on decomposition (MOEA/D [8]) on the multi-objective portfolio selection model whose data are taken from the historical data of the Shanghai Stock Exchange Market.

Table 1. The metrics C and HV obtained by AMOEA/D, NSGAII and MOEA/D on MVS (A represents the algorithm AMOEA/D, and B represents the algorithms NSGAII and MOEA/D)

		AMOEA/D	NSGAII	MOEA/D
$C(A,B)$	mean	NA	0.9333	0.8667
	std	NA	0.0160	0.0241
$C(B,A)$	mean	NA	0	0
	std	NA	0	0
HV	mean	0.7483	0.6354	0.5640
	std	0.0235	0.0267	0.0258

4.1. **Data processing.** In the experiments, the 12 candidate assets are chosen from Shanghai Stock Exchange. The exchange codes of these 12 assets are 601098, 601880, 600563, 600038, 601888, 601377, 600721, 600681, 600571, 600419, 600570, 600201, respectively. The original data of these assets is the weekly sampled in three years from January 2012 to January 2015. Using the simple estimation method in Vercher et al. [18], the statistics of the historical data of 12 rates are got. The parameters $(c_i, d_i, \delta_i\theta_i)$ of these 12 assets [19] are (0.0416 0.0662 0.0224 0.01315), (0.0434 0.0639 0.0352 0.2148), (0.0526 0.0657 0.0290 0.0599), (0.0508 0.0723 0.0338 0.0994), (0.0220 0.0278 0.0124 0.0571), (0.0449 0.0699 0.0239 0.1900), (0.0723 0.0990 0.0481 0.1264), (0.0708 0.0954 0.0426 0.1297), (0.0499 0.0820 0.0315 0.0965), (0.0705 0.0970 0.0515 0.2344), (0.0299 0.0503 0.0194 0.0875) and (0.0290 0.0379 0.0164 0.0590).

4.2. **Parameters setting.** Real vectors are used to code these three algorithms. The parameters of NSGAII and MOEA/D are the same as the setting in the original literature to. The initial population sizes of all algorithms are set to 105 and 105 initial weight vectors are generated; each algorithm is run 30 times with the maximal number of function evaluations 100 000 on all test problems. For AMOEA/D, the size of neighborhood list is set to $0.1N$, $p1$ and CR are set to 0.8 and 0.6, respectively.

4.3. **Performance metrics.** In this paper, the true Pareto optimal fronts of the MVS problems are unknown. Therefore, to quantificational compare with the performances of algorithms hyper-volume indicator (HV) [20] and coverage metric [21] (C metric) are used. The hyper-volume indicator is used widely in evolutionary multi-objective optimization to evaluate the performance of algorithms. It computes the volume of the dominated portion of the objective space relative to a reference point. Higher values of this performance indicator imply more desirable solutions. The hyper-volume indicator measures both the convergence and diversity of the obtained solutions.

4.4. **Numerical results.** Table 1 shows the mean and standard deviation of the C and HV values obtained by AMOEA/D, NSGAII and MOEA/D in the 30 independent runs. From Table 2, according to the HV, it can conclude that the final solutions by AMOEA/D are not dominated those obtained by MOEA/D and NSGAII, and most of final solutions obtained by NSGAII and MOEA/D are dominated the final solutions by AMOEA/D, these indicate that the convergence performance of AMOEA/D is better than NSGAII and MOEA/D; according to the HV, it is obvious that the mean values of HV obtained by AMOEA/D are larger than those obtained by NSGAII and MOEA/D, which shows that AMOEA/D performs better than NSGAII and MOEA/D on MVS problem and the solutions obtained by AMOEA/D has a better diversity than those obtained by NSGAII and MOEA/D.

To visually compare the performance of the three algorithms, the solutions obtained by them on MVS problem are shown in Fig. 1. Obviously, the convergence and diversity of solutions obtained by AMOEA/D are better than those obtained by NSGAII and MOEA/D. These compare results illustrate that AMOEA/D performs better than other two algorithms on MVS problem and the proposed algorithm can well solve the MVS problem.

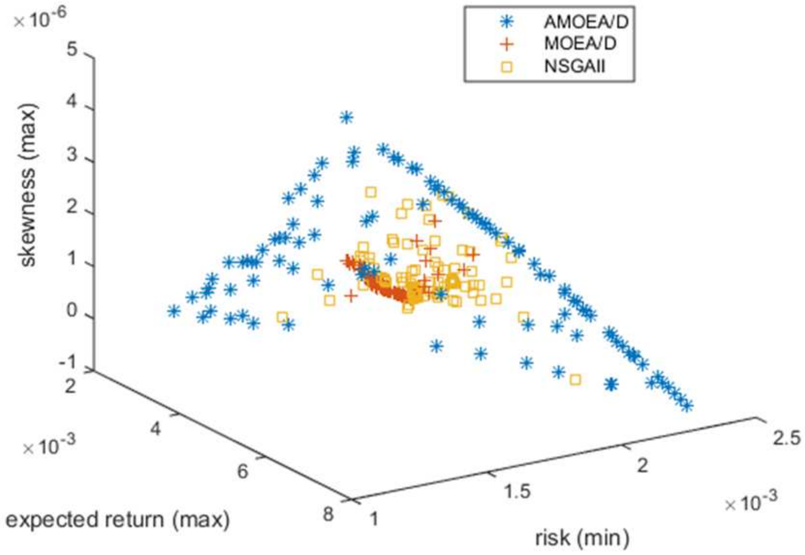

FIGURE 1. Solutions obtained by AMOEA/D, NSGAII and MOEA/D on MVS problem

5. **Conclusions.** This work focuses on the study of the fuzzy portfolio selection that explicitly involves skewness in the multiobjective framework. To solve this multi-objective portfolio models, a decomposition-based multi-objective evolutionary algorithm with adaptive weight vector adjustment (MOEA/DA) is especially designed to solve this problem. An adaptive weight vector adjustment strategy which some weight vectors are adaptively deleted or added according to the distances of obtained non-dominated solutions is proposed to solve this problem with unknown PF, a crossover operator based on uniform design is designed to generate feasible solutions for portfolio selection problems, and a selection strategy is used to help crossover operators to improve the search efficiency. Finally, some numerical examples are presented to illustrate the practicality and effectiveness of the proposed algorithm based on the data from Shanghai Stock Exchange. For the future research, the multi-objective fuzzy portfolio selection problem and MOEAs will be applied to other asset allocation problems, mutual fund portfolio selection problems, combinational optimization models and multi-period problems.

Conflict of interest. The authors have declared that no conflict of interest exists.

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