Journal of Information Hiding and Multimedia Signal Processing ©2025 ISSN 2073-4212
Ubiquitous International Volume **16**, Number **2**, June **2025**

An Intelligent Optimization Method for Product Design Based on Optimal Density Peak Clustering

Ting Wang[1,*]

[1]School of Architectural Engineering,
Xuzhou Vocational College of Industrial Technology, Xuzhou 221140, P. R. China
717483823@qq.com

Xiang-Zhen Zhou[a]

[a]Faculty of Information Science and Technology,
National University of Malaysia, Selangor 43600, Malaysia
619543699@qq.com

*Corresponding author: Ting Wang

Received June 2, 2024, revised July 3, 2024, accepted July 4, 2024.

Abstract. *In modern product design, complex modular structures and variable dependencies pose new challenges to design efficiency and quality. Traditional design structure optimisation methods, such as modular design, quality function development and constraint-based optimisation techniques, although simplifying the design process to a certain extent, have limited effectiveness in coping with high-dimensional, multi-module and dynamically changing complex systems. To overcome these limitations, this paper proposes an intelligent optimisation method for product design based on Optimized Density Peaks Clustering (ODPC). Firstly, ODPC is able to automatically identify critical modules and high-density regions in the design structure by combining the analysis of local density and relative distances in order to optimise the complex interactions between modules. By dynamically adjusting the bandwidth (using the silver disc criterion) and truncation distance (using K-NN) for density calculation, ODPC can flexibly adapt to different design scenarios and complexities. In addition, this paper introduces the distance metric adjustment based on information theory, which combines the metrics of mutual information and KL dispersion to capture the complex relationships between modules more precisely, and improves the modularity level and design quality of the system. In order to verify the practicability of the ODPC method, this paper carries out an application analysis through the design case of a smart air purifier.*
Keywords: product design optimization; density peaks clustering; design structure matrix; automated parameter selection; modular management

1. **Introduction.** In today's rapidly evolving technological environment, organisations are faced with increasing challenges as product design complexity and competitive pressures continue to grow. Product design requires not only excellence in functionality and performance, but also cost-effectiveness, manufacturing feasibility and market adaptability. Therefore, how to optimise the product design process and reduce development time and cost while maintaining high quality has become a key factor in the competitiveness of enterprises [1, 2, 3]. Intelligent optimisation technology provides a new path to this goal, and through automation and intelligent means, it can significantly improve design efficiency and innovation. In the context of intelligent manufacturing and Industry 4.0,

intelligent optimisation of product design not only enhances the R&D capability of enterprises, but also has great significance for the upgrading and transformation of the whole industry [4, 5]. It can help enterprises quickly respond to market changes and shorten the time-to-market, while ensuring product reliability and quality.

Clustering technique, as an important data analysis tool [6, 7], is widely used in various fields. In intelligent optimisation of product design, clustering methods can be used to automatically identify and group functional modules and components in a design to effectively manage and simplify the design structure. Through clustering analysis, designers can better understand the distribution of functions within a product and the interdependencies between modules, so as to optimise the design solution and reduce system complexity [8]. Especially for complex products with highly modular and diverse components, clustering techniques can reveal potential design patterns and optimisation paths, helping design teams to make more informed decisions during the development process. Density Peak Clustering (DPC), as an advanced clustering method [9, 10], can accurately identify key modules and optimisation opportunities in a design by combining the analysis of local densities and relative distances, which provides a strong support for intelligent optimisation of product design. The objective of this study is to construct an efficient design optimisation framework by combining DPC with Design Structure Matrix (DSM), which is able to automatically identify critical modules and optimisation paths in complex designs, thus enhancing product design efficiency and performance.

1.1. **Related work.** Optimisation of product design structures has always been an important research topic in the field of engineering, and the traditional methods mainly include modular design, Quality Function Development (QFD) [11, 12] and constraint-based optimisation techniques. These methods have been successful in improving the efficiency and quality of product design to some extent, but there are still many challenges in dealing with complex systems.

Modular design is a widely used approach to simplify the design process and manage complexity by breaking down a product into relatively independent modules. Helo [13] proposes a methodology for analysing and optimising the modular design of a product by means of the Design Structure Matrix (DSM) approach. The DSM presents the modules and their interactions in the system in the form of a matrix, which provides the designer with a clear DSM provides designers with a clear perspective to identify and optimise the coupling between modules. It is shown that DSM can effectively simplify the complexity of product design, but when dealing with multi-dimensional and dynamically changing systems, the relationships between modules may become difficult to manage.

QFD is another commonly used product design optimisation method that guides the design process by translating customer requirements into engineering characteristics. Liu [14] showed that QFD can significantly improve product marketability and customer satisfaction. However, QFD has limitations in dealing with modern complex product designs, especially when the design needs to balance multiple conflicting objectives, which may not be adequately addressed by traditional QFD methods.

Constraint-based optimisation techniques are also widely used in product design optimisation. Kreng and Lee [15] proposed a framework for multi-objective optimisation using genetic algorithm (GA) for solving complex constrained problems in product design. It was found that GA performs well in dealing with high-dimensional optimisation problems, but further improvement of the algorithm's adaptability and robustness is still needed when facing uncertainty and dynamically changing environments. In addition, empirically based heuristics have been widely used in product design optimisation. For

example, Meng [16] used the simulated annealing algorithm (SA) to optimise an aero-engine design. It is shown that SA can effectively search the complex design space, but its convergence speed and local optimal solution problems limit its wide use in practical applications.

Currently, research on product design structure optimisation focuses on how to better deal with multi-dimensional data and dynamically changing dependencies of complex systems [17, 18]. Modern product design involves the integration of multiple technology domains and functional modules, and traditional optimisation methods are overwhelmed in dealing with these complexities. To overcome these challenges, researchers have proposed many new methods and techniques, but these methods also face their own limitations [19].

Firstly, systems engineering methods have been widely used in modern product design. Pessoa and Becker [20] discuss the application of systems engineering in complex product design, emphasising the importance of comprehensive requirements analysis and functional decomposition at the early stages of design. However, the systems engineering approach requires a great deal of upfront work and cross-team coordination, and its application efficiency is somewhat limited in an environment of rapid iteration and flexible design.

Secondly, methods based on network analysis have been used to optimise complex dependencies between product modules. Dong et al. [21] used Social Network Analysis (SNA) to study interactive recommendations in product design and found that the design structure can be better understood and optimised by analysing the network relationships between modules. However, SNA may be difficult to adapt to real-time design adjustments when facing highly dynamic and changing dependencies.

1.2. **Motivation and contribution.** To address these challenges, this paper proposes an intelligent optimisation method for product design based on Optimized Density Peaks Clustering (ODPC). The main innovations and contributions of this study include:

(1) ODPC introduces a mechanism for dynamic parameter tuning, including automatic adjustment of bandwidth and adaptive selection of truncation distance. By adopting the Silver Plate criterion to optimise the bandwidth for density computation and the K-Nearest Neighbors (K-NN) [22, 23] to automatically select the truncation distance, ODPC is able to flexibly adapt to different design scenarios and complexities. This feature enables ODPC to provide robust and efficient optimisation solutions in the face of changing design requirements and systems of varying complexity.

(2) An information theory-based distance metric adjustment is introduced in ODPC to optimise the similarity and difference metrics between data points by combining mutual information and KL dispersion. With this integrated distance metric, ODPC is able to more accurately reflect the complex relationships between design modules, thus improving the modularity level and design quality of the system. This improvement demonstrates strong adaptability and performance enhancement when dealing with design optimisation problems with high-dimensional, non-linear data.

2. **Peak density clustering algorithm and its optimisation.**

2.1. **principles and analysis.** dpc is a novel algorithm for identifying the centres of clusters based on the local densities and relative distances of the data points. the dpc algorithm does not require any prior knowledge of the number of clusters in the data, and it can efficiently find clusters of arbitrary shapes. Therefore, it performs well in dealing with complex high-dimensional datasets.

The core idea of the DPC algorithm is to use the local density and relative distance of the data points to identify the clustering centres. Specifically, the algorithm identifies clustering centres using two main metrics: 1. local density (ρ): a measure of how dense the area around each data point is; 2. relative distance (δ): a measure of the minimum distance between each data point and a data point with a higher density.

For each point x_i in the dataset, the local density ρ_i can be calculated by.

$$\rho_i = \sum_j \chi\left(d(x_i, x_j) - d_c\right) \tag{1}$$

where χ is an indicator function.

If $d(x_i, x_j) < d_c$ then $\chi = 1$, otherwise $\chi = 0$. The $d(x_i, x_j)$ denotes the distance between the data points x_i and x_j, and d_c is a truncation distance used to define the extent of the neighbourhood. In this way, ρ_i indicates how many data points are within distance d_c.

For each data point x_i, its relative distance δ_i is defined as the distance between x_i and the denser nearest neighbour data point.

$$\delta_i = \min_{j:\rho_j > \rho_i} d(x_i, x_j) \tag{2}$$

For the data point with the highest density, define its δ value as the maximum distance from that point to all other points to ensure that it can be used as a clustering centre.

2.2. **Strategy of ODPC.** In the ODPC algorithm, several improvements are made to improve the accuracy and efficiency of clustering. Specifically, this paper focuses on the following aspects of optimisation: density calculation, tuning of the distance metric and automated parameter selection. These optimisation strategies are described in detail below.

2.2.1. *Optimization of density calculation.* In the classical peak density clustering algorithm, the calculation of the local density (ρ) relies on a truncation distance (d_c), i.e., the density is only accumulated if the distance between two points is less than d_c. This method is concise but has the following shortcomings: (1) The choice of truncation distance is sensitive to the result: the choice of d_c has a great influence on the calculation of density, and if it is not chosen properly, it may lead to a decrease in the accuracy of the clustering result. (2) Insensitivity to outliers: Classical methods tend to ignore outliers (i.e., isolated points), which may adversely affect the identification of clustering centres.

To overcome these problems, the optimised density calculation method improves the density calculation by introducing Kernel Density Estimation (KDE) and Gaussian kernel function. The specific optimisation strategy is as follows:

KDE is a parameter-free density estimation method that smoothly estimates the probability density of the data by placing a kernel function at each data point location. The density estimation method of KDE is shown as follows.

$$\rho_i = \sum_j K\left(d(x_i, x_j), h\right) \tag{3}$$

where K denotes the kernel function, usually a Gaussian kernel function is chosen.

$$K\left(d(x_i, x_j), h\right) = \exp\left(-\frac{d(x_i, x_j)^2}{2h^2}\right) \tag{4}$$

where h is a bandwidth parameter that controls the width of the kernel function, i.e. the range of influence of each data point on its neighbourhood.

The choice of bandwidth h is critical. A smaller value of h will result in an estimated density with higher resolution but possibly with noise, while a larger value of h will result in a density estimate that is too smooth. The bandwidth can be chosen automatically by methods such as cross-validation or Silverman's rule of thumb to balance the smoothness and accuracy of the estimate.

The use of a Gaussian kernel function to compute the density allows the effect of each data point on other points in its neighbourhood to decay gradually, rather than being binarised as in the case of truncated distances. This approach makes the density calculation smoother and more stable, and better reflects the local densities of the data. With these improvements, the density calculation becomes more flexible and robust, with better adaptability to clusters of different shapes and densities. In addition, the optimised density calculation method enables ODPC to identify cluster centres more accurately and be more sensitive to outliers, thus improving the overall quality of the clustering results.

2.2.2. *Information theory-based distance metric tuning.* In Optimal Density Peak Clustering (ODPC) algorithm, the choice of distance metric has a crucial impact on the performance of the algorithm and the clustering effect. In order to better handle complex data structures and high-dimensional datasets, we introduce a distance metric adjustment method based on information theory. The method utilises Mutual Information (MI) and KL Scatter (Kullback-Leibler Divergence (KLD)) in information theory to measure the similarity and difference between data points, thus improving the accuracy and robustness of clustering.

Information theory provides a measure of uncertainty and similarity from the perspective of probability distributions. In data analysis and clustering, the use of information-theoretic metrics allows for a deeper understanding of the interrelationships between data, with significant advantages especially when dealing with nonlinear and high-dimensional data.

Mutual information is a measure of the degree of interdependence between two random variables. For two random variables X and Y, the mutual information is defined as follows:

$$I(X;Y) = \sum_{x\in X}\sum_{y\in Y} p(x,y)\log\frac{p(x,y)}{p(x)p(y)} \tag{5}$$

where $p(x,y)$ is the joint probability distribution; $p(x)$ and $p(y)$ are the marginal probability distributions of X and Y respectively. Mutual information reflects the amount of information shared between the variables X and Y, with larger values indicating greater interdependence between the two.

KL scatter is an asymmetry measure for the difference between two probability distributions P and Q. For discrete distributions P and Q, KL scatter is defined as follows:

$$D_{KL}(P\|Q) = \sum_{x\in X} P(x)\log\frac{P(x)}{Q(x)} \tag{6}$$

The KL scatter measures the information loss of P with respect to Q, with the KL scatter value being smaller as P and Q get closer, and vice versa.

In order to effectively apply information-theoretic metrics in ODPC, we propose a distance metric adjustment strategy based on mutual information and KL scatter. First, the joint probability distribution and edge probability distribution of the data points are calculated using the KDE method described above. For each data point x_i and x_j in the

dataset, we first construct its joint probability distribution $p(x_i, x_j)$ by KDE.

$$p(x_i, x_j) = \frac{1}{N} \sum_{k=1}^{N} K(x_i, x_k) K(x_j, x_k) \tag{7}$$

where N is the number of data points.

Calculate the edge probability distributions $p(x_i)$ and $p(x_j)$ for each data point separately. The marginal distribution can be obtained by integrating the joint distribution over another variable.

$$p(x_i) = \int_{-\infty}^{\infty} p(x_i, x_j) dx_j \tag{8}$$

$$p(x_j) = \int_{-\infty}^{\infty} p(x_i, x_j) dx_i \tag{9}$$

Using mutual information as a distance metric, the similarity between data points is measured by calculating their mutual information. For data points x_i and x_j, their distance metric based on mutual information is defined as follow:

$$d_{\mathrm{MI}}(x_i, x_j) = -I(x_i; x_j) \tag{10}$$

where $I(x_i; x_j)$ is the mutual information of the data points x_i and x_j. The negative sign is used to convert the similarity metric to a distance metric; the greater the mutual information, the smaller the distance.

The KL scatter is used to measure the difference between the probability distributions of the data points. For the data points x_i and x_j, their distance measure based on KL scatter is defined as follow:

$$d_{\mathrm{KLD}}(x_i, x_j) = D_{\mathrm{KL}}(p(x_i) \parallel p(x_j)) + D_{\mathrm{KL}}(p(x_j) \parallel p(x_i)) \tag{11}$$

This symmetric KL scatter measure ensures that the asymmetric effects of distance are eliminated and that the distance between two data points is the sum of their mutual information loss.

Finally, a comprehensive information-theoretic distance metric is used to measure the similarity between data points. In order to combine the advantages of mutual information and KL dispersion, we propose an integrated distance metric. For data points x_i and x_j, the combined information theoretic distance metric is defined as follow:

$$d_{\mathrm{IT}}(x_i, x_j) = \alpha d_{\mathrm{MI}}(x_i, x_j) + \beta d_{\mathrm{KLD}}(x_i, x_j) \tag{12}$$

where α and β are weight parameters to balance the contribution of mutual information and KL scatter.

A comprehensive distance metric is used to replace the traditional Euclidean distance in the process of clustering centre identification and data point assignment, thus improving the accuracy and robustness of clustering. The schematic diagram of distance metric adjustment based on information theory is shown in Figure 1.

In this schematic, we show the concept of distance metric adjustment based on information theory. The relationship between these data points and the clustering centres is represented by different distance metrics (mutual information and KL scatter). The blue dots indicate the position of each data point in the dataset. The coordinates of the data points (e.g., A, B, C, etc.) are used to identify the locations of these points. The red "X" markers indicate the cluster centres determined by the ODPC algorithm. The location of the cluster centre is calculated based on the density and relative distance of the data points. The dotted line connections indicate the distances between the data points and the clustering centres, and these distance measures can be calculated based on mutual information and KL scatter. Each dashed line shows how the data points

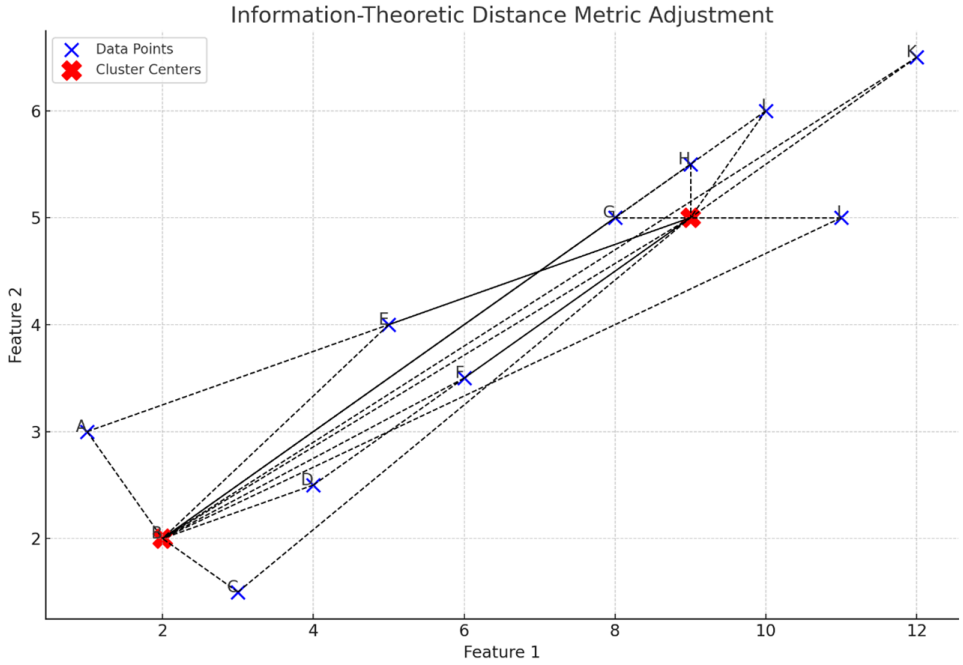

FIGURE 1. Adjustment of distance metrics based on information theory

are connected to the clustering centres by information-theoretic metrics, reflecting their similarities and differences. With this information-theoretic based adjustment of distance metrics, ODPC is able to capture the complex relationships between data points more accurately, especially when dealing with high-dimensional and non-linear data, and this approach significantly improves the performance of clustering and the stability of results.

2.2.3. *Automated parameter selection.* In the ODPC algorithm, parameter selection has a crucial impact on the accuracy and stability of the clustering results. In traditional methods, parameters are usually determined by relying on expert experience or through multiple trials, which is time-consuming and may lead to inconsistent clustering results. In order to enhance the robustness and adaptability of ODPC, this paper proposes a set of automated parameter selection strategies, including automatic adjustment of bandwidth, automatic selection of truncation distance, and data-driven weight optimisation.

The bandwidth h is a key parameter in kernel density estimation (KDE), which determines the degree of smoothing of the kernel function. In ODPC, a reasonable choice of bandwidth can significantly improve the accuracy and clustering effect of density estimation. We use Silverman's Rule of Thumb to automatically adjust the bandwidth h. Silverman's Rule of Thumb is a rule of thumb for estimating the bandwidth h, which is calculated as follows:

$$h = 0.9 \cdot \min\left(\sigma, \frac{\text{IQR}}{1.34}\right) \cdot n^{-1/5} \tag{13}$$

where σ is the standard deviation of the dataset, IQR is the interquartile range of the data, which represents the middle 50% range of the data, and n is the sample size.

With Silverman's Rule of Thumb, the bandwidth h is dynamically estimated and adjusted so that the density estimation captures the main features of the data while avoiding loss of detail due to excessive smoothing.

The truncation distance d_c is a key parameter in the calculation of the local density (ρ) and determines over what extent the data points contribute to the density. Automatic selection of d_c can be achieved by K-NN. The steps for K-NN selection of d_c are as follows:

(1) Calculate the distance to the nearest k-neighbour for each data point. For each point x_i in the data set, calculate its distance to all other points and find the nearest k-neighbours. Let the distances to these neighbours be $d_{i1}, d_{i2}, \ldots, d_{ik}$.

(2) Choose the average distance as the cut-off distance. For each data point x_i, calculate the average distance of its nearest k neighbours:

$$d_{c,i} = \frac{1}{k} \sum_{j=1}^{k} d_{ij} \tag{14}$$

where d_{ij} is the distance between the point x_i and its j-th nearest neighbour.

(3) Determine the global truncation distance. The final truncation distance d_c can be chosen as the average of all $d_{c,i}$:

$$d_c = \frac{1}{n} \sum_{i=1}^{n} d_{c,i} \tag{15}$$

This ensures that d_c is applicable to density calculations for most data points.

In the integrated information theory distance metric, the weighting parameters α and β determine the contribution of mutual information and KL scatter to the distance metric. To make the ODPC more flexible and adaptable to the characteristics of the data, we use a data-driven approach to automatically adjust these weights. The data-driven weight selection strategy is as follows:

First, the density variation and similarity distribution of the data are calculated. The density variation V_ρ is used to measure the difference in density at different points in the dataset and can be quantified by the density variance:

$$V_\rho = \frac{1}{n} \sum_{i=1}^{n} (\rho_i - \overline{\rho})^2 \tag{16}$$

where ρ_i is the local density of data point x_i and $\overline{\rho}$ is the average density of all data points.

The similarity distribution V_{sim} is used to measure the variation in similarity between data points and can be quantified by the variance of the similarity matrix. Similarity is assumed to be represented by the reciprocal of the distance $d(x_i, x_j)$:

$$V_{sim} = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{1}{d(x_i, x_j)} - \overline{sim} \right)^2 \tag{17}$$

where $\overline{sim}$ is the average similarity of all pairs of points.

Then, the weighting parameters are dynamically adjusted. The weighting parameters α and β are adjusted according to the relative size of the density change and similarity distribution.

$$\alpha = \frac{V_\rho}{V_\rho + V_{sim}} \tag{18}$$

$$\beta = \frac{V_{sim}}{V_\rho + V_{sim}} \tag{19}$$

3. **Product Design Optimization Model Based on ODPC.**

3.1. **Application of DSM in product design.** DSM is a powerful tool for representing and analysing the structure of complex product designs [24]. DSM presents the interrelationships between modules and components of a system in the form of a matrix. DSM is widely used in systems engineering and product design to optimise the design process, manage inter-module dependencies, and improve the maintainability of designs.

DSM is a square matrix of $n \times n$ [25], where n denotes the number of modules or components in the system. The rows and columns of the DSM represent the different modules in the system, and the elements of the matrix represent the interrelationships or dependencies between these modules. The basic form of the DSM is as follows:

$$DSM = \begin{bmatrix} M_1 & M_2 & M_3 & L & M_n \\ 0 & a_{12} & a_{13} & L & a_{1n} \\ a_{21} & 0 & a_{23} & L & a_{2n} \\ a_{31} & a_{32} & 0 & L & a_{3n} \\ M & M & M & O & M \\ a_{n1} & a_{n2} & a_{n3} & L & 0 \end{bmatrix}$$

where M_i and M_j denote the i-th and j-th modules in the system; and the element a_{ij} denotes the dependency or interaction between modules M_i and M_j.

By analysing the DSM, it is possible to find out which modules have strong dependencies on each other, thus guiding the partitioning and integration of modules. The values a_{ij} in the matrix can be either binary (0 or 1) or quantitative values that measure the strength of relationships between modules. An example of the DSM matrix is shown in Figure 2.

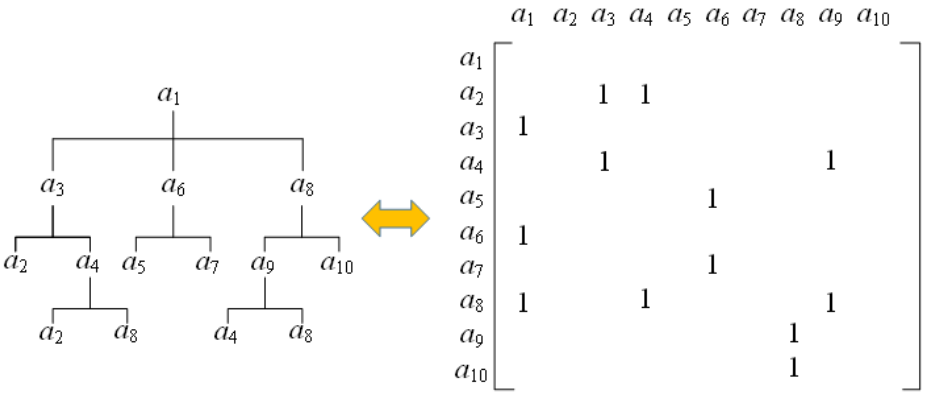

FIGURE 2. Examples of DSM matrices

In ODPC, DSM is not only used to represent the structure of product design, but also to optimise the clustering process of the design structure. First, the design structure matrix of the product is constructed. The DSM is built by defining all the modules in the system and the dependencies between them. This process usually involves listing all the modules in the system and identifying the direct dependencies or interactions between each module. Finally, these relationships are represented as matrix elements in the DSM. For example, for a complex mechanical system, a DSM can be constructed to represent the connections and interactions between different mechanical components.

3.2. **ODPC-based structural clustering model for product design.** In complex product design, it is crucial to optimise the dependencies and functional integration between modules. The ODPC-based product design structure clustering model proposed in this paper effectively identifies and optimises the modular structure of products by combining with DSM to improve design efficiency and quality.

3.2.1. *DSM-based Product Design Structure Representation.* The DSM is defined as a square matrix of $n \times n$, where n denotes the number of modules in the system and the elements of the matrix d_{ij} denote the dependencies or interactions between modules M_i and M_j. The matrix D is represented as follow:

$$D = \begin{bmatrix} d_{11} & d_{12} & d_{13} & L & d_{1n} \\ d_{21} & d_{22} & d_{23} & L & d_{2n} \\ d_{31} & d_{32} & d_{33} & L & d_{3n} \\ M & M & M & O & M \\ d_{n1} & d_{n2} & d_{n3} & L & d_{nm} \end{bmatrix}$$

where d_{ij} denotes the degree of dependency or interaction strength of module M_i on M_j.

3.2.2. *ODPC-based clustering process.* The ODPC algorithm automatically identifies the clustering centres and assigns modules to the most suitable clusters by analysing the densities and relative distances in the DSM to optimise the design structure. The following are the specific steps for implementing the ODPC-based clustering model:

Step 1: Identify Cluster Centres. In DSM, the local density ρ_i and relative distance δ_i of each module are calculated using the ODPC algorithm. The $\rho - \delta$ graph identifies the modules with high density and large distance as the clustering centres.

Step 2: Module Assignment. Assign each module M_i to the nearest clustering centre C_j. This assignment process is based on the distance and density relationship between modules and ensures that each module is assigned to the most appropriate cluster.

Step 3: Optimise inter-module dependencies. Use the dependency information in DSM to further optimise the clustering structure. The optimisation objectives include minimising the interaction complexity between clusters and maximising the functional consistency within clusters.

In order to minimise the interaction complexity between clusters, the dependencies between different clusters need to be reduced, so the optimisation objective function is defined as.

$$F_1 = \min \sum_{i \neq j} d_{ij} \cdot x_{ij} \tag{20}$$

where x_{ij} is a binary variable indicating whether modules M_i and M_j are located in different clusters (1 for yes, 0 otherwise).

Also in order to maximise the consistency within the clusters, the inter-module dependency within the same cluster needs to be enhanced with an optimisation objective function:

$$F_2 = \max \sum_{i,j \in C_k} d_{ij} \tag{21}$$

where C_k is the set of modules within cluster k and d_{ij} denotes the dependency strength between M_i and M_j.

Step 5: Generate optimised design structure. The final output of the optimised design structure matrix D' shows the optimised clustering structure and inter-module dependencies. The optimised DSM can better reflect the modular structure and functional integration of the system, and improve the design efficiency and maintainability of the system.

Next, we give an example of a application. Suppose we have a product design structure containing 5 modules with the following DSM:

$$D = \begin{bmatrix} 0 & 1 & 0.5 & 0.2 & 0.3 \\ 1 & 0 & 0.7 & 0.8 & 0.4 \\ 0.5 & 0.7 & 0 & 0.6 & 0.2 \\ 0.2 & 0.8 & 0.6 & 0 & 0.9 \\ 0.3 & 0.4 & 0.2 & 0.9 & 0 \end{bmatrix} \tag{22}$$

Using the above steps, we can cluster and optimise it by ODPC. A new design structure matrix D' is generated and the optimised clustering results and inter-module dependencies are shown as follows.

$$D' = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0.7 & 0.8 & 0 \\ 0 & 0.7 & 0 & 0.6 & 0 \\ 0 & 0.8 & 0.6 & 0 & 0.9 \\ 0 & 0 & 0 & 0.9 & 0 \end{bmatrix} \tag{23}$$

4. **Application case of ODPC.** In order to better illustrate the application of Optimal Density Peak Clustering (ODPC) in real product design, we choose a simpler household appliance, a smart air purifier. This case demonstrates how ODPC can improve design efficiency, performance and user experience by optimising the product design structure.

First, the DSM of the smart air purifier is established to represent the interrelationships between the functional modules and their subcomponents. For example, the DSM of a smart air purifier may include the following modules: fan system, cartridge system, sensor system, and control system. Assume that the initial DSM is as follows:

$$D = \begin{bmatrix} 0 & 0.8 & 0.5 & 0.3 \\ 0.8 & 0 & 0.4 & 0.7 \\ 0.5 & 0.4 & 0 & 0.6 \\ 0.3 & 0.7 & 0.6 & 0 \end{bmatrix} \tag{24}$$

In this matrix, the strength of the dependency between the fan system and the cartridge system is 0.8 and the strength of the dependency between the cartridge system and the sensor system is 0.4.

Local densities ρ_i and relative distances δ_i are calculated for each module using kernel density estimation methods to identify potential clustering centres. The calculation results are as follows.

$$\rho = [1.6, 1.9, 1.5, 1.6] \tag{25}$$

$$\delta = [0.3, 0.2, 0.4, 0.5] \tag{26}$$

where density ρ and relative distance δ denote the importance of each module in the design structure and its relative isolation from other modules, respectively.

On the $\rho - \delta$ graph, modules with high density and large relative distances are selected as clustering centres. This case will control the system (M_4) which can be identified as the centre of clustering because it has high density and relative distance.

Assign the other modules to the most appropriate cluster centres. Based on the distance and density relationships between modules, the fan system (M_1) and the sensor system (M_3) are assigned to the control system clusters, while the cartridge system (M_2) is also assigned to the same clusters due to its high dependency with the fan system. Based on the initial clustering results, the dependencies between modules are further optimised with the goal of minimising the interaction complexity between clusters and maximising

the functional consistency within clusters. Complex interactions between different modules are reduced by adjusting the assignment of modules and optimising the clustering structure.

The optimised DSM is as follows.

$$D' = \begin{bmatrix} 0 & 0.8 & 0.5 & 0 \\ 0.8 & 0 & 0.4 & 0 \\ 0.5 & 0.4 & 0 & 0.6 \\ 0 & 0 & 0.6 & 0 \end{bmatrix} \tag{27}$$

The optimisation has resulted in enhanced dependencies between the fan system, cartridge system and sensor system, while the control system has fewer direct dependencies with other systems, thus simplifying the complex interactions between the systems.

5. **Conclusion.** In this study, an intelligent optimisation method for product design based on ODPC is proposed. Firstly, the core algorithm and optimisation strategy of ODPC are discussed in detail, including the optimisation of density calculation, the adjustment of distance metric based on information theory, and the automated parameter selection method. Through these improvements, ODPC is able to automatically identify key modules in the design structure, optimise inter-module dependencies, and maintain efficient adaptability in different design scenarios. These features of ODPC enable it to perform well in coping with modular management and optimisation of complex systems.

ODPC has demonstrated its strong potential in the application of smart air purifiers. By analysing and optimising the design structure of the air purifier, ODPC successfully simplifies the complex interactions between modules and improves the performance and design efficiency of the system. This case demonstrates the effectiveness and feasibility of the ODPC methodology in real product design, and proves its advantages in dealing with complex design problems. By dynamically adjusting and optimising key parameters, ODPC is able to maintain robust performance and efficient optimisation in a changing design environment.

Although ODPC achieved significant results in the use cases in this paper, future research could further explore its potential and scalability for application in other complex systems. For example, the performance of ODPC in larger and higher dimensional datasets could be investigated, or its application in real-time design optimisation could be explored.

Acknowledgment. This work is partially supported by the Key projects of Xuzhou Vocational College of Industrial Technology (No. XGY2023ZXZD03).

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