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A High Robust and Blind Image Watermarking Using Arnold Transform Mapping in the DCT Domain of YCbCr Color Space

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Abstract. *This study proposes a robust and blind watermarking scheme, based on Arnold transform and DCT in YCbCr color space. Before embedding the watermark, the host image is scrambled using Arnold transform, and then the scrambled RGB image is converted into YCbCr color space. To improve the security of the scheme, the watermark is encrypted with a private key. The encrypted watermark is then embedded into the channels of YCbCr by the proposed method, which has low computation complexity. Experimental results reveal that the proposed scheme not only has good perceptual quality after embedding watermark, but also has satisfactory performance of robustness. More importantly, we verify that embedding the watermark into different channels can resist different attacks. This study also develops an expanded scheme including a voting mechanism, which improves the robustness compared with state-of-the-art methods.*
Keywords: Watermarking, Arnold Transform Mapping (ATM), Discrete Cosine Transform (DCT), YcbCr color space.

1. **Introduction.** The rapid development of Internet or IOT technologies has resulted in multimedia files, such as videos, audios and images, becoming increasingly prevalent in our lives, resulting in an increasing need for attention to copyright [1]. In this scenario, digital watermarking is a common way to protect the copyright and restrict copying.

A watermarking scheme for embedding a logo or watermark into an image to protect copyright needs to satisfy two requirements; namely perceptual quality and robustness. Perception is a model for evaluating human information processing and recognition and the perceptual quality depends on the significance of the change in the stimulus material. The change in perceptual quality means a minimal difference between the host image before and after embedding watermark into it, minimizing interest from malicious attackers. Robustness refers to whether the watermark can be extracted in a satisfactory visual quality after the watermarked image is attacked using methods such as JPEG compression, rotation, cropping or adding noise.

Many watermarking schemes have been proposed in recent years. Some of these embed a watermark by modifying the pixel intensities in the spatial domain of the host image, while others modify the coefficients in the transformed domain. The former techniques in the spatial domain such as self-reference watermarking [2] or direct current (DC) coefficient based watermarking [3] are simple to implement because of low computational complexity, but are not very robust. In 2020, Mei et al. [4] proposed a high robust watermark scheme which applies improved weight template to enhance the imperceptibility of the watermark and performs double autocorrelation to resist the attacks such as collages, cutouts, etc. On the contrary, the watermarking methods in the transform domain have higher performance and robustness, so these methods are more popular. These frequency-based watermarking schemes can adopt many transform domains, including singular value decomposition (SVD) [5–7], discrete Fourier transform (DFT) [8], discrete wavelet transform (DWT) [9], and discrete cosine transform (DCT) [8, 10–12]. In particular, the DCT-based watermarking schemes have gained much attention because DCT is applied in JPEG and MPEG compression standards, which are important in image and video manipulation. DCT can be applied either on the whole image or on a specific size block of the image, such as 4×4, 8×8 or 16×16. For the energy compaction and computational complexity of different block sizes, an 8×8 sized block is most popular in watermarking techniques. Lin et al. [12] proposed a scheme that embeds a watermark into a host image by changing the DCT low-frequency coefficients with the concept of mathematical remainder. Phadikar et al. [13] have proposed embedding watermarks into visually significant regions of the host image using quantization index modulation. Ma et al. [14] proposed a DCT domain-based watermarking method with Arnold transform mapping. The watermark is scrambled using the ATM method before performing the embedding procedure, which decreases the correlation between pixels and thus improves the security of the scheme. However, the perceptual quality of the watermarked image is not satisfactory. To improve the security of the watermarking scheme, Guo et al. [15] proposed a method for encrypting the host image as well as the watermark before embedding. Although the Guo et al.' method achieves a high level of security, experimental results reveal that it has poor robustness to signal attacks, particularly JPEG compression. Khalili [16] proposed a DCT-Arnold chaotic watermarking scheme in 2015. Unlike other methods, Khalili embeds watermark into one channel of the YCbCr color space rather than the RGB space. It can produce a watermarked image with satisfactory perceptual quality, but is difficult to implement due to high computation cost complexity. Parah et al. [17] presented a scheme using inter-block coefficient differencing to embed a watermark into the DCT domain. However, their scheme is still vulnerable to some signal attacks, such as rotation, sharpening and noise addition.

This study presents a blind robust watermarking scheme that embeds watermarks into YCbCr channels. The effect of using different channels on robustness to signal attacks is discussed. The proposed scheme is then expanded by incorporating a voting mechanism to improve robustness. Experimental results indicate that the proposed scheme not only has high perceptual quality, but also resists signal attacks better than state-of-the-art schemes. The rest of this paper is organized as follows. Section 2 introduces the inter-conversion between the RGB and YCbCr color spaces, as well as the Arnold Transform Mapping. Section 3 describes the proposed watermarking method. Section 4 presents experimental results and a brief discussion. Finally, conclusions are drawn in Section 5.

2. **Related Work.** This section introduces necessary knowledge of embedding watermark into YCbCr color space. Specifically, Subsection 2.1 introduces the transform from RGB to YCbCr and the reverse process. Subsection 2.2 presents the Arnold Transform Mapping.

2.1. **YCbCr color space.** YCbCr is an important color space, which is used as a part of color image pipeline in video and digital photograph systems. Y represents the luminance component, while Cb and Cr stand for the blue and red difference chroma components, respectively. After converting RGB image into the YCbCr color space, the human visual system (HVS) can easily tolerate the distortion which confirms that humans resolve reference incrementally in the presence of constraining visual context [18]. Embedding watermark into different channels can resist different signal attacks [19]. To verify the different channels' robustness to a specific attack, the proposed scheme embeds the watermark into different channels using the proposed method, and then tests the performance after different image processing attacks. The transform between RGB and YCbCr is shown as Formula (1) and Formula (2), in which the (1) is RGB to YCbCr while (2) is the backward transformation.

$$\begin{bmatrix} Y \\ Cb \\ Cr \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix} + \begin{bmatrix} 0.29890 & 0.58660 & 0.11450 \\ -0.16874 & -0.33126 & 0.50000 \\ 0.50000 & -0.41869 & -0.8131 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1.40200 \\ 1 & -0.34414 & -0.71414 \\ 1 & 1.7720 & 0 \end{bmatrix} \times \begin{bmatrix} Y \\ Cb \\ Cr \end{bmatrix} - \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix} \tag{2}$$

2.2. **Arnold transform mapping.** The generalized 2D Arnold Transform Mapping (ATM) [20] is an image scrambling technique making area preserving and invertible without loss of information. It is also known as cat map which transforms each pixel with coordinate (x, y) in an $L \times L$ image into a new location (x', y'). The coordinates are then transformed using the scheme of Andalibi and Chandler [21] as shown in Formula (3):

$$\begin{cases} x' = a_1 x + a_2 y + KL \bmod L \\ y' = a_3 x + a_4 y + KL \bmod L \end{cases} \tag{3}$$

where a_1 to a_4 must satisfy $|a_1 \times a_4 - a_2 \times a_3| = 1$ and the parameter K is obtained by $K = \max\{|a_1|, |a_2|, |a_3|, |a_4|\}$. The aim of applying the ATM is to scramble the image into a progressively disordered image. After ATM is applied a different number of times, it leads to differently disordered images. Increasing the security of the watermarking scheme is absolutely the reason to perform ATM operations.

For a watermark image, an important feature of the transform is the periodicity, which means that performing the ATM operation p times restores the scrambled image. The period p should not exceed $3L$ according to the suggestion in [21].

The proposed method performs ATM operations using the following formula:

$$\begin{cases} x' = x + 2y \bmod L \\ y' = x + y \bmod L \end{cases} \tag{4}$$

i.e. $a_1 = a_3 = a_4 = 1$ and $a_2 = 2, K = 2$. By using above transform formula, we find that if the size of image is 512×512, that is $L = 512$, then $p = 384$, which means that the original image is obtained by performing the ATM operation 384 times. To obtain a specific status of scrambled image, the iteration needs to be performed a specific number of times.

3. **Proposed Scheme.** This section described the proposed watermarking scheme in three stages. The first stage is the preliminary phase, in which the Arnold Transform Mapping (ATM) operation is performed on the host image for m times, and the scrambled RGB image is then converted into the YCbCr color space, which contains the three channels into which a watermark is embedded. In addition to the operation on the host image, the watermark is encrypted using an encryption key Key. In the second stage, the most important stage, the encrypted watermark bits are embedded into the YCbCr channels. The final stage takes the inverse operations of the first stage. A watermarked image in YCbCr color space is transformed into RGB space. Then a watermarked image is transformed back to its original form after the ATM is performed for $(p - m)$ times.

Figure 1 illustrates the flowchart of watermarking embedding procedure, and the detailed steps are shown as follows:

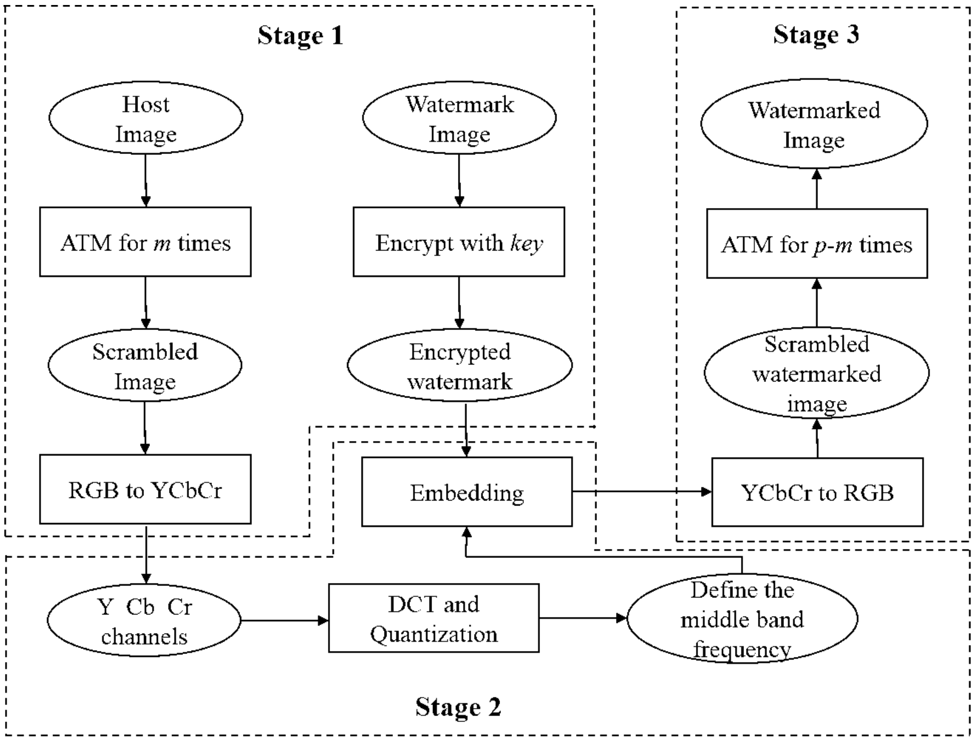

FIGURE 1. The flowchart of the proposed watermark embedding scheme.

3.1. Watermark embedding phase.

First Stage:

Step 1. We have a square host image with the size of $M \times N$ where $M = N$. Assume the periodicity of the image is p, which indicates that the host image can be scrambled to its original form after p times of ATM transforms. Scramble the host image m times using the ATM operation, $(m < p)$.

Step 2. Convert the scrambled RGB image into YCbCr color space according to Formula (1); that is, yielding an image with three channels: Y, Cb, and Cr.

Step 3. To protect the watermark, the watermark should be encrypted using an encryption key Key prior to its embedding. Generate a random bit stream with the Key, and then perform an *Exclusive-OR* operation on the watermark bit stream to obtain a string of encrypted watermark bits.

Second stage:

Consider the host image with Y channel as an example:

Step 1. Divide the host image with Y channel into non-overlapping blocks of sized 8×8, and then perform the DCT transformation on each block.

Step 2. Quantify each DCT-transformed block with the quantization table as shown in *Table 1*, which is a typical quantization table in the JPEG standard.

Step 3. Embed the encrypted watermark bits into the middle-band frequency of the quantified block simply by replacing the original value with the to-be-embedded data.

The same operations are performed on the other two channels, Cb and Cr.

Third stage:

The purpose of this stage is to restore the scrambled and converted host image back to the original RGB image. The steps are follows:

Step 1. Convert the watermarked image in the YCbCr color space to its RGB format using Formula (2).

Step 2. Re-scramble the RGB image using the ATM method (p–m) times to obtain the watermarked image, in which p denotes the number of times that the ATM transform was performed on a host image.

TABLE 1. The quantization table in the JPEG standard

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

3.2. **Watermark extracting phase.** The extracting watermark algorithm is almost the same as the embedding procedure. First, scramble the watermarked image using the ATM method m times and then convert the scrambled RGB image into YCbCr color space using Formula (1). Divide the watermarked channel into blocks and each block has size of 8×8. Then the DCT transform is performed followed by quantization. Finally, obtain the data in the defined middle-band frequency where the encrypted watermark bits are embedded. The encryption key Key is used to decrypt the encrypted bits to get the watermark bits. Figure 2 illustrates the flowchart of the watermark extracting procedure.

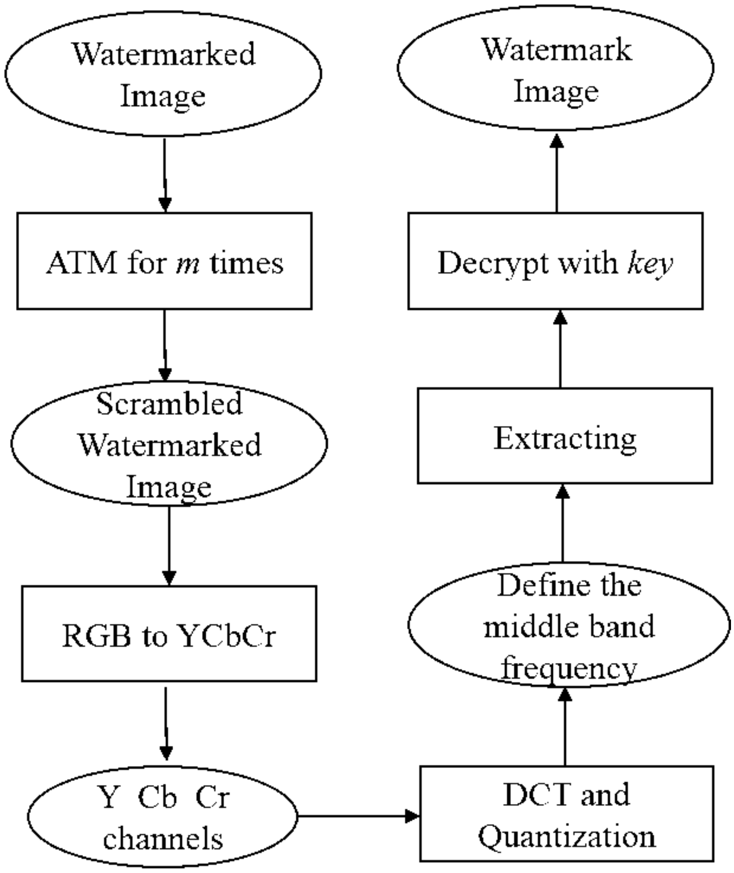

FIGURE 2. The flowchart of the watermark extracting procedure.

4. **Experimental Results.** We evaluate the performance of the proposed scheme from two main aspects; that is, perceptual quality and robustness. According to perceptual quality, *Peak Signal to Noise Ratio* (PSNR) is universally adopted to measure the visual quality of images. *Normalized cross correlation* (NC) is regarded as the generally accepted metric to measure the robustness. The adopted metrics *PSNR* and *NC* are defined as below:

As the most important and commonly used image visual quality metric, PSNR is used to evaluate the quality between the watermarked image and the original cover image, defined as

$$PSNR = 10\log_{10}(\frac{255^2}{MSE})dB \tag{5}$$

where MSE is the mean square error of an image which has $M \times N$ pixels:

$$MSE = \frac{1}{M \times N}\sum_{u=1}^{M}\sum_{v=1}^{N}(p_{uv} - p'_{uv})^2 \tag{6}$$

where p_{uv} is the pixel value of cover image and p'_{uv} is the pixel value of the watermarked image.

Normalized cross correlation is a standard metric for evaluating the similarity between original watermark and extracted watermark, and is defined as

$$NC = \frac{\sum_i \sum_j W(i,j)\overline{W}(i,j)}{\sqrt{\sum_i \sum_j W(i,j)^2}\sqrt{\sum_i \sum_j \overline{W}(i,j)^2}} \tag{7}$$

where W is the original watermark and is the extracted watermark.
Besides *NC* as the evaluation of robustness, *Bit Error Rate (BER)* is defined as the ratio of error bits of the extracted watermark compared with the original one. Less *BER* is, more robust of the watermarking scheme is. *BER* is defined as

$$BER(\%) = \frac{1}{M \times N}[\sum_{k}^{M\times N} W(k) \oplus \overline{W}(k)] \times 100 \tag{8}$$

Alike with *NC*, the variables in the formula stand for same meanings.

4.1. **Experimental results.** The proposed scheme was tested on four color images, *Lena, Baboon, Peppers* and *F16*, which are among the most common images adopted in watermarking schemes. Figure 3 shows the watermark has size of 32×32. Figure 4 displays the original and watermarked images embedded in different channels. Specifically, Figures. $4(a_1), (b_1), (c_1), and (d_1)$ represent four original images, while Figures $4(a_2), (b_2), (c_2), (d_2)$, Figures $4(a_3), \ldots, (d_3)$, and Figures $4(a_4), \ldots, (d_4)$ stand for the watermarked image after embedding the watermark into the Y, Cb and Cr channels, respectively. The images in Figure 4 reveal that the perceptual quality of the watermarked images is satisfactory. In particular, embedding the watermark into Cr channel produces a watermarked image with a high *PSNR* value.

FIGURE 3. A watermark image means "blessing" or "happiness".

In addition to the perceptual quality of the watermarking scheme, robustness to attacks is an indispensable element for evaluation. Extracting watermark from a watermarked image without attacks yields a lossless watermark can be obtain, where the NC and BER of the extracted watermark are 1 and 0%, respectively.

To test the robustness of our proposed scheme, and to test whether different channels can resist different attacks, some signal attacks were added to the watermarked image Lena, which was selected as the test image. Figure 5 shows the test results against different signal attacks, which are JPEG 90, Cropping 25%, Scaling 75%, respectively for Y, Cb, and Cr channels. Experimental results demonstrate that the watermark embedding into the Y channel performed best against JPEG compression attack; Cb channel performed well against resist cropping attacks, while the Cr-channel had best resistance against scaling attacks. Analytical results clearly indicate that for embedding the watermark in a different channel yields different robustness against JPEG compression, cropping and scaling attacks, and each channel performs best against a different attack type. However, the proposed watermarking scheme has worse NC values than other watermarking schemes. To improve the robustness of the proposed scheme, a voting mechanism was added when embedding the watermark into different channels. Each encrypted watermark bit was embedded for three times instead of one time. Consequently, when extracting watermark from attacked watermarked image, a voting mechanism is utilized to set the extracted bit to 1 or 0. Figure 6 demonstrates the results following the addition of the watermark. A comparison of Figure 5 and Figure 6 clearly reveals that the quality of extracted watermark improves significantly when assisting the same attack. The values of NC also provide proof. However, adding the voting mechanism triples the capacity, reducing the perceptual quality of the watermarked image. Figures 7 and 8 display the values of PSNR

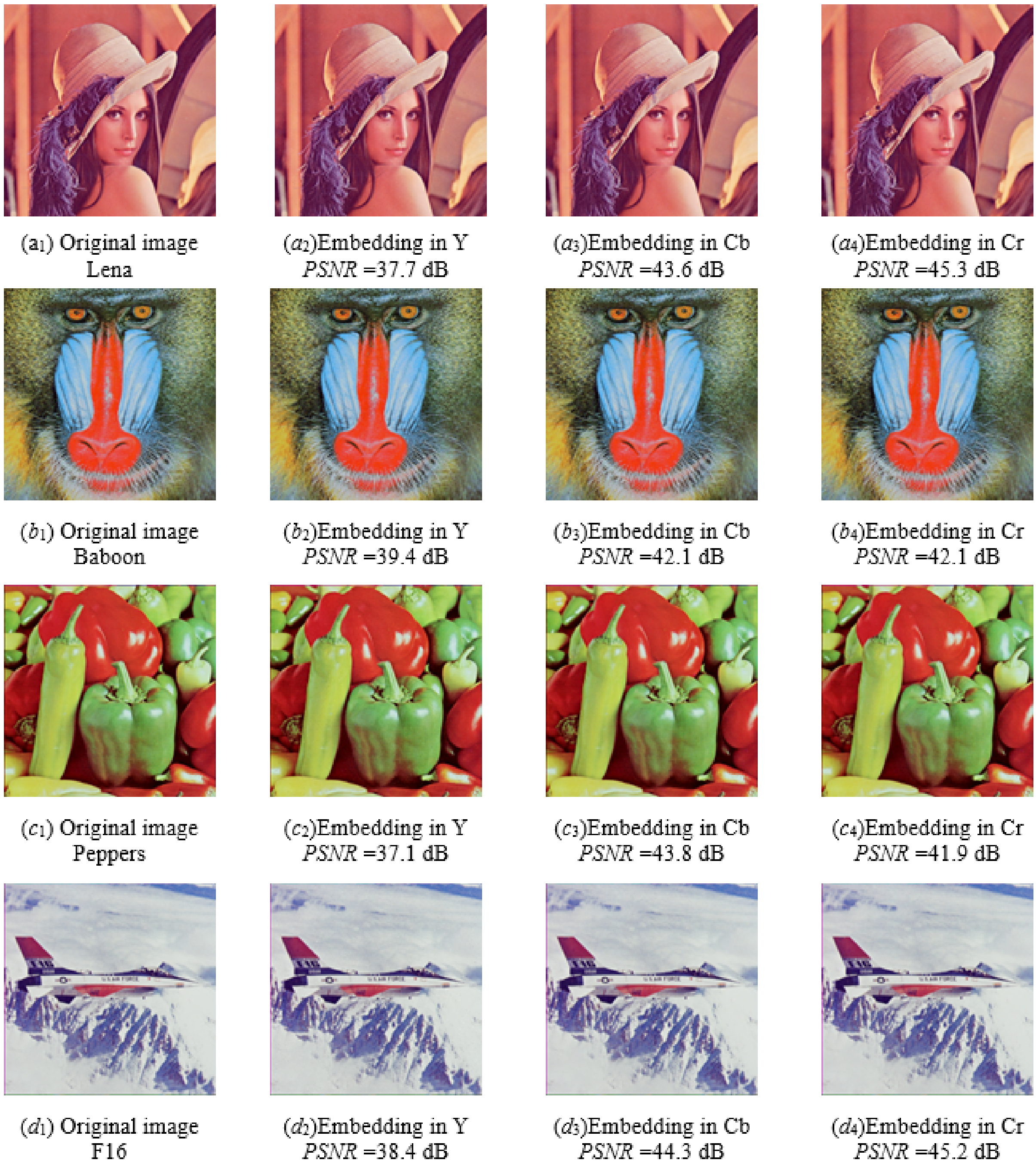

FIGURE 4. Four original images and their associated watermarked images with the PSNR values.

of watermarked images in the two proposed schemes, and compares them with the state-of-the-art watermarking schemes in [14,15,17]. Figures 7 and 8 show the proposed scheme yields a satisfactory PSNR when embedding a watermark once time, and has better performance than other schemes. In particular, the value of PSNR reaches 45 dB when the watermark is embedded into the Cr-channel. However, adding the voting mechanism affects the PSNR of the watermarked image. Compared with other schemes, in addition to Y channel, PSNRs of the other two channels are acceptable.

Whether adopting the one-time embedding scheme or three-time embedding scheme which has voting mechanism, the attack type needs to be known to determine which

FIGURE 5. Attacked watermarked Lena and extracted watermarks from different channels.

channel to embed the watermark. However, this cannot be predicted in practice. Therefore, the proposed scheme is extended by embedding a watermark into each channel with a voting mechanism, in which adopting the watermark is embedded in each channel. Lena was again selected to demonstrate the robustness of the watermark under different attacks. Figure 9 shows the attacked watermarked images and extracted watermarks. Table 2 lists the NC and BER values compared with Parah et al.'s scheme [17]. The data in Figure 9 and Table 2 indicate that the proposed scheme has higher NC values than Parah et al's scheme, indicating that it has better performance of robustness.

Figure 6. Attacked watermarked Lena and extracted watermarks from different channels with voting mechanism .

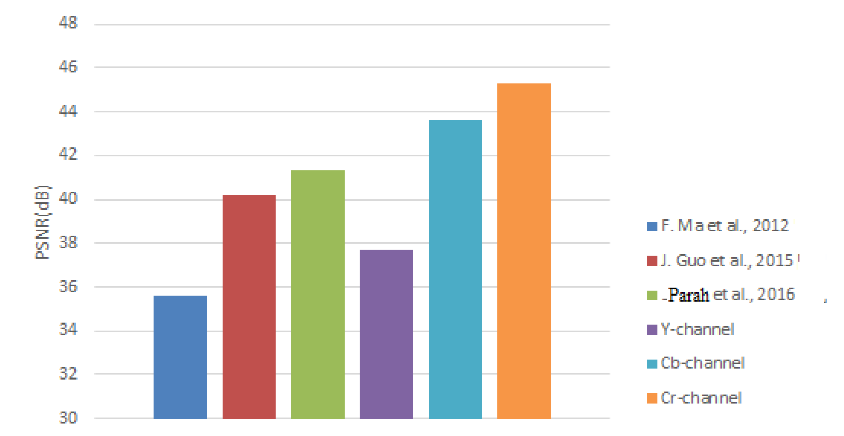

Figure 7. The image quality PSNR comparison for Lena without voting mechanism.

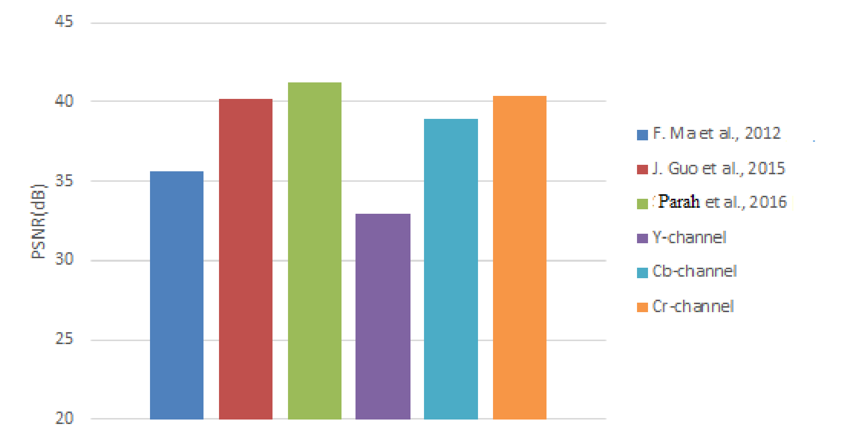

FIGURE 8. PSNR comparison for Lena with voting mechanism.

FIGURE 9. Watermarked images Lena and the extracted watermarks under various attacks.

Table 2. The comparisons between Parah et al.'s scheme and ours under various attacks

Attacks	Parah et al.'s method [17]		Ours	
	NC	BER(%)	NC	BER(%)
Salt and pepper noise(0.01)	0.8598	15.6	0.9642	4.10
Histogram equalization	0.9665	3.87	1	0
Gaussian noise (var = 0.001)	0.9375	8.66	0.9983	0.20
Sharpening	0.9731	2.98	0.9923	0.88
Rotation (1 degree)	0.9980	0.37	1	0
Rotation (5 degree)	0.9867	1.88	0.9991	0.09
Rotation (45 degree)	0.9628	6.23	0.9226	8.89
JPEG (QF = 90)	0.9841	1.56	0.9974	0.29

4.2. **Discussion.** Experimental results reveal that adopting the voting mechanism improves the robustness when meeting the same attacks. The proposed scheme adopts a three-time voting method. Obviously, increasing the number of embedding processes run, for example, by embedding the watermark five times in each channel, would enhance the watermarking robustability. Increasing the voting time means embedding more data into a host image, which would hurt the PSNR of the watermarked image. This trade-off is an inevitable issue and might need to address in the future.

5. **Conclusion.** This study proposes a blind and robust watermarking method that embeds a watermark into YCbCr color space after a DCT transform operation. Before embedding procedure, an Arnold transform mapping (ATM) and a secret key are applied to the host image and watermark, respectively. Experimental results reveal that the proposed method has satisfactory perceptual quality compared with other schemes. More importantly, we also prove that embedding watermark into different channels can resist different attacks. For instance, the Y channel best resists JPEG compression; the Cb channel resists cropping, while Cr channel has best robustness to scaling attacks. An expanding method is also developed to enhance the robustness by adding a voting mechanism. Experimental results reveal that the expanded method performs well in resisting attacks.

Digital image watermarking technology has been researched and developed for many years, and various embedded technologies have been proposed. However, even though the current AI technology has gradually become an obvious one, there are still very few AI-based methods to protect copyright. Combining the quantization index modulation scheme and the chaotic Logistic mapping, the CNN model is trained and the watermark is embedded in the weights of the CNN model [22]. Therefore, there will be more issues to be addressed for weighting the watermark in a network model.

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