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Joint Optimal Sensing Time and Power Allocation for Hybrid Spectrum Sharing based on Spectrum-Energy Efficiency

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Abstract. *Green cognitive radio is designed for alleviating the spectrum scarcity problem by sensing and allocating the idle spectrum bands. However, the existing design of CR systems needs to be optimized to increase the Spectrum Efficiency (SE) as well as Energy Efficiency (EE), which may not be obtained concurrently while using minimum transmit power. To implement this, this study proposes a joint design concerning spectrum sensing time as well as power allocation optimization issues suited to hybridize spectrum sharing based on SE-EE trade-off. It also considers the parameters like Quality of Service (QoS) requirement, PU's average interference power constraint, and SU's average transmit power constraint. To begin with, this problem has been formulated as a problem related to fractional programming that can be addressed based on the Dinkelbach method and Lagrangian duality theory. Subsequently, an iterative algorithm has been proposed for deriving optimal sensing time and relevant power allocation strategy that enable SU to achieve maximum EE and SE while ensuring QoS of PU. Finally, numerical simulations are presented for verifying the proposed power allocation strategy's performance in practice.*
Keywords: Green cognitive radio, sensing time, power allocation, spectrum efficiency, energy efficiency

1. Introduction.

The demand for high data rates and user scale has been boosted by the rapid increase in wireless services. Owing to this, the limited radio spectrum has become more and more crowded, and the energy consumption of the wireless system [1, 2] is growing substantially. Despite this situation, the Federal Communications Commission (FCC) has stated that many allocated spectrums experience low utilization in the daytime [3]. According to other research works [4, 5], 2% to 10% global energy consumptions and 2% of greenhouse gases originate from information and communication technology. Hence, by using 'spectrum holes', the inefficiency of radio spectrum and energy creates possibilities for the novel communication system known as Cognitive Radio (CR) [6, 7]. In CR, the Secondary Users (SUs) can use the unused spectrum of the Primary User (PU) without causing degradation to PU's QoS. Hence, CR is considered as a prominent radio technique to improve spectrum efficiency.

On the one hand, the issue of power allocation has received considerable critical attention as it seems to be a high-efficient solution used for improving the performance of the CR system, offering a fairly high rate of transmission to the SUs under power limits, along with maintaining the PU's QoS [8]. On the other hand, recent developments in the field of green communication have led to an increased interest in improving the spectrum and energy efficiency in green CR systems. Therefore, the primary challenge for the green CR system is how to appropriately formulate the optimal power allocation strategy while taking both SE and EE into consideration.

1.1. Literature Review.

SE maximization: SE has been used as a key performance indicator in the design and analysis of the CR system. Several researchers have reported that the SE maximization (throughput maximization) by optimizing the cooperative spectrum sensing as well as transmission parameters [9-13]. Researchers have presented a power control scheme to optimize both sensing time as well as fusion rule in Fusion Center (FC) and achieved the optimization of SU's throughput [9] . The novel CR system has been investigated to improve the SUs' throughput, where the data transmission time of SU is the whole sensing frame [10]. CR's transmit power control scheme is fully investigated under PR receiver's average interference power constraint [11]. A power allocation optimization problem has been proposed for underlying spectrum sharing under the transmit power constraint of SU [12]. This work was extended further [13], where the authors studied the issue of

determining the optimal sensing time and power allocation scheme to maximize SU's throughput under PU's average interference power constraint as well as SU's transmit power constraint. However, these studies primarily focus on maximizing or improving SE, devoid of any general agreement about EE maximization.
EE maximization: Focusing solely on maximizing the SE will lead to the higher energy consumption of the green CR system. EE, which is generally required to extend the lifetime networks, has been widely recognized as a key measure for designing the CR system. A large and growing body of literature has investigated EE maximization schemes. The optimization of sensing time to maximize EE has also been studied [12], while the fusion rule has been optimized to design the maximal EE for cooperative spectrum sensing systems [15]. Power allocation schemes have been observed for maximizing EE with various spectrum sharing mechanisms (i.e., overlay, underlay, and hybrid mechanism) [16-18]. A resource allocation scheme to maximize EE for SUs with underlay access was addressed previously [16], along with the investigation of EE maximization in CR system under SUs' transmit power in the overlay spectrum access scenarios [17]. To augment SUs' transmission chances, a hybrid spectrum mechanism had been developed [18] , which allows the SUs to transmit optimal power according to the spectrum sensing result. Moreover, there is research works [19] that provide ways to find the optimal power allocation scheme capable of maximizing EE for hybrid spectrum transmission subject to two power constraints (the average transmit power and the interference power).
SE-EE trade-off: There are only few researchers have paid attention to the problem of balancing SE and EE in the CR system. An optimal sensing time has been designed, and the final decision threshold is determined based on the SE-EE trade-off under the constraint of detection probability [20]. Also, research [21] is done to optimize both sensing time and final decision threshold for energy harvesting of CR networks.

From what has been discussed above, different optimization parameters, including sensing time, transmission time, fusion rules, sensing strategies and power allocation, are considered for overlay, underlay and hybrid spectrum sharing mechanism based on SE maximization and EE maximization . In addition, some works have been done to optimize the sensing time, decision threshold and constraint of detection probability based on SE-EE trade-off. However, the power allocation strategy considering the SE-EE trade-off has not been sufficiently addressed in the green CR system. Therefore, we will address the power allocation strategy based on SE-EE trade-off in the green CR system in this paper.

1.2. **Main Contributions and Organization.** Motivated by the discussions in Section 1.1, SE-EE power allocation scheme for hybrid spectrum sharing is put forward during this research. Specifically, our major concern is to optimize spectrum sensing time and power allocation for maximizing SE and EE under the constraint on SU's average transmit power and interference power of PU. During the optimization process, target detection probability has also been taken into consideration. The main contributions made by the research can be summarized as below:

(i) Consider the SE and EE maximization problem which comes under SU's average transmit power constraint and PU's average interference power constraint based on the hybrid spectrum sharing paradigm. In addition, we explicitly give the definition of SE and EE under the hybrid spectrum sharing in the formulations. The SE is defined as the average data rate per unit bandwidth, while EE is defined as the achievable CR transmitted data volume per consumed energy unit.
(ii) The joint SE and EE maximization problem is not a nonlinear concave fractional problem under transmit and an interference power constraint, which is converted

into an equivalent concave form. Lagrangian method, Dinkelbach theory, and subgradient scheme are applied to search the optimal transmit power as well as sensing time.

(iii) In this paper, the impact of imperfect sensing decisions, the SU's transmit power constrain, and the PU's interference power constraint on SE and EE are discussed.

This work is structured as below. The system model and the definition of SE and EE based on hybrid spectrum sharing are presented in Section II. In Section III, joint maximization of EE and SE is formulated under SUs' transmit power and PU's interference power. Section IV analyzes the optimization problem. Subsequently, numerical results are listed for discussion in Section VI. Eventually, Section VII concludes the entire study.

2. **System Model.** A network comprising the PU system and SU system (Fig. 1) has been proposed, where two secondary transceivers access the spectrum band licensed to relative primary transceivers with the hybrid spectrum sharing model. g_{pp}, g_{ss}, g_{sp}, g_{ps} are the instantaneous channel power gains from Primary Transmitter (PT) to Primary Receiver (PR), Secondary Transmitter (ST) to Secondary Receiver (SR), ST to PR, and PT to SR, separately. Besides, it is assumed that these channels are flat fading and channel power gains seem ergodic and stable at SU [9, 10, 13].

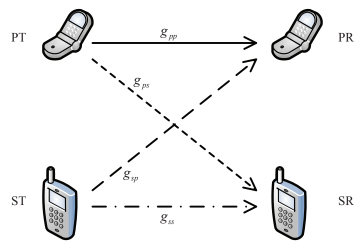

FIGURE 1. System model of CR network

Fig. 2 illustrates the frame structure of PU and SU system. Specifically, each cognitive frame consists of two phases, i.e., the spectrum sensing phase with the duration τ and the data transmission phase with the duration $T - \tau$. In hybrid spectrum sharing model, SUs can adjust the transmission power according to the corresponding sensing outcomes. When the target frequency band of PU is recognized as idle by spectrum sensing, SU can have a greater power $P^{(0)}$ in the data transmission phase. Similarly, when the PU channel has been recognized to be busy, SU transmits at a relatively lower power $P^{(1)}$ in the data transmission phase.

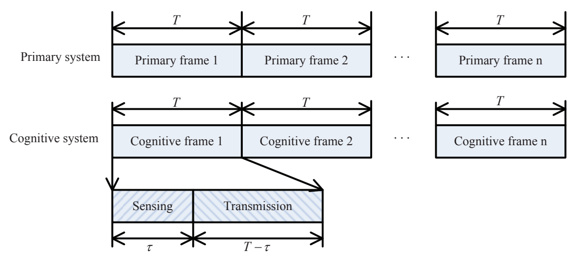

FIGURE 2. Frame structure of CR network

2.1. **Spectrum-Sensing Model.** In the spectrum sensing phase, every SU performs the spectrum sensing independently. According to PU's state, the received signal at SU is indicated by:

$$\begin{array}{ll} r(n) = w(n), & (\text{under } H_0) \\ r(n) = s(n) + w(n), & (\text{under } H_1), \end{array} \tag{1}$$

where $r(n)$ and $w(n)$ represent the receive signal at the SU and the additive white Gaussian noise at the SR. It is assumed that noise $w(n)$ belongs to an Independent and Identically Distributed (i.i.d) circularly symmetric complex Gaussian, which has a zero mean and variance σ_n^2. This belongs to a known prerequisite. Meantime, $s(n)$ represents the PU signal with all the samples following the i.i.d random process with zero mean and variance σ_s^2. Hypothesis H_0 and H_1 highlight absent PU and active PU, respectively. According to the energy detection scheme, relevant test statistics of energy detector should be contrasted with the detection threshold ε for deciding if the PU is absent or active. γ denotes the average Signal to Noise Ratio (SNR) of PU's received signal measured by the secondary detector under H_1. Under the Additive White Gaussian Noise (AWGN) fading channel hypothesis, SU's detection probabilities and false alarm can be calculated as [9]

$$P_d = Q\left(\left(\frac{\varepsilon}{\sigma_n^2} - \gamma - 1\right)\sqrt{\frac{\tau f_s}{2\gamma + 1}}\right) \tag{2}$$

$$P_{fa} = \left(\sqrt{2\gamma + 1}Q^{-1}(P_d) + \sqrt{\tau f_s}\gamma\right), \tag{3}$$

where τ indicates the sensing time, f_s suggests the sampling frequency and $Q(\cdot)$ refers to the complementary distribution function of standard Gaussian.

2.2. **SE of CR System.** Under the hybrid spectrum sharing model, SU adapts its transmit power in accordance with spectrum sensing decisions. This means that if SU detects the status of the frequency band to be active, it transmits data at a lower power $P^{(1)}$. Similarly, if SU is recognized as idle, it transmits data at a higher power $P^{(0)}$. Generally, $P^{(0)} > P^{(1)}$. Besides, P_p has been denoted as the received variance from the PU. According to PU's real status (active/idle), which is affected by the imperfect spectrum sensing, four scenarios of transmission rate exist. Final sensing results from SU concerning PU's presence or absence are as follows:

$$r_{00} = \log_2\left(1 + \frac{g_{ss}P^{(0)}}{N_0}\right), \tag{4}$$

$$r_{01} = \log_2\left(1 + \frac{g_{ss}P^{(1)}}{N_0}\right), \tag{5}$$

$$r_{10} = \log_2\left(1 + \frac{g_{ss}P^{(0)}}{g_{ps}P_p + N_0}\right), \tag{6}$$

$$r_{11} = \log_2\left(1 + \frac{g_{ss}P^{(1)}}{g_{ps}P_p + N_0}\right), \tag{7}$$

where the first index number illustrates PU's real status ('0' means the idle status and '1' suggests the active status) and the second index number denotes SU's decisions ('0' means absence and '1' means presence). Table 1 enlists four kinds of possibilities of the transmission model under the real-time environment depending on PU's real status and SU's decisions.

Table 1. Possible transmit power and rate for SE

PU's actual status	Sensing results	Related probability	Transmit power	Transmit rate
1	1	$P(H_1)P_d$	$P^{(1)}$	r_{11}
1	0	$P(H_1)(1-P_d)$	$P^{(0)}$	r_{10}
0	1	$P(H_0)P_f$	$P^{(1)}$	r_{01}
0	0	$P(H_0)(1-P_f)$	$P^{(0)}$	r_{00}

This infers that when the PU's channel is occupied but the sensing result made by the SU is vacant, which is called the undetected case, the SU continually transmits data at a higher power $P^{(0)}$, for it has no idea about the presence of PU. Moreover, if PU's spectrum band is vacant and corresponding sensing results show occupied, which can be called a false alarm case, the SU continually transmits data at a lower power $P^{(1)}$ despite the availability of the entire channel to the SU. Therefore, the average transmit bits of SU are indicated by:

$$R_{avg} = (T-\tau)\,E\left[(a_0r_{00}+a_1r_{01}+b_0r_{10}+b_1r_{11})\right], \tag{8}$$

where $a_0 = P(H_0)(1-P_f)$, $b_0 = P(H_1)(1-P_d)$, $a_1 = P(H_0)P_f$, $b_1 = P(H_1)P_d$. $P(H_0)$ and $P(H_1)$ denote the idle and busy probability for the PU channel, respectively.

In wireless communication, SE (measured in bits/s/Hz) considered the average data rate per unit bandwidth, quantifies the utilization rate of the available spectrum. Therefore, the average SE of the CR system can be represented as:

$$\eta_{SE} = \frac{T-\tau}{T}E\left\{[a_0r_{00}+a_1r_{01}+b_0r_{10}+b_1r_{11}]\right\}. \tag{9}$$

2.3. **EE of CR System.** In the hybrid spectrum sharing mechanism, SU's energy consumption comprises three sections, i.e., circuit consumption power P_c, spectrum sensing power P_s and data transmit power $P^{(0)}$ and $P^{(1)}$. In the imperfect spectrum sensing case, there are four kinds of scenarios depending on PU's real status as well as SU's decisions, which can be inferred from Table 2.

Table 2. Possible transmit energy for EE

PU's actual status	Sensing results	Related probability	Energy consumption
1	1	$P(H_1)P_d$	$\tau(P_s+P_c)+(T-\tau)\left(P^{(1)}+P_c\right)$
1	0	$P(H_1)(1-P_d)$	$\tau(P_s+P_c)+(T-\tau)\left(P^{(0)}+P_c\right)$
0	1	$P(H_0)P_f$	$\tau(P_s+P_c)+(T-\tau)\left(P^{(1)}+P_c\right)$
0	0	$P(H_0)(1-P_f)$	$\tau(P_s+P_c)+(T-\tau)\left(P^{(0)}+P_c\right)$

It can be concluded from Table 2 that if the PU channel is recognized to be absent, the SU will transmit data at a higher transmission power $P^{(0)}$. This can be computed from the equation of energy consumption- $\tau(P_s+P_c)+(T-\tau)\left(P^{(0)}+P_c\right)$. On the contrary, when the channel of PU is considered present, the SU will transmit data at a lower transmission power $P^{(1)}$, and its energy consumption can be calculated as $\tau(P_s+P_c)+$

$(T-\tau)\left(P^{(1)}+P_c\right)$. Therefore, the average energy consumption is given by:

$$\begin{aligned} E_{avg} &= E\left\{(a_1+b_1)\times\left[\tau\left(P_s+P_c\right)+(T-\tau)\left(P^{(0)}+P_c\right)\right]\right. \\ &\quad \left.+(a_0+b_0)\times\left[\tau\left(P_s+P_c\right)+(T-\tau)\left(P^{(1)}+P_c\right)\right]\right\} \\ &= E\left\{\tau\left(P_s+P_c\right)+(a_0+b_0)(T-\tau)\left(P^{(0)}+P_c\right)\right. \\ &\quad \left.+(a_1+b_1)(T-\tau)\left(P^{(1)}+P_c\right)\right\}. \end{aligned} \tag{10}$$

In wireless communication, the EE (measured in bits/Joule/Hz), which is the successfully transmitted information bits per unit energy from the transmitter to the receiver, can quantify energy utilization efficiency. EE of the CR system can be expressed as:

$$\eta_{EE} = \frac{R_{avg}}{E_{avg}} = \frac{\eta_{SE}\cdot T}{E_{avg}}. \tag{11}$$

3. **Problem Formulation.** In the hybrid spectrum sharing mechanism, interference on the PU channel is divided into two cases, namely, missed detection and correct detection. In the case that PU is wrongly recognized as absent (missed detection), the SU transmits with $P^{(0)}$. On the contrary, when the PU is recognized as present (correct detection), the lower transmit power $P^{(1)}$ will be adopted. Thus, PU's average interference power constraint is given as below:

$$C_1:\ \ \frac{T-\tau}{T}E\left[b_0 g_{sp}P^{(0)}+b_1 g_{sp}P^{(1)}\right]\le\Gamma, \tag{12}$$

where Γ suggests the maximum average interference power that is tolerable to PU of the frequency channel.

In the power allocation scheme, for controlling the SU's transmission power, it is mandatory to confine the SU's transmit power to a given threshold. SU's average transmission power is indicated as below:

$$C_2:\ \ \frac{T-\tau}{T}E\left[a_0P^{(0)}+a_1P^{(1)}+b_0P^{(0)}+b_1P^{(1)}\right]\le P_{av}, \tag{13}$$

where P_{av} indicates SU's maximum average transmit power.

Eventually, since the priority of the CR system is protecting the PU's QoS, a high detection probability will be necessary. In this research, target detection probability is set as P_d^{th}. Thus, the detection constrain can be formatted as follows:

$$C_3:\ \ P_d\ge P_d^{th}, \tag{14}$$

where P_d^{th} is the target detection probability of PU.

This research aims at designing a power allocation strategy that can maximize SE-EE under the average transmission and interference power constraints. Therefore, the optimization issue for SE-EE can be given by the following formula:

$$\begin{aligned} GP \qquad & \underset{\left\{\tau,P^{(0)},P^{(1)}\right\}}{\text{maximize}}\ \rho\eta_{SE}+(1-\rho)\,\eta_{EE} \\ & = \rho\eta_{SE}+(1-\rho)\,\tfrac{T\eta_{SE}}{E_{avg}} \\ & \text{subject to}\ \ C_1, C_2, C_3,\ P^{(0)}\ge 0,\ P^{(1)}\ge 0, \end{aligned} \tag{15}$$

where ρ represents the balancing factor, with $0\le\rho\le 1$. In the case of $\rho=1$, the problem mentioned above is about SE maximization. In the case of $\rho=0$, it can be considered as the EE problem.

4. **Solutions of the Formulation Problem.** When $0 < \rho < 1$, the optimization problem (15) belongs to a non-convex problem concerning $P^{(0)} and\, P^{(1)}$, but in no relation to the sensing time τ. Whereas, given that the sensing time falls into the range $[0, T]$, a one-dimension exhaustive search method is adopted for determining the optimal sensing time, which is indicated by:

$$\tau_{opt} = \arg\ \max_{\tau}\ \ \rho\eta_{SE} + (1-\rho)\,\frac{T\eta_{SE}}{E_{avg}}. \tag{16}$$

Then the optimized power allocation $P_{opt}^{(0)}$ and $P_{opt}^{(1)}$ can be obtained by:

$$\left[P_{opt}^{(0)},\ P_{opt}^{(1)}\right] = \left[P^{(0)},\ P^{(1)}\right]_{\tau=\tau_{opt}}. \tag{17}$$

Table 3. Design of the optimal sensing time

Design of the optimal sensing time
For $\tau = 0 : T$;
Calculate $P^{(0)}$ and $P^{(1)}$ using the method in table. 4;
End
The optimal sensing time $\tau_{opt} = \arg\ \max_{\tau}\ \ \rho\eta_{SE} + (1-\rho)\,\frac{T\eta_{SE}}{E_{avg}}$.

Table 3 highlights the design of the optimal sensing time where the one-dimension exhaustive search method is applied. Consequently, a method will be explored to seek the optimal power allocation strategy to maximize the SE-EE. In conclusion, the optimization problem GP may be transformed to the corresponding convex optimization problem, concerning the transmit power $P^{(0)}$, $P^{(1)}$ under the given sensing time τ. Hence, for the given sensing time, the design of the optimal sensing time τ and the optimization problem GP are transformed into the convex problem GP1 as below:

$$\begin{aligned} &GP1 \quad (\text{Given } \tau) \\ &\qquad \underset{\left\{P^{(0)},P^{(1)}\right\}}{\text{maximize}}\ \eta_{total}{=}\rho\eta_{SE} + (1-\rho)\,\eta_{EE} \\ &\qquad\qquad\qquad\quad = \rho\eta_{SE} + (1-\rho)\,\tfrac{T\eta_{SE}}{E_{avg}} \\ &\qquad \text{subject to}\ \ C_1,\, C_2 C_3,\, P^{(0)} \geq 0,\ P^{(1)} \geq 0. \end{aligned} \tag{18}$$

Problem GP1 belongs to a nonlinear fractional programming problem that seeks solutions from $P^{(0)}$, $P^{(1)}$. Following the fractional program-related theory and Dinkelbach's algorithm [22], GP1 can be transformed into the linear parametric problem GP2 through the introduction of another parameter. Problem GP2 can be expressed as:

$$\begin{aligned} &GP2 \quad (\text{Given sensing time } \tau) \\ &\qquad \underset{\left\{P^{(0)},P^{(0)},\xi\right\}}{\text{maximize}}\ \ f(\xi) = \rho\eta_{SE} + (1-\rho)\,T\eta_{SE} - \xi E_{avg} \\ &\qquad\qquad\qquad\quad = \left[T + (1-T)\,\rho\right]\eta_{SE} - \xi E_{avg} \\ &\qquad \text{subject to}\ \ C_1,\, C_2 C_3,\, P^{(0)} \geq 0,\ P^{(1)} \geq 0 \end{aligned} \tag{19}$$

where ξ indicates a non-negative parameter which may be explained as the pricing factor representing the energy consumption of SU. In terms of the optimal values for GP1 and GP2, results are obtained as follows:

Theorem 4.1. *The optimal SE-EE value for GP1 can be obtained as long as there is one optimal parameter ξ_{opt} in GP2 when $f(\xi_{opt}) = 0$ holds. Besides, the optimal SE-EE* *should equal to* ξ_{opt}

$$\xi_{opt} = \frac{[T+(1-T)\rho]\eta_{SE}\left(P_{opt}^{(0)}, P_{opt}^{(1)}\right)}{E_{avg}\left(P_{opt}^{(0)}, P_{opt}^{(1)}\right)} = \text{maximize}\left\{\frac{[T+(1-T)\rho]\eta_{SE}\left(P^{(0)}, P^{(1)}\right)}{E_{avg}\left(P^{(0)}, P^{(1)}\right)}\right\} \tag{20}$$

and

$$\begin{aligned} f\left(\xi_{opt}\right) &= f\left(\xi_{opt}, P_{opt}^{(0)}, P_{opt}^{(1)}\right) \\ &= \text{maximize}\left\{\left[T+(1-T)\rho\right]\eta_{SE}\left(P^{(0)}, P^{(1)}\right) - \xi_{opt}E_{avg}\left(P^{(0)}, P^{(1)}\right)\right\} \\ &= 0 \end{aligned} \tag{21}$$

For a given ξ, problem GP2 with respective transmit power $P^{(0)}$, $P^{(1)}$ are considered as a convex problem. Therefore, the Lagrange duality approach can be used to solve the problem of GP2 [20] with the duality gap is zero. Concerning the transmit powers $P^{(0)}$, $P^{(1)}$, the Lagrangian can be expressed as:

$$\begin{aligned} & L\left(P^{(0)}, P^{(1)}, \lambda, \mu\right) \\ & \quad = \left[T+(1-T)\rho\right]\eta_{SE} - \xi E_{avg} \\ & \quad -\lambda\left\{\tfrac{T-\tau}{T}E\left[a_0P^{(0)} + a_1P^{(1)} + b_0P^{(0)} + b_1P^{(1)}\right] - P_{av}\right\} \\ & \quad -\mu\left\{\tfrac{T-\tau}{T}E\left[b_0P^{(0)} + b_1P^{(1)}\right] - \Gamma\right\}, \end{aligned} \tag{22}$$

where λ and μ denote non-negative Lagrangian multipliers related to SUs' transmit power constraint and PU's interference constraint. The Lagrange dual optimization problem GP2 is expressed as below:

$$g\left(\lambda, \mu\right) = \max\ L\left(P^{(0)}, P^{(1)}, \lambda, \mu\right) \tag{23}$$

Thus, the Lagrange dual optimization problem is now given by

$$\begin{aligned} GP3 \quad & \text{minimize}\ \ \mathrm{g}\left(\lambda, \mu\right) \\ & \text{subject to}\ \lambda > 0,\ \mu > 0 \end{aligned} \tag{24}$$

For the given $P^{(0)}$, $P^{(1)}$, the supremum of the Lagrangian associated with transmit powers $P^{(0)}$ and $P^{(1)}$ need to be found for calculating the dual function $\mathrm{g}\left(\lambda, \mu\right)$. The joint optimization problem GP3 in relation to these two transmit powers is composed of two optimization subproblems, with one about $P^{(0)}$ and the other about $P^{(1)}$. This can be represented as below:

$$\begin{aligned} & \text{subproblem 1 (SP1)}: \underset{P^{(0)}\geq 0}{\text{maximize}} \\ & \quad Lop1 = \tfrac{[T+(1-T)\rho](T-\tau)}{T}E\left[a_0r_{00} + b_0r_{10}\right] - \xi E\left[\left(a_0+b_0\right)\left(T-\tau\right)P^{(0)}\right] \\ & \qquad -\lambda\tfrac{T-\tau}{T}E\left[a_0P^{(0)} + b_0P^{(0)}\right] - \mu\tfrac{T-\tau}{T}E\left[b_0P^{(0)}\right] \end{aligned} \tag{25}$$

$$\begin{aligned} & \text{subproblem 2 (SP2)}: \underset{P^{(1)}\geq 0}{\text{maximize}} \\ & \quad Lop2 = \tfrac{[T+(1-T)\rho](T-\tau)}{T}E\left[a_1r_{01} + b_1r_{11}\right] - \xi E\left[\left(a_1+b_1\right)\left(T-\tau\right)P^{(1)}\right] \\ & \qquad -\lambda\tfrac{T-\tau}{T}E\left[a_1P^{(1)} + b_1P^{(1)}\right] - \mu\tfrac{T-\tau}{T}E\left[b_1P^{(1)}\right] \end{aligned} \tag{26}$$

The aforementioned subproblems (SP1 and SP2) belong to the convex optimization problems. Karush-KuhnTucker (KKT) conditions can be employed for seeking the optimal solution. The first-order partial derivative for variables Lop1 and Lop2 concerning variables $P^{(0)}$ and $P^{(1)}$ can be given by:

$$\begin{aligned} \tfrac{\partial Lop1}{\partial P^{(0)}} = & \tfrac{[T+(1-T)\rho](T-\tau)}{T}a_0\tfrac{1}{N_0+g_{ss}P^{(0)}}\tfrac{1}{\ln 2} + \tfrac{[T+(1-T)\rho](T-\tau)}{T}b_0\tfrac{1}{N_0+g_{ps}P_p+g_{ss}P^{(0)}}\tfrac{1}{\ln 2} \\ & -\xi\left(a_0+b_0\right)\left(T-\tau\right) - \lambda\tfrac{T-\tau}{T}\left(a_0+b_0\right) - \mu\tfrac{T-\tau}{T}b_0 \end{aligned} \tag{27}$$

$$\begin{aligned} \tfrac{\partial Lop2}{\partial P^{(1)}} = & \tfrac{[T+(1-T)\rho](T-\tau)}{T}a_1\tfrac{1}{N_0+g_{ss}P^{(1)}}\tfrac{1}{\ln 2} + \tfrac{[T+(1-T)\rho](T-\tau)}{T}b_1\tfrac{1}{N_0+g_{ps}P_p+g_{ss}P^{(1)}}\tfrac{1}{\ln 2} \\ & -\xi\left(a_1+b_1\right)\left(T-\tau\right) - \lambda\tfrac{T-\tau}{T}\left(a_1+b_1\right) - \mu\tfrac{T-\tau}{T}b_1 \end{aligned} \tag{28}$$

Let $\frac{\partial Lop1}{\partial P^{(0)}} = 0$ and $\frac{\partial Lop2}{\partial P^{(1)}} = 0$, quadratic equations can be gained as below:

$$\left(P^{(0)}\right)^2 - A_0 P^{(0)} + B_0 = 0, \tag{29}$$

$$\left(P^{(1)}\right)^2 - A_1 P^{(1)} + B_1 = 0, \tag{30}$$

where

$$A_0 = \frac{\log_2^e \left[T + (1-T)\,\rho\right](a_0 + b_0)}{(\lambda + \xi T)(a_0 + b_0) + \mu b_0} - \frac{2N_0 + g_{ps}P_p}{g_{ss}}, \tag{31}$$

$$B_0 = \frac{N_0(N_0+g_{ps}P_p)}{g_{ss}^2} - \frac{\log_2^e[T+(1-T)\rho][a_0(N_0+g_{ps}P_p)+b_0N_0]}{g_{ss}[(\lambda+\xi T)(a_0+b_0)+\mu b_0]} \tag{32}$$

$$A_1 = \frac{\log_2^e \left[T + (1-T)\,\rho\right](a_1 + b_1)}{(\lambda + \xi T)(a_1 + b_1) + \mu b_1} - \frac{2N_0 + g_{ps}P_p}{g_{ss}}, \tag{33}$$

$$B_1 = \frac{N_0(N_0+g_{ps}P_p)}{g_{ss}^2} - \frac{\log_2^e[T+(1-T)\rho][a_1(N_0+g_{ps}P_p)+b_1N_0]}{g_{ss}[(\lambda+\xi T)(a_1+b_1)+\mu b_1]}\ . \tag{34}$$

In case that there are multiple solutions for quadratic equations (29) and (30), real solutions for the equations can be considered as optimal allocations. Further, the presence of roots in equations (29) and (30) need to be proved. The function for the quadratic equations (29) and (30) that correspond to the discriminant root can be given by:

$$\begin{aligned}\Delta_0 &= A_0^2 - 4B_0 \\ &= \left(\frac{\log_2^e[T+(1-T)\rho](a_0+b_0)}{[(\lambda+\xi T)(a_0+b_0)+\mu b_0]} - \frac{g_{ps}P_p}{g_{ss}^2}\right)^2 + \frac{\log_2^e[T+(1-T)\rho]a_0 g_{ps}P_p}{[(\lambda+\xi T)(a_0+b_0)+\mu b_0]g_{ss}^2}\end{aligned} \tag{35}$$

$$\begin{aligned}\Delta_1 &= A_1^2 - 4B_1 \\ &= \left(\frac{\log_2^e[T+(1-T)\rho](a_1+b_1)}{[(\lambda+\xi T)(a_1+b_1)+\mu b_1]} - \frac{g_{ps}P_p}{g_{ss}^2}\right)^2 + \frac{\log_2^e[T+(1-T)\rho]a_1 g_{ps}P_p}{[(\lambda+\xi T)(a_1+b_1)+\mu b_1]g_{ss}^2}\end{aligned} \tag{36}$$

It can be analyzed from (35) and (36) $\Delta_0 > 0$, $\Delta_1 > 0$, so equations (29) and (30) correspond to two unequal real roots. Moreover, the solution can be achieved through the standard quadratic-root formula. With the predetermined sensing time, the optimal power allocation can be indicated by:

$$P_{opt}^{(0)} = \left[\frac{A_0 + \sqrt{\Delta_0}}{2}\right]^+, \quad P_{opt}^{(1)} = \left[\frac{A_1 + \sqrt{\Delta_1}}{2}\right]^+ \tag{37}$$

where $[x]^+ = \max(0,\, x)$. For determining the optimal power allocation strategy $P_{opt}^{(0)}$, $P_{opt}^{(1)}$, optimal values of λ and μ, which can maximize the dual function $\mathrm{g}(\lambda,\, \mu)$, should be correctly identified. Here, the gradient projection approach is applied for finding the optimal solution, and it demands to calculate the sub-gradients of λ and μ. These sub-gradients of λ and μ can be expressed as $\Delta\lambda = P_{av} - \left[(a_0 + b_0)\, P^{(0)} + (a_1 + b_1)\, P^{(1)}\right]$ and $\Delta\mu = \Gamma - \left(b_0 P^{(0)} + b_1 P^{(1)}\right)$, respectively. λ and μ can be obtained from the following equations.

$$\lambda^{(k+1)} = \left[\lambda^k - s\Delta\lambda\right]^+, \tag{38}$$

$$\mu^{(k+1)} = \left[\mu^k - s\Delta\mu\right]^+, \tag{39}$$

where s represents the step size, k denotes the number of iterations. As inferred from some previous studies, when the step size s is small enough, dual variables will be converted to the optimal value λ_{opt}, μ_{opt} in a small area [22]. As a consequence, $P_{opt}^{(0)}$ and $P_{opt}^{(1)}$ can be achieved by substituting (31)-(36) into (37).

For solving the problem GP, the value ξ should be found. The Dinkelbach algorithm is suitable for solving the fractional programs, which can be converted into the optimal value at a superlinear rate [22]. Dinkelbach method based on iterative power allocation algorithm in SE-EE maximization has been listed in Table 4.

Table 4. Proposed iterative power allocation algorithm.

Algorithm 1 power allocation that maximizes SE-EE of CR system
1. **Given**: the maximum iteration number $L_{\max}$ and the error tolerance $\delta_1,\ \delta_2,\ \delta_3$;
2. **Initialize**:Calculate $\xi^{(0)} = \xi_0$, dual variables $\lambda^{(0)} = \lambda_0$ and $\mu^{(0)} = \mu_0$, the iteration index $n = 0$ and $k = 0$;
3. **While** $\left(\left\|f\left(\xi^{(n)}\right)\right\| \le \delta_3$ and $n \le L_{\max}\right)$ **do**
4. Calculate $P^{(0)}$ and $P^{(1)}$ using (31)-(37);
5. Update λ and μ via the subgradient method as follows:
6. **Repeat**
7. Calculate subgradients $\Delta\lambda$ and $\Delta\mu$ using (38) and (39) and update $\lambda^{(k+1)}$, $\mu^{(k+1)}$ as follows:
8. $\lambda^{(k+1)} = \left[\lambda^k - s\Delta\lambda\right]^+$,
9. $\mu^{(k+1)} = \left[\mu^k - s\Delta\mu\right]^+$,
10. $k = k + 1$,
11. where k is iteration number, s is the step size;
12. **until** $\left(\left(\left\|\lambda^{(k)}\left(\Delta\lambda\right)\right\| \le \delta_1\right.\right.$ and $\left.\left.\left\|\mu^{(k)}\left(\Delta\mu\right)\right\| \le \delta_2\right)\right)$,
13. **Set** $n = n + 1$ and $\xi^{(n)} = \frac{[T+(1-T)\rho]\eta_{SE}\left(P_k^{(0)}, P_k^{(1)}\right)}{E_{avg}\left(P_k^{(0)}, P_k^{(1)}\right)}$;
14. **End While**
15. **Return** $\left[P_{opt}^{(0)}, P_{opt}^{(1)} 1\right] = \left[P_k^{(0)}, P_k^{(1)}\right]_{\tau=\tau_{opt}}$ and $\xi_{opt} = \xi^{(n)}$, respectively.

In this algorithm, there are two nested loops. For the outer loop, the Dinkelbach method can iteratively solve SE-EE problem by addressing an array of parameterized concave problems. It is shown that the Dinkelbach method possesses a superlinear convergence rate, and the sequence can be converted into an optimal solution through limited iterations. For the inner loop, Lagrange multipliers are updated with the sub-gradient approach, including sub-gradient calculations and other easy projection operations. This sub-gradient approach has been extensively adopted for seeking Lagrange multipliers owing to its simplicity, convenient implementation, direction computing speed, and global convergence property. Therefore, the presented algorithm has impressive computation efficiency.

5. **Simulation Results.** Simulations have been illustrated in this section for evaluating the performance of presented iterative power allocation strategies. Here, the measured SE and EE values need to be normalized relative to the maximum SE and EE values. These are identified as the normalized SE $\eta_{SE}^{norm} = \frac{\eta_{SE}}{max(\eta_{SE})}$ and the normalized EE $\eta_{EE}^{norm} = \frac{\eta_{EE}}{max(\eta_{EE})}$. The results can be averaged with 1000 Monte Carlo simulations. Relevant parameter settings are enlisted in Table 5.

Fig. 3 illustrates the effects of the sensing time and $P\left(H_0\right)$ on the η_{SE}^{norm} (when $\rho = 1$) and η_{EE}^{norm} (when $\rho = 0$). The threshold corresponding to average interference power tolerated to PU has been set as $\Gamma = -10$ dB and the total transmit power of SU is set as $P_{av} = 15$ dB. For any $P\left(H_0\right)$, there is an optimal value τ_{SE}^{opt} that maximizes η_{SE}^{norm} and

TABLE 5. List of simulation parameters.

Parameter name	Value
Sensing time T	100 ms
Sampling frequency f_s	6 MHz
Probability that PU is idle $P(H_0)$	$\{0.6, 0.7, 0.8, 0.9\}$
Detection probability P_d	0.9
Maximum average transmit power P_{av}	$\{5 \sim 20\}$ dB
Maximum average interference power Γ	$\{-20 \sim -5\}$ dB
Sensing power P_s	40 mW
Transmit power P_p	180 mW
Circuit power P_c	80 mW
Step size for updating λ and μ	0.1
Error tolerance $\delta_1,\ \delta_2,\ \delta_3$	10^{-5}

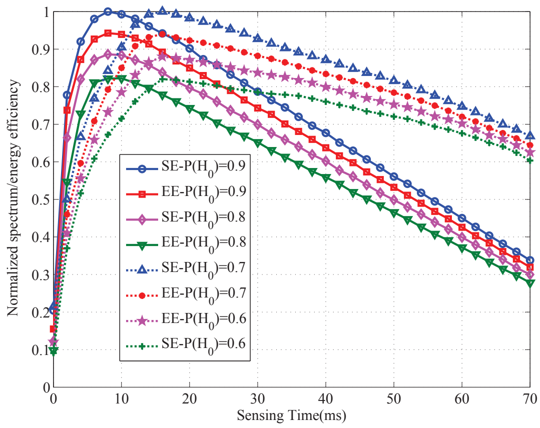

FIGURE 3. The normalized SE and EE versus sensing time for different values of $P(H_0)$.

one optimal value τ_{EE}^{opt} that maximizes η_{EE}^{norm}. These maximum values of τ_{EE}^{opt} and η_{EE}^{norm} cannot be achieved simultaneously. The maximum η_{SE}^{norm} can be obtained when $\tau = 8\ ms$, whereas the maximum η_{EE}^{norm} can be obtained when $\tau = 16\ ms$. It is evident from Fig. 3 that the normalized SE and EE increase as $P(H_0)$ increases. The phenomenon should be considered rational, for a higher $P(H_0)$ corresponds to a higher probability when PU's spectrum band is available, and more chances will be provided for SU to have data transmission at a higher transmit power.

Fig. 4 shows the variations in η_{SE}^{norm} and η_{EE}^{norm} with ρ, when the optimal sensing time τ and power allocation $P^{(0)}, P^{(1)}$ are determined to maximize $\rho\eta_{SE} + (1-\rho)\,\eta_{EE}$. It is observed that the curves of η_{SE}^{norm} and η_{EE}^{norm} are not smooth because different pairs of τ and $P^{(0)}, P^{(1)}$ are chosen for different values of ρ. Moreover, at $\rho = 0.2$, η_{SE}^{norm} and η_{EE}^{norm} values are about 0.72 and 0.73, respectively, while at $\rho = 0.4$, η_{SE}^{norm} and η_{EE}^{norm} values are about 0.93 and 0.37, respectively. As a result, the exact value of ρ can be determined only based on the requirements proposed by the CR system.

Fig. 5 reveals variations in η_{SE}^{norm} and η_{EE}^{norm} versus the maximum average transmit power P_{av} with different $P(H_0)$ values with identical maximum average interference power Γ and ρ for hybrid and underlay spectrum sharing. It can be concluded that the normalized SE and EE increase as $P(H_0)$ value. This outcome is rational because a higher $P(H_0)$

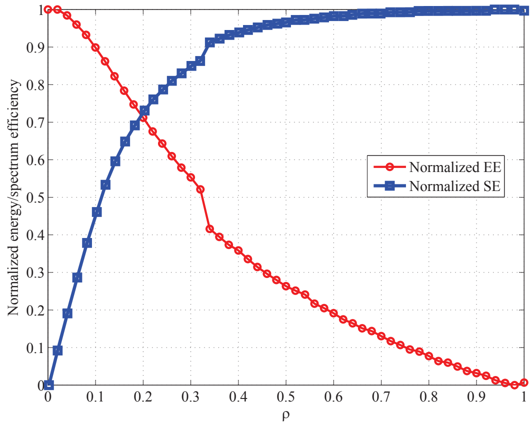

FIGURE 4. The normalized SE and EE versus ρ when $P^{(0)}, P^{(1)}$ and τ are jointly optimized to maximize $\rho\eta_{SE} + (1-\rho)\,\eta_{EE}$.

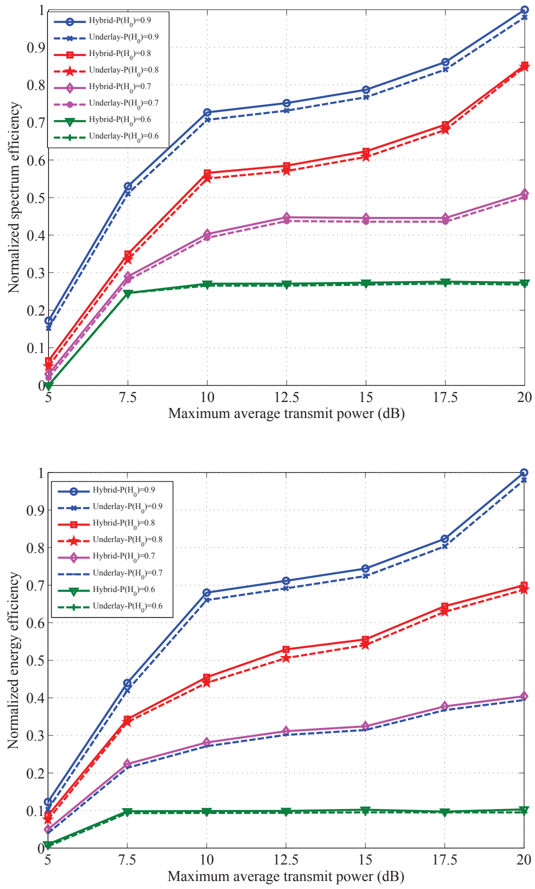

FIGURE 5. The normalized SE and EE versus the maximal average transmit power P_{av} for different values of $P\left(H_0\right)$.

increases the probability of the availability of spectrum band of PU, and more chances are provided for SUs' transmission of data qat a higher transmit power. It is also shown that the values of η_{SE}^{norm} and η_{EE}^{norm} increase with the increase of average transmit power.This phenomenon should be ascribed to the fact that a higher P_{av} allows the SU to allocate transmit power more flexibly, and thus, a higher EE and SE can be achieved. This shows that SE and EE for the hybrid spectrum sharing are higher than those for the underlay spectrum sharing, and this may be ascribed to the point that SU can adapt transmit power according to the sensing result.

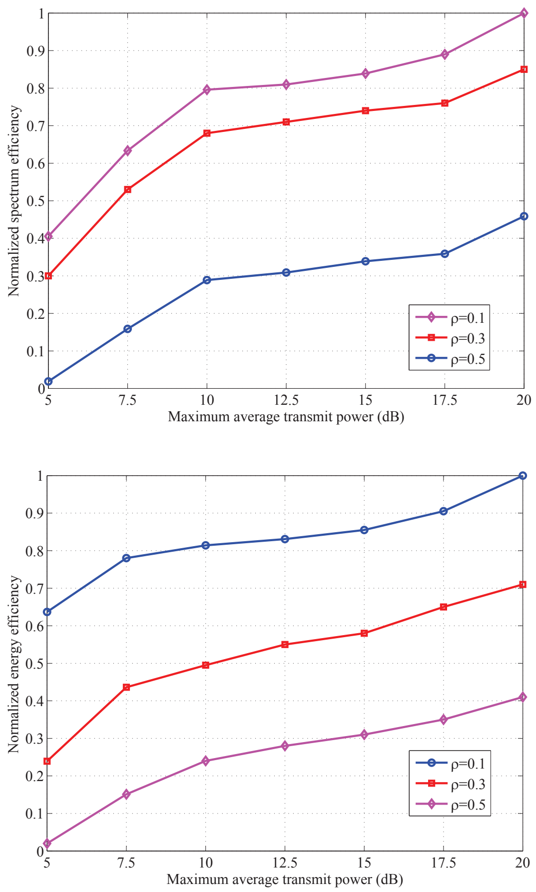

FIGURE 6. The normalized SE and EE versus the maximal average transmit power P_{av} for different values of ρ.

Fig. 6 highlights the normalized SE and EE versus the maximum total transmit power P_{av} with different values of ρ under identical maximum average interference power Γ. The maximum average interference power of PU has been determined as $\Gamma = -10$ dB. And the values of ρ are set as $\rho = 0.1$, $\rho = 0.3$ and $\rho = 0.5$. As shown in Fig. 6, both η_{SE}^{norm} and η_{EE}^{norm} values increase as maximum average transmit power increase. Moreover, under the same condition of average transmit power of SU, the η_{SE}^{norm} decreases with the increase of ρ, whereas the η_{EE}^{norm} increase with the increase of ρ. Eventually, it can be concluded form 6 that the gap of η_{SE}^{norm} between $\rho = 0.1$ and $\rho = 0.3$ is smaller than that between $\rho = 0.3$ and $\rho = 0.5$ and the gap of η_{EE}^{norm} between $\rho = 0.1$ and $\rho = 0.3$ is almost the same as that between $\rho = 0.3$ and $\rho = 0.5$. This thing has also been highlighted in Fig. 4.

Fig. 7 shows the normalized SE and EE versus the maximum total transmit power P_{av} for different values of Γ under the same $P(H_0)$, respectively. $P(H_0)$ value is set as 0.8 and the balance factor is set as $\rho = 0.2$. A similar results that both η_{SE}^{norm} and η_{EE}^{norm} values increase with the increase in average transmit power constraint as well as the interference power constraint can be obtained from Fig. 7. This similar phenomenon can be similarly explained by the fact that the transmit power of SU can increase due

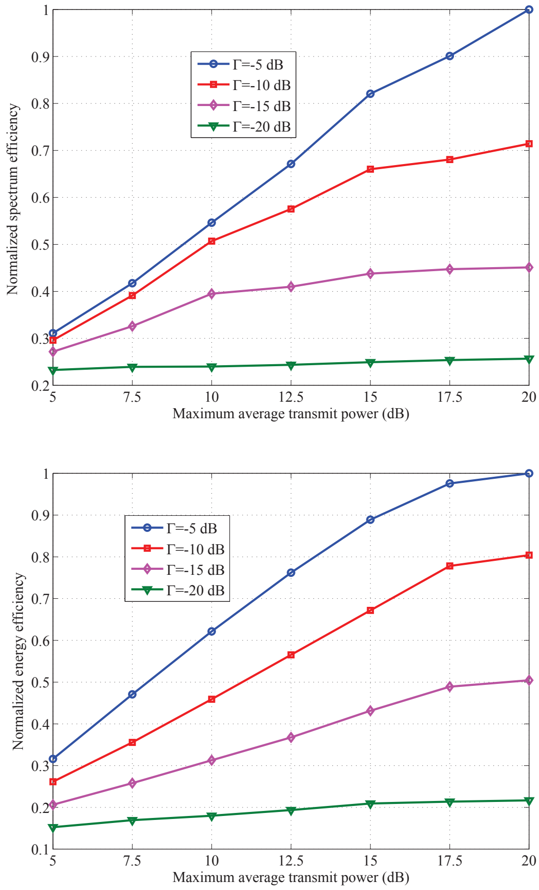

FIGURE 7. The normalized SE and EE versus the maximal average transmit power P_{av} for different values of Γ.

to the increasingly loose constraints. When Γ becomes sufficiently large, the transmit power constraint is the main factor affecting the η_{SE}^{norm} and η_{EE}^{norm}. For example, when $\Gamma = -5$ dB, the η_{SE}^{norm} and η_{EE}^{norm} of the SU depend on the transmit power constraint since the transmit power constraint is the dominant constraint. On the contrary, when $\Gamma = -20$ dB, the interference power constraint is the main constraint.

6. **Conclusion.** The joint optimal sensing time and power allocation policies based on the hybrid spectrum sharing, which can maximize the SE and EE for the CR system, have been addressed in this paper. Meanwhile, SU's average transmit power constraint, PU's average interference power constraint, and the protection of PU constraint have been considered in the SE-EE optimization problem. The outcome of simulations has proved the performance of the presented iterative power allocation scheme. In the future, the power allocation scheme capable of maximizing SE and EE in the energy harvesting cooperative CR system can be extended.

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