

Community Detection Method for Complex Power Optical Fiber Network Based on Improved Local Edge Centrality

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ABSTRACT. *The layout of the power optical fiber network is intricate and complicated. In order to avoid the defect of relying on administrative divisions for management, it is necessary to divide the entire network into communities. In view of the importance of edge strength weighting in the complex network model, local edge centrality is suggested from a local perspective. Furthermore, based on improved local edge centrality, we propose a new algorithm for community detection in power optical fiber networks. Combining local edge betweenness and the idea of structural hole, the algorithm first divides the original network into some strongly connected communities with optimal modularity through an improved edge deletion process. The isolated nodes and some communities are then reconnected to the initial community to optimize the community structure and employs edge density to get the final partition for the network. The experiments are conducted on the Jilin Province power optical fiber network. Compared with that of other four typical methods, the proposed algorithm obtains the best results in the number of community, community structure. Especially, compared with the method based on administrative detection, the tenacity is improved by 60% and the connectivity within each community is realized. Meanwhile, the proposed algorithm gets good performance in the modularity and effectiveness.*

Keywords: Complex network; Power optical fiber; Backbone network; Community detection; Local edge centrality; Edge density

1. **Introduction.** In recent years, complex networks have attracted the attention of many disciplines [1, 2], including mathematics, computer science, sociology, biology, etc. They are usually represented by networks, the nodes in the network refer to various entities in systems, and edges or links represent relationships between these entities. Different from general network nodes with equal or similar degrees, the degree distribution of real-world networks follows a power-law distribution [3]. There are a large number of nodes with lower degrees in a network, and a small number of nodes with higher degrees [4], which is called small-world and scale-free properties in complex networks. However, in real-world networks, neither node degree distribution nor edge distribution is uniform. A common phenomenon is that nodes in a group are a subset of nodes in a given social network that are tightly connected, while nodes between groups are loosely connected. This characteristic of a given social network is called community structure [5], which is a popular problem and has been extensively studied in various works [6].

In community detection of complex networks, a modularity optimization method is proposed [7] to evaluate the quality of community partitioning. The greater the modularity is, the higher the quality of the community in general. FastGreedy [8] is a greedy algorithm that successively joins groups of vertices to form larger communities, and then selects the community partition with the greatest modularity. The algorithm has low time complexity, so it can be applied to large complex networks. But it has poor accuracy. Clauset [9] proposed a FastGreedy-based CNM algorithm that computes modularity using a heap structure to achieve an optimal community structure. It has obvious improvements in efficiency and accuracy, but it brings the disadvantage of unbalanced distribution of community size. To solve the problem of unbalanced community size distribution, Kong et al. [10] designs a greedy optimization method which can discover the hierarchical structure of the network and perform well in both effectiveness and efficiency. The main problem with modularity optimization methods is the resolution limitation, which means that small size communities cannot be identified.

The divisive method is a commonly used technique that divides the network into several parts by removing the edges between nodes in the network [11]. Radicchi et al. [12] proposed a new measure, called the clustering coefficient of edges. According to the measure, the higher the centrality value of an edge, the more likely the edge is a collection of edges

within a community. Furthermore, by modifying the marginal clustering coefficients, the label propagation algorithm LPA [13] is proposed to group community members together through the propagation of the same information in the network. But the disadvantage is that the randomness of label propagation leads to unstable community division. CO-LEC [14] is a stable community detection algorithm based on local edge centrality that uses modularity maximization. However, the network study above does not take into account the diversion of neighbor nodes. As a typical complex network system, the power optical fiber network has been widely accepted [15]. These methods also doesn't take into account the particularity of nodes and edges in real power optical fiber network, and does not apply to community detection of power optical fiber network.

Aiming at the intricate layout of the power optical fiber network links, in order to avoid the defect of relying only on administrative divisions for management, it is necessary to divide the entire network into communities. In this paper, we propose a novel community detection method for complex power optical fiber network based on our improved local edge centrality. Based on the model, first we extract the backbone networks from the network to get the initial community and define local edge centrality by using local edge betweenness and global adjacent betweenness. Then we calculate the local edge centrality of all edges in network, and delete the edges according to threshold E . Finally, we add the communities to the backbone network according to the edge density and test the proposed algorithm on the network. Results on Jilin power optical fiber network are achieved good results with the tenacity and effectiveness. The algorithm solved the real problem of disconnection within the administrative district, which enables the internal connectivity of communities and facilitates the management of provincial administrative regions.

The paper is organized as follows. Section 2 introduces complex power optical fiber network model. Section 3 define local edge centrality. On the basis of that, the community detection algorithm is presented in section 4. The experiments is described in section 5. Finally, Section 6 concludes this study.

2. Power optical fiber network model. Large-scale power optical fiber network has been laid for power communication network. The power optical fiber network is divided into primary level, secondary level and third level, that is the three-levels power optical fiber network. The backbone network is composed of primary level and secondary level, which is the main route to connect various regions. In order to better manage the power optical fiber network, the corresponding complex power optical fiber network model should be analyzed and established, the following assumptions are made:

1): Each station is regarded as one node in the three-levels power optical fiber network. The communication stations of substations of 220 kV and above 220 kV, power plants and dispatching centers are regarded as the standard nodes with the node weight of 1 and the remaining stations are regarded as the nodes with weight of 0.5.

2): Each cable link between stations is considered as one edge of the three-levels power optical fiber network. Ignoring the length, number of cores and voltage level of each cable link, it supports indiscriminate bidirectional transmission. Therefore, the edges are non-directional in the power optical fiber network model.

3): Multiple cable links with the same starting point and end point are combined into one edge in the model, then we eliminate the special network structure such as self-loop in the power optical fiber network model.

According to the actual three-levels fiber network topology, and based on complex network theory, it is abstracted as an undirected weighted network.

Therefore, a complex power optical fiber network model can be constructed as $G(V, E, W, B)$. The node set $V = \{v_i | i = 1, 2, \dots, N\}$, N is the number of nodes of G . The edge set $E = \{e_{ij} | i = 1, \dots, N, j = 1, \dots, N, i \neq j\}$, where $e_{ij} = (v_i, v_j)$ is the edge from node v_i to v_j , $e_{ij} = e_{ji}$. The node weight set $W = \{W_{v_i} | i = 1, 2, \dots, N\}$, where W_{v_i} is the weight of node v_i . Subordination set $b_{v_i} = \{0, 1\}$, if the station is a node on the provincial primary and secondary backbone network, the value is 1, otherwise is 0.

For the stations that the voltage level are above 220 kV, their bearing modes are all dedicated channels, the delay and error rate are far less than the rest of the sites, so the stations that voltage level of 220 kV above are set to the weight of 1, the rest of the stations are set to the weight of 0.5. Then the weight of each edge e_{ij} is computed by following formula:

$$W_{e_{ij}} = \frac{W_{v_i} + W_{v_j}}{2} \quad (1)$$

Jilin Province power optical fiber network model is used to describe the process of the model construction, which are listed as follows.

There are uneven connections between parts in the model and the ring-shaped and star-shaped network are mixed. The model is abstracted from three-levels topology of power optical fiber network, which consists of 256 nodes and 346 edges. Here each node in the network represents an optical fiber station and each edge in the network represents a connection between stations. And the different voltage of the station determines whether it is a dedicated channel, so we take the voltage into account.

The primary and secondary power optical fiber network is defined as the power fiber backbone network, which consists of 50 nodes and 55 edges. There is at least one connection edge in the backbone network that connects two different administrative regions. The power fiber backbone network is composed of most stations above 220 kV, some stations below 220 kV and a few power plants. The model of Jilin power optical fiber network is shown in Figure 1, rough red lines represent backbone links, red circles represent nodes with weights of 1, and white circle represent nodes with weights of 0.5.

3. Local Edge Centrality Measure. For community detection, the centrality measure of an edge in the network depends not only on the single property of the edge such as betweenness, but also on the role and influence of the edge in the whole network and the influence of the adjacent nodes. Therefore, we propose a new local edge centrality to determine which edge should be removed when partitioning community.

3.1. Local edge betweenness. The traditional edge betweenness can detect the ability of edge e_{ij} to control the transmitting information along the shortest path between edges in the network. Among them, the edge betweenness within three-hop has the greatest influence, the three-hop is the shortest paths composed of three edges. Therefore, we give the definition of local edge betweenness LEB_{ij}^s of edge e_{ij} , where LEB_{ij} is the ratio of the number of two-hop passing edge e_{ij} to the total number of all two-hop paths in G , and LEB_{ij}^t is the ratio of three-hop passing edge e_{ij} to the total number of all three-hop paths.

$$LEB_{ij}^s = \sum_{s \neq t \in V} \frac{\sigma_{st(2)}(e_{ij})}{\sigma_{st(2)}} \quad (2)$$

In the Equation (2), $\sigma_{st(2)}$ is the number of all two-hop in G , and $\sigma_{st(2)}$ denotes the number of two-hop passing edge e_{ij} starting from node v_s and ending from node v_t .

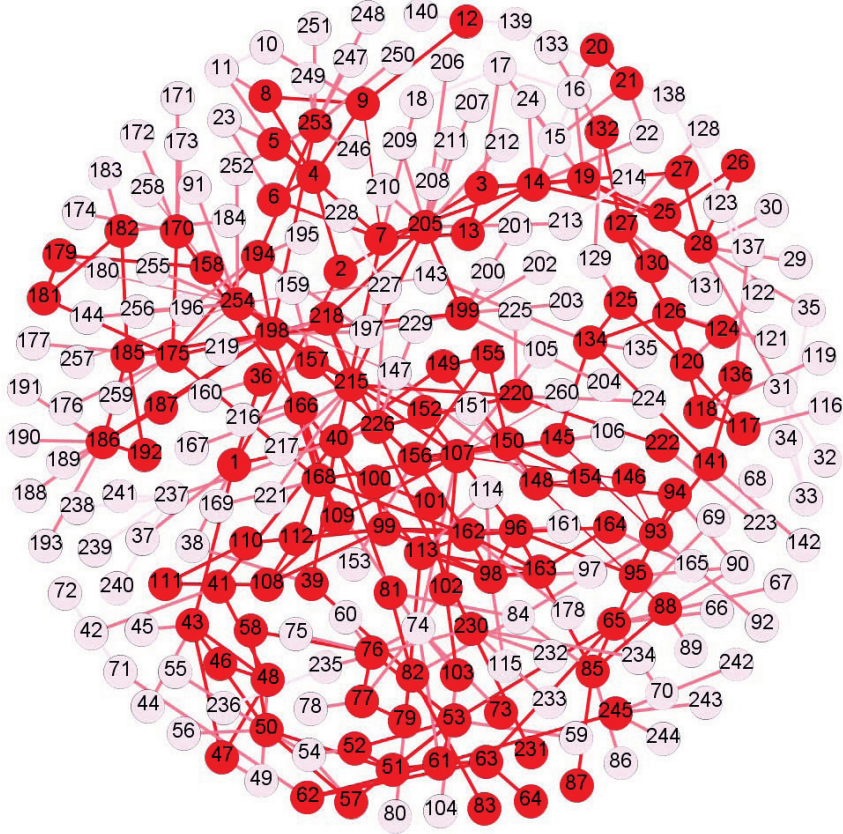


FIGURE 1. Jilin power optical fiber network model

$$LEB_{ij}^t = \sum_{s \neq t \in V} \frac{\sigma_{st(3)}(e_{ij})}{\sigma_{st(3)}} \quad (3)$$

In the Equation (3), $\sigma_{st(3)}$ is the number of all three-hop in G , and $\sigma_{st(3)}$ denotes the number of three-hop passing edge e_{ij} and starting from node v_s and ending from node v_t .

Based on the LEB_{ij}^s and LEB_{ij}^t , local edge betweenness LEB_{ij} is defined as follows:

$$LEB_{ij} = \alpha \times LEB_{ij}^s + \beta \times LEB_{ij}^t \quad (4)$$

In the Equation (4), α and β are the weight coefficients of LEB_{ij}^s and LEB_{ij}^t respectively, and the importance of two hops is greater than that of three hops, that is, $\alpha > \beta$. And in the calculation of the two-hop shortest path, each edge occupies 1/2 of the two-hop shortest path, that is, the importance of this edge in the entire two-hop path is 1/2; in the calculation of the three-hop shortest path In the process, each edge occupies 1/3 of the shortest path of three hops. Therefore, in the above formula, $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ are taken as the weight coefficients of LEB_{ij}^s and LEB_{ij}^t respectively.

3.2. Global adjacent betweenness based on structural hole. With the emergence of triangular structure, the idea of structural hole [16] is used to measure the global influence of edge e_{ij} in complex network, and the degree of closeness between its edge direct connected nodes and adjacent nodes is comprehensively measured as the global adjacent betweenness. The greater the degree of closeness is, the weaker the hub role of

nodes is. The hub roles of nodes v_i and v_j on both sides of edge e_{ij} are weakened, and the influence of e_{ij} will be increased at the same time.

For any node v_i in $G(V, E, W, B)$, $|V| = n$, $|E| = m$, the neighborhood set $N(i) = \{v_j | e_{ij} \in E\}$ refers to the set of adjacent nodes of v_i , and the closed neighborhood set of node v_i is defined as $N[i] = N(i) \cup \{v_i\}$.

First, we propose the general betweenness degree as follows:

$$B_{ij}^g = \left\{ \sum_{k \in V/N[j]} (A_{ik}) + \sum_{k \in N[j]} (A_{ik}) \right\} + \left\{ \sum_{l \in V/N[i]} (A_{jl}) + \sum_{l \in N[j]} (A_{jl}) \right\} \quad (5)$$

where $A = (A_{ij})_{N \times N}$ indicates the adjacent matrix of network. $(A_{ik}) = 1$, if there is one edge connects nodes v_i and v_k ; otherwise, $(A_{ik}) = 0$.

Here we replace A_{ik} and A_{jl} with $A_{ik} \times B'(k)$ and $A_{jl} \times B'(l)$ respectively to define the global adjacent betweenness based on structural hole, hence:

$$B_{ij}^w = \sum_{k \in V/N[j]} (A_{ik} \times B'(k)) + \sum_{k \in N[j]} (A_{ik} \times B'(k)) + \sum_{l \in V/N[i]} (A_{jl} \times B'(l)) + \sum_{l \in N[i]} (A_{jl} \times B'(l)) \quad (6)$$

where $B'(k) = \frac{B(k)}{\max_{s \in v} \{B(s)\}}$ is the normalized betweenness centrality of node v_k . Here, $B(k)$ is the betweenness centralities of nodes v_k , and:

$$B(k) = \sum_{s \neq t \neq k \in V} \frac{\sigma_{st}(k)}{\sigma_{st}} \quad (7)$$

where σ_{st} is the number of shortest paths between nodes v_s and v_t , and $\sigma_{st}(k)$ is the number of shortest paths between nodes v_s and v_t that pass through node v_k .

3.3. Local Edge centrality. In the process of community detection, we define local edge centrality (shorted as *LEC*) to determine which edge to remove when partitioning community, and the LEC_{ij} of edge e_{ij} is computed based on local edge betweenness and global adjacent betweenness of e_{ij}

$$LEC_{ij} = LEB_{ij} + B_{ij}^w \quad (8)$$

4. Community detection algorithm.

4.1. The algorithm design. In this section, on the basic of the proposed local edge centrality, we design the improved local edge centrality algorithm (denoted as ILEC) based on power optical fiber network model with four steps.

In the first step, with the primary and secondary backbone network of each administrative regions, the initial community and the backbone network model are extracted from $G(V, E, W, B)$. The stations and links within each administrative region in the backbone network are retained to form the corresponding initial community. As a result, the backbone network model $G_0(V^{(0)}, E^{(0)}, W)$ is get, the node set $V^{(0)} \in V$ and the edge set $E^{(0)} \in E$. We denote the degree of v_i by d_i , i.e., $d_i = |N(i)|$.

In the second step, according to equation (8), LEC_{ij} of each edge e_{ij} in G can be calculated as the importance weight of e_{ij} . The quality of network partitions is then assessed by a quantitative metric using modularity [17] as shown in the following formula.

$$Q = \frac{1}{2} \sum_i \sum_j (A_{ij} - P_{ij}) \delta(c_i, c_j) \quad (9)$$

Here, P is the adjacent matrix of null model of network, and $P_{ij} = \frac{d_i d_j}{2m}$. c_i is the community which v_i belongs to.

The LEC values and threshold θ are used to distinguish the the importance of all links, determining whether the edge will be keep or not in community detection. All edges in the edge set $E^{(0)}$ are retained. When LEC value of one edge in the edge set $E/E^{(0)}$ is greater than the θ , the edge is removed. Repeating division until the modularity is maximum, the partitioned community model $G_1(V^{(1)}, E^{(1)}, W)$ is found.

In the third step, G_1 is partitioned into community set $C^{(1)}$, including communities with only one isolated node. In the process of dealing with isolated nodes, there are two situations. When all neighbors of an isolated node are also isolated nodes, we merge the isolated node with the neighbor with the largest degree among all its neighbors. For example, for isolated node v_i , $\arg \max \{d_i\}$ means the node v_i with the largest degree among the neighbor nodes of node v_i . If all nodes in $N(i)$ are also isolated nodes, then we merge v_i and $\arg \max \{d_i\}$ as a new community. When there are at least two communities that can connect to isolated node v_i , we add v_i to the community with largest density ρ . And the density of community c_j is defined as follows.

$$\rho_j = \frac{2m_j}{|c_j|(|c_j| - 1)} \quad (10)$$

where m_j is the number of edges in c_j .

In the fourth step, with p communities with nodes and edges in G_0 and the other q communities are obtained, the network is expressed as G_2 , and the communities in G_2 are $C^{(2)} = \{c_1, \dots, c_p, c_{p+1}, \dots, c_{p+q}\}$. The first p communities are communities including nodes and edges in backbone network model G_0 , and $C^{(0)} \in \{c_1, \dots, c_p\}$. According to the edge density, the communities $\{c_{p+1}, \dots, c_{p+q}\}$ in $C^{(2)}$ and communities $\{c_1, \dots, c_p\}$ are combined, and the p communities cannot be combined with each other until all nodes in G_2 are connected to the backbone network model G_0 .

Specifically, we introduce a relationship matrix $R = (R_{ab})_{q \times p}$, which is defined as follows:

$$R = \begin{bmatrix} R_{1,p+1} & \cdots & R_{p,p+1} \\ \vdots & \ddots & \vdots \\ R_{1,p+q} & \cdots & R_{p,p+q} \end{bmatrix} \quad (11)$$

$$R_{ab} = \frac{E(c_a, c_b)}{|c_a|} \quad (12)$$

In the Equation (12), R_{ab} is called edge density. $|C_a|$ is the number of edges inside the community, $E(c_a, c_b)$ is the number of edges connecting communities in the network G of communities c_a and c_b .

In order to get the final community result, according to the Equation (11), we find the maximum value of R_{ab} in R , merge the corresponding communities C_a and C_b . Updating the R result from the mergence until the matrix becomes a zero matrix of a q row p column, the community set $C^{(3)} = \{c_1, \dots, c_p\}$ is obtained and $C^{(0)} \in C^{(3)}$.

Keeping the nodes community relationship in $C^{(3)}$ unchanged, all nodes in $C^{(3)}$ are connected to the original edges of network G . The final community detection set $C^{(4)} = \{c_1, \dots, c_p\}$ is obtained and $C^{(3)} \in C^{(4)}$.

4.2. Community detection algorithm. The algorithm ILEC for complex power optical fiber network model is described as follows, the algorithm is a four-step process. Step 1 and step 2 are the process of edge-deleting. Then reconnects some communities and employs the edge density to get the final partition in the step 3 and step 4.

Algorithm ILEC

Input: the network model $G(V, E, W, B)$

Output: the final community set $C^{(4)} = \{c_1, \dots, c_p\}$

Step 1:

- (1) extract backbone network $G_0(V^{(0)}, E^{(0)}, W) \leftarrow G(V, E, W, B)$
- (2) get the backbone network community of the administrative district $C^{(0)}$
- (3) calculate local edge centrality of each edge e_{ij} in G
- (4) for each edge $e_{ij} \in E$
- (5) calculate LEC_{ij} according to Equation (8)
- (6) end for

Step 2:

- (7) sort LEC_{ij} in a non-increasing order
- (8) for each edge $e_{ij} \in E/E^{(0)}$ do
- (9) if $LEC_{ij} > \theta$
- (10) remove the edge e_{ij} from G
- (11) else
- (12) keep the edge e_{ij} in G
- (13) end if
- (14) end for
- (15) get partitioned community $G_1(V^{(1)}, E^{(1)}, W)$ when modularity is maximum
- (16) get community set is $C^{(1)} = \{c_1, c_2, \dots, c_u\}$

Step 3:

- (17) for $i = 1$ to u do //handle the isolated nodes
 - (18) if $c_i == 1$ then
 - (19) $C_{iso} = C_{iso} \cup \{c_i\}, C^{(1)} = C^{(1)} / \{c_i\}$
 - (20) end if
 - (21) end for
 - (22) if $C_{iso} \neq \theta$ then
 - (23) for each node v_i in C_{iso} do
 - (24) if all of v_i 's neighbors are in C_{iso} then
 - (25) $C^{(1)} = C^{(1)} \cup \{v_i, v_j\}$, $\arg \max \{d_i\}$ where $v_i \in N(i)$
 - (26) $C_{iso} = C_{iso} / \{\{v_i\}, \{v_j\}\}$
 - (27) else if there exist node v_i 's degree is 1 in G then
 - (28) add v_i to the original connected node in G
 - (29) else
 - (30) add v_i to the community that maximizes density ρ in G
 - (31) end if
 - (32) end for
 - (33) end if
 - (34) get the network G_2 . The community $C^{(2)} = \{c_1, \dots, c_p, c_{p+1}, \dots, c_{p+q}\}, C^{(0)} \in \{c_1, \dots, c_p\}$
- Step 4:
- (35) establish a matrix $R = (R_{ab})_{q \times p}$

- (36) while $R \neq R_{zero}$ do
(37) calculate R_{ab} according to Equation (12)
(38) find the maximum R_{ab} in R
(39) merge c_a and c_b as a new community, update $C^{(2)}$
(40) end while
(41) return the resulting communities set $C^{(3)} = \{c_1, \dots, c_p\}$, $C^{(0)} \in C^{(3)}$
(42) keeping the nodes in $C^{(3)}$ unchanged, $C^{(3)}$ is connected to the original edge of network G to obtain the final community set $C^{(4)} = \{c_1, \dots, c_p\}$, $C^{(3)} \in C^{(4)}$

The flowchart of the algorithm ILEC is described as Figure 2.

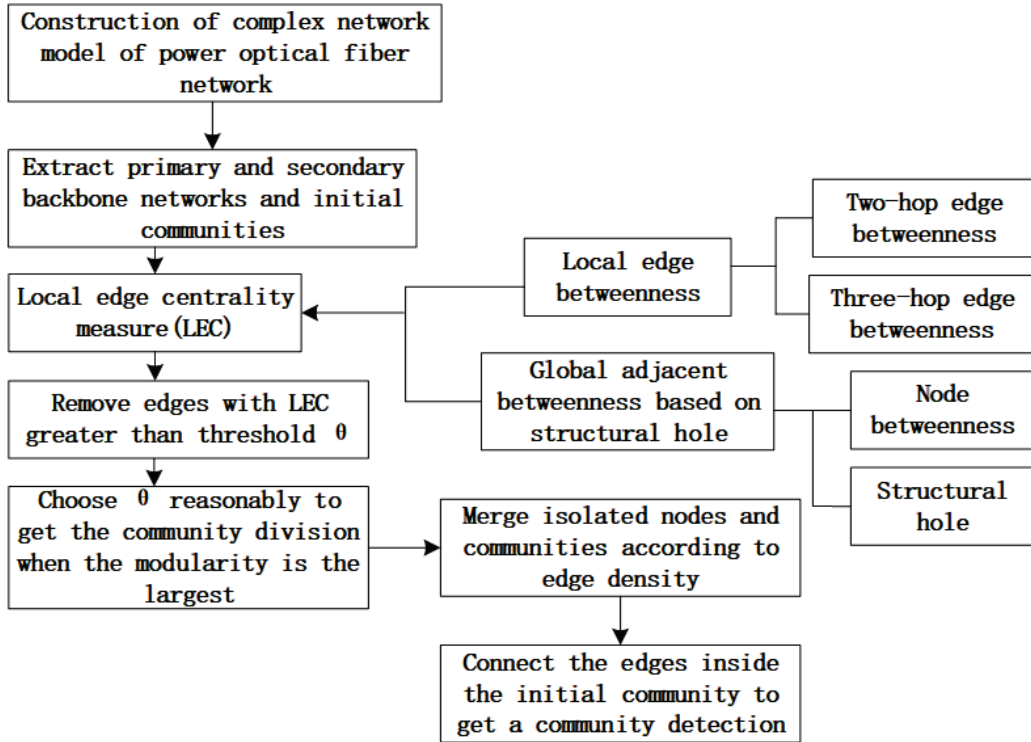


FIGURE 2. The flowchart of the algorithm ILEC

5. Experiments. In this section, by using Jilin Province complex power optical fiber network model, the experiments of community detection of the proposed algorithm ILEC are discussed. Then we compare the performance of the algorithm ILEC with GN [18], CO-LEC [14], EDCD [19] and the community detection method based on administrative regions shorted as AR.

5.1. Experiment on Jilin Province power optical fiber network model. The power optical fiber network model is constructed for Jilin Province power optical fiber network. On the basis of that, the process of community detection of the proposed algorithm ILEC is discussed.

First of all, we extract nodes and edges from the backbone network within each administrative region, and obtain the corresponding basic framework of each community.

In the second step, we calculate LEC of each edge in the network according to Equation (8). The top 30% part of result is shown as Table 1:

TABLE 1. The part LEC of each edge in the network

No.	LEC	No.	LEC	No.	LEC	No.	LEC	No.	LEC
1	13.70	11	5.86	21	0.54	31	3.87	41	3.65
2	7.35	12	6.14	22	5.79	32	11.21	42	4.84
3	7.35	13	7.7	23	15.49	33	9.09	43	4.87
4	9.20	14	4.29	24	8.79	34	7.54	44	5.57
5	15.82	15	3.08	25	19.53	35	7.88	45	5.78
6	16.60	16	5.69	26	1.97	36	8.37	46	15.23
7	12.04	17	11.82	27	0.96	37	4.01	47	0.53
8	10.36	18	5.16	28	2.24	38	3.55	48	0.71
9	4.92	19	4.69	29	2.89	39	3.99	49	4.24
10	7.28	20	6.34	30	1.23	40	5.16	50	3.73

Then importance weight of all the edges in the network are measured by the corresponding *LEC* value. For example, edge e_{12} is the same as the neighbor node of edge e_{13} , but since edge e_{12} is connected to a structural hole in the second-step and third-step neighbor nodes, the weight of edge e_{12} is weakened, edge e_{13} has a larger connection function and the weight of edge e_{12} is less than e_{13} . Since there are sparse edges between communities in each administrative region, this part of the edges plays a greater role in connection. The larger value of *LEC* indicates that this edge has a greater weight in the network. Therefore, we remove these edges to imitate the division between administrative regions. We can see that the larger edges of *LEC* are e_6, e_{25} and so on. These edges are the backbone of the three-levels power optical fiber network model and are the links between administrative regions.

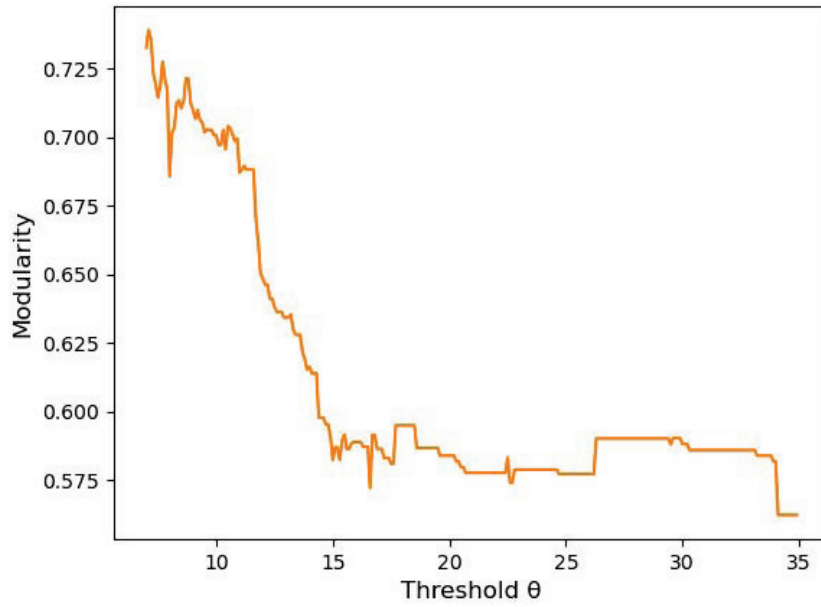
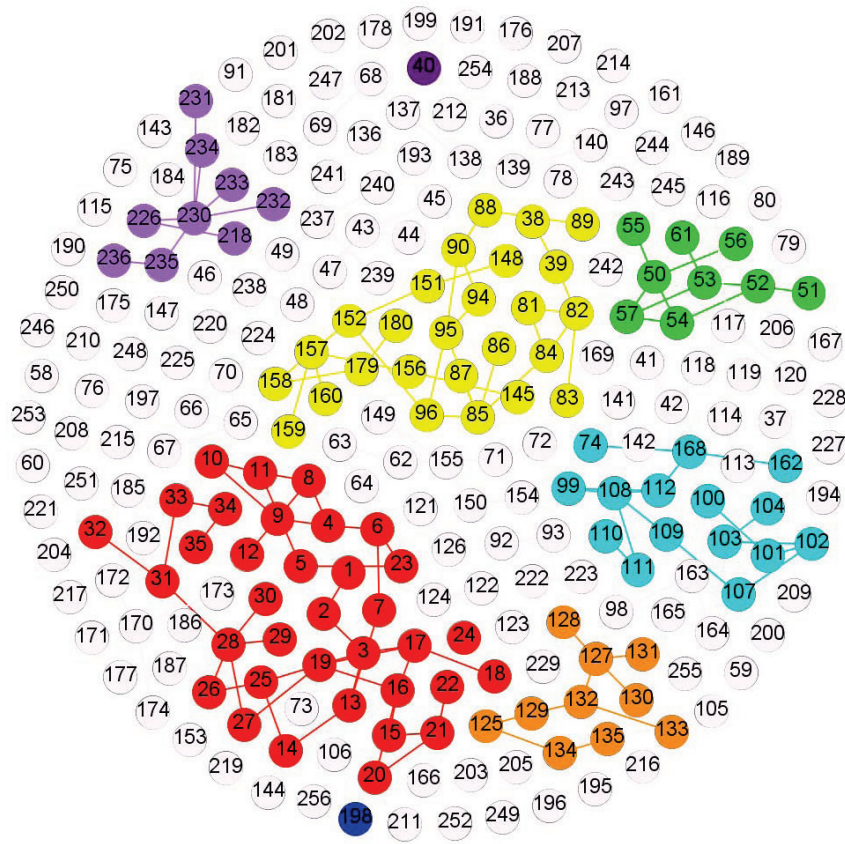
The threshold θ is used to distinguish the links, determining whether the edge will be keep or not. Set different θ from 7 to 35 (step length is 0.1) to divide G as shown in Figure 3. For edge larger than θ in G , remove the edge, calculate the modularity Q corresponding to each θ and select the θ to the network with the largest modularity as the threshold of G . In this network, we choose 7.6 as the threshold θ .

In the process of partition, once *LEC* value of the edge is greater than θ , the edge should be removed and many small communities with stable internal structure were obtained. except the backbone network, It is found that Baishan and Tonghua are still closely connected and cannot be splited. Therefore, nodes in Baishan and Tonghua are combined into one community. The number of communities in the set $C^{(1)}$ is determined to be 8, and the first community partition $G_1(V^{(1)}, E^{(1)}, W)$ is obtained as shown in Figure 4.

In the third step, the network is partitioned into community set $C^{(1)}$, including communities with only one isolated node. After merging the isolated nodes, the network is represented as G_2 . The communities in G_2 are $C^{(2)} = \{c_1, \dots, c_8, c_9, \dots, c_{26}\}$, where the first 8 communities are communities including nodes and edges in backbone network model G_0 , and $C^{(0)} = \{c_1, \dots, c_8\}$. And $C^{(2)}$ is shown in Figure 5.

In the fourth step, according to Equation (11) and Equation (12), add the communities $\{c_9, \dots, c_{26}\}$ in $C^{(2)}$ into $\{c_1, \dots, c_8\}$ until all nodes in G_2 are connected to the backbone network G_0 , but the communities in $\{c_1, \dots, c_8\}$ cannot be combined with each other. The community set $C^{(3)} = \{c_1, \dots, c_8\}$ is obtained and $C^{(0)} \in C^{(3)}$.

Keeping the nodes community relationship in $C^{(3)}$ unchanged, all nodes in $C^{(3)}$ are connected to the original edges of network G . The final community set $C^{(4)} = \{c_1, \dots, c_8\}$

FIGURE 3. Relationship between threshold θ and modularityFIGURE 4. The initial community partition G_1

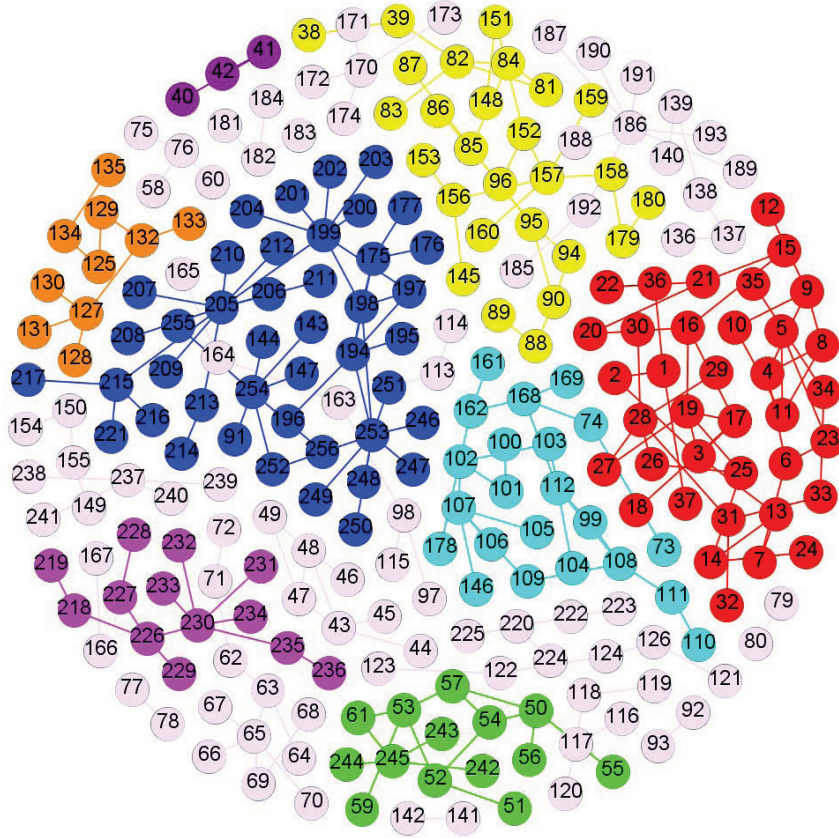


FIGURE 5. The community partition result after collection of isolated nodes

is obtained and $C^{(3)} \in C^{(4)}$. With all this done, we obtain the community detection as shown in Figure 6.

5.2. Evaluation Metric. A. Community profile

Some network models have the same coritivity but different connectivity, so John Matta et al. proposed tenacity [20]. Therefore, we focus on community number, max community size and min community size, and propose a measure of edge-tenacity which can roughly reflect the distribution of the communities.

$$T'(G) = \min\left\{\frac{|S| + \tau(G - s)}{\omega(G - s)} : S \in E(G)\right\} \quad (13)$$

Here, $\omega(G - s)$ denotes the number of connected components and $\tau(G - s)$ is replaced to $\frac{\max(c_i) + \min(c_j)}{2}$. When G is not a complete graph and $(G - s)$ is not connected, then S is called a cut set of G . In this, $|S|$ represents the number of removed edges.

B. Efficiency

Efficiency [19] is used to measure the speed of information transmission between nodes. The definition of network average global efficiency is as follow.

$$E = \frac{1}{N(N-1)} \sum_{v_i \neq v_j \in V} \frac{1}{d_{ij}} \quad (14)$$

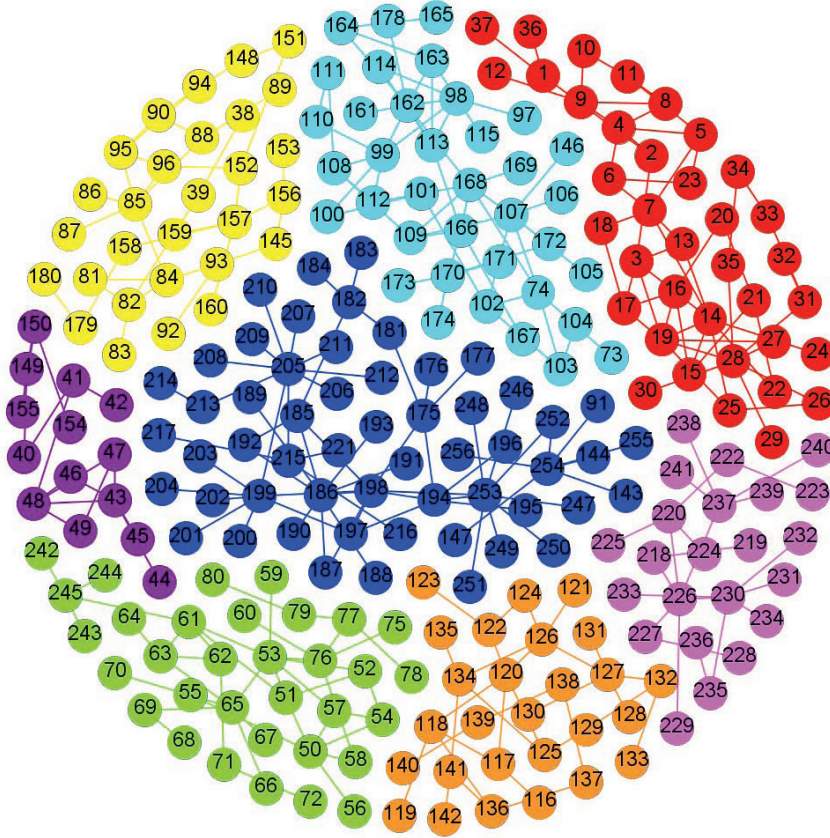


FIGURE 6. The final community detection with ILEC algorithm

The distance d_{ij} between node v_i and v_j in the network is defined as the number of edges on the shortest path connecting these two nodes, and its reciprocal $\frac{1}{d_{ij}}$ is called the efficiency between node v_i and v_j . When there is no path connectivity between nodes, $d_{ij} = \infty$, $\frac{1}{d_{ij}} = 0$. The average global efficiency of the network is the average efficiency of all pairs of nodes.

C. Modularity

Modularity [17] is a commonly used method to measure the strength of community structure as shown in Equation (9). The larger the modularity is, the stronger the strength of the community structure is.

5.3. Experiments and Analysis under different algorithms. First we analyze the characteristics of the power optical fiber network model of Jilin Province. According to the method of administrative detection, there are internal associations that are not connected, therefore the number of administrative community is 13 shown in Table 2. For example, the interior of Baicheng City is composed of three administrative communities. The number of nodes of these three communities are 49, 3, 4 and the number of edges is 48, 1, 3, respectively. And the layout of the Baicheng city is star-shaped, almost no ring. On the contrary, the ring-shaped phenomenon is serious in Tonghua City and Baishan City, and the connection between the two cities is very close and cannot be separated. If forced division is made, it is a kind of harm to community structure. In our method, it is found that the backbone networks of the two cities are closely linked, so the two cities are analyzed as one community.

From this point of view, although the power optical fiber network model in Jilin Province consistent with the small world and scale-free, it's topological structure has certain unique characteristic. It presents the coexistence of star-shaped and ring-shaped distribution, and the edge connection of each part is uneven. The whole network topology is complex.

Then we compare the community profile among ILEC, GN, CO-LEC, EDCD and AR algorithms shown in the Table 2.

TABLE 2. Community profile of ILEC, GN, CO-LEC, EDCD and AR algorithms

Profile	GN	CD-LEC	EDCD	ILEC	AR	
Community number	8	22	1	8	13	
Max community	Node size	45	45	256	56	49
	Edge size	44	44	312	63	48
Min community	Node size	9	2	–	14	11
	Edge size	8	1	–	17	12
Edge number	322	271	312	322	310	
Tenacity	6.25	4.43	–	8.00	5.08	

It can be seen that ILEC is more realistic and GN (as shown in Figure 7) can control the number of community. Specifically, CO-LEC tends to give the community structure with a large community number and a relatively small community size in most cases. Otherwise, EDCD prefers a large community size but a small community number. Algorithms ILEC and GN give a compromise between these algorithms and more suitable with the actual situation of the 9 administrative regions.

In addition, the algorithm ILEC also obtains the best tenacity. Compared with the algorithm AR based on administrative detection, the tenacity is improved by 60%, and the connectivity within each community is realized. This is because ILEC uses the information of the backbone network in the division process, the approximate distribution of the backbone network and the small change of the other nodes to obtain the best detection effectiveness. The GN, CO-LEC, EDCD and AR algorithms add edge or remove edge based on the weight of the edge, resulting in a large difference with the actual situation. The actual management allocation can't be achieved in accordance with other methods. If taking the community profile into consideration, the proposed method ILEC achieves the balance between the size and the number of community compared with other algorithms.

Finally we compare the tenacity, effectiveness and modularity between different algorithms shown in Figure 8, where the y axis is the result of normalizing these indexes. It can be seen that EDCD has the best effectiveness. However, according to EDCD, there is only one community, and the division is meaningless as shown in Figure 9. Compared with the other methods, our method ILEC ranks first in overall efficiency and achieves better results.

For the modularity, it can be seen that CO-LEC has the best performance as shown in Figure 10. There is no significant difference in GN, ILEC and administrative detection that all obtain a high modularity. These algorithms are basically consistent with community partitioning except EDCD. The EDCD algorithm performs worst.

According to the above experimental results, the algorithm ILEC effectively solves the internal connectivity problem of the Jilin Province power optical fiber network, and improves the toughness by 60%.

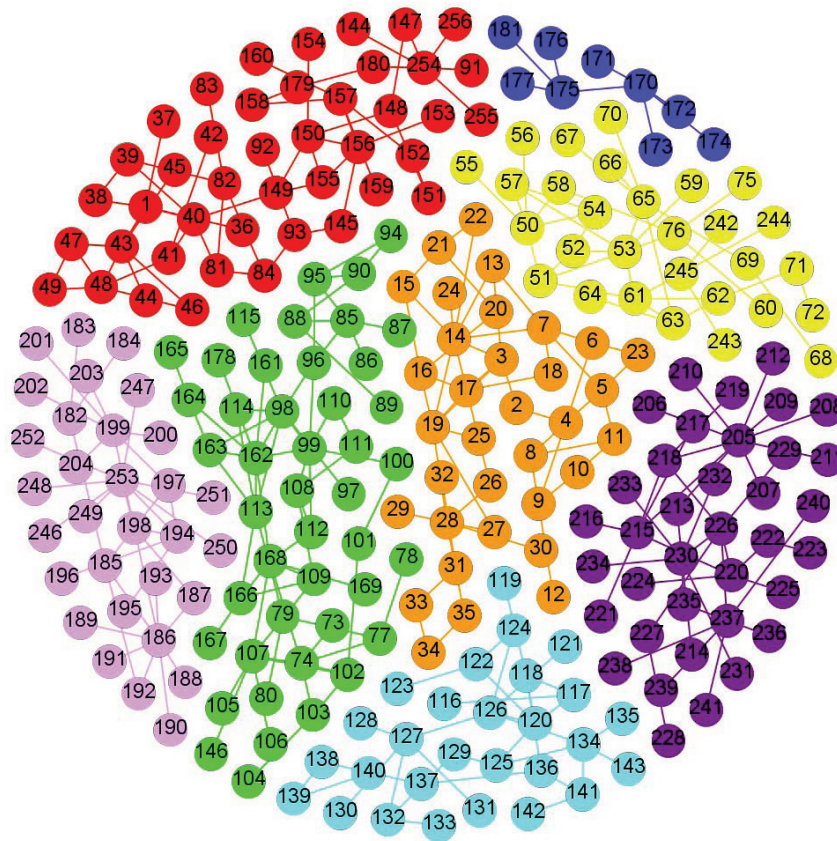


FIGURE 7. The community detection result with GN algorithm

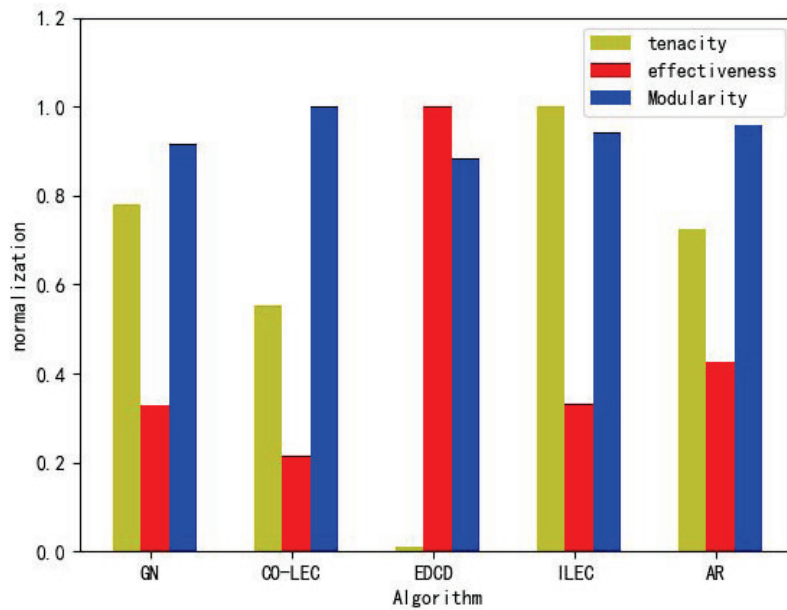


FIGURE 8. Tenacity, effectiveness and modularity on the network

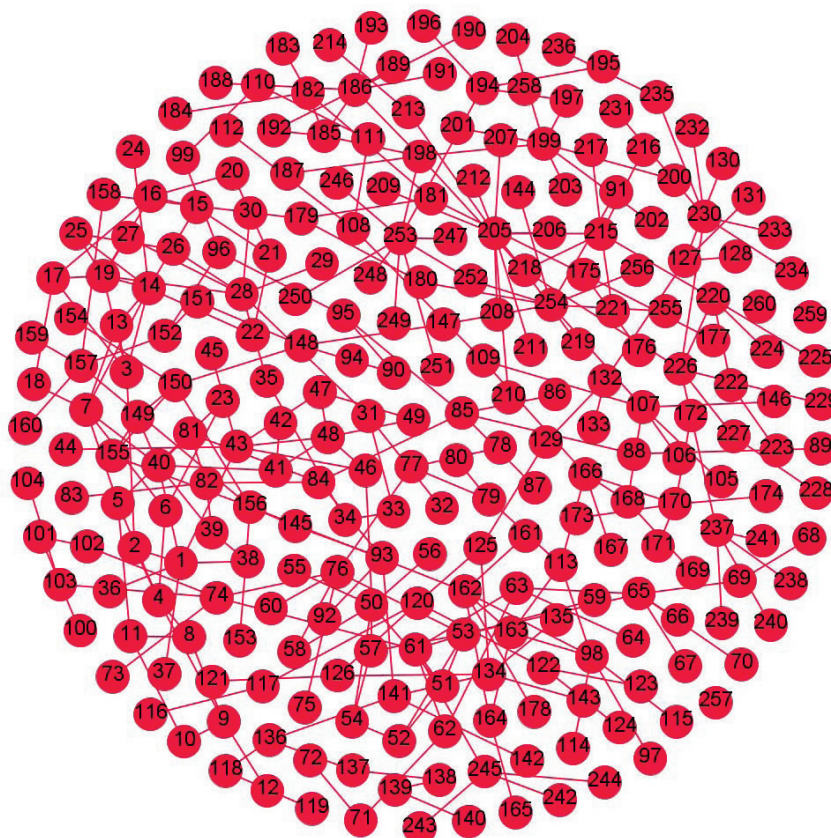


FIGURE 9. The community detection result with EDCD algorithm

6. Couclusion and future work. Community detection in complex power optical fiber network is a valuable research topic and has practical significance. Therefore, a large number of community detection methods have been investigated. Base on the divisive method, in this paper, we propose a new method of community detection based on improved local edge betweenness ILEC. Empirical analysis shows that the proposed method successfully solves the problem of intra-community disconnection and achieves good tenacity. The method is based on the backbone network, by changing the nodes ownership of three-levels networks, the internal connection of each community is made, which is beneficial of the management of provincial administrative regions and avoids the defects of relying solely on administrative divisions. According to the experimental analysis, all parts of the divided region are connected, and the overall tenacity increases by 60%.

Till now our method is only suitable for dealing with the community detection of power optical fiber network problems. However, in the actual power optical fiber network management, it is necessary to add some edges or remove some edges to improve the overall efficiency of the network and enhance the anti-risk ability of the network. In the future, we intend to extend our algorithm to the edge-adding and edge-removing process of power optical fiber network in order to further strengthen the management efficiency and anti-risk ability of power optical fiber network.

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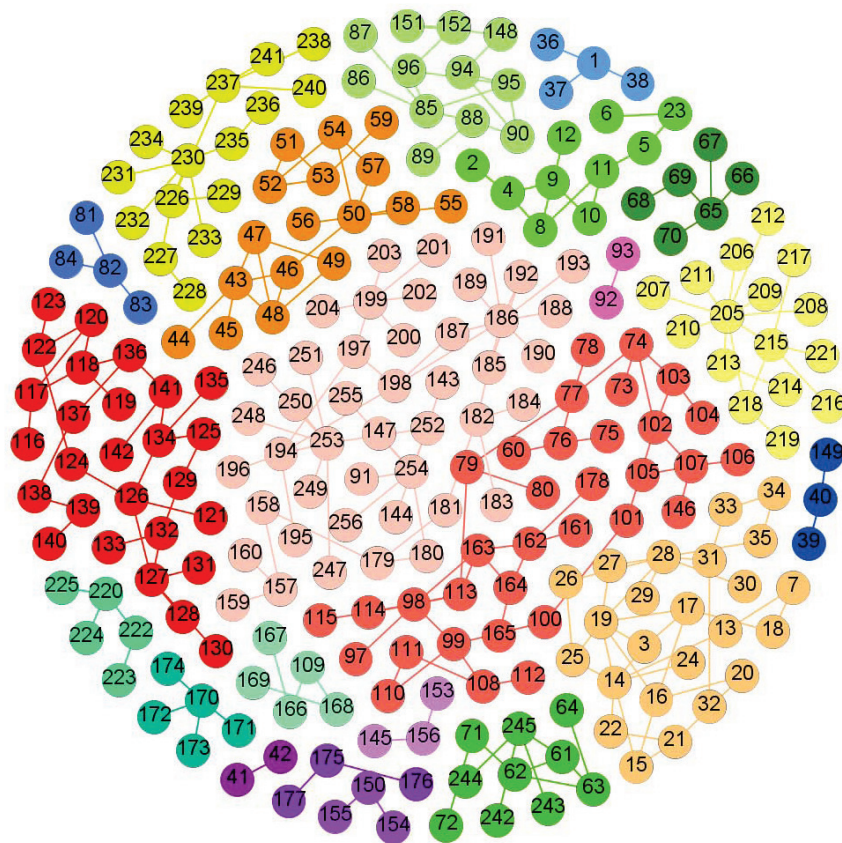


FIGURE 10. The community detection result with CO-LEC algorithm

REFERENCES

- [1] F. Liu, Z. Wang, Y. Deng, GMM: A generalized mechanics model for identifying the importance of nodes in complex networks, *Knowledge-Based Systems*, vol. 193, pp. 1-17, 2020.
- [2] Z. H. Wang, C. Wang, X. Yang, Dynamic synchronization of extreme heat in complex climate networks in the contiguous United States, *Urban Climate*, vol. 38, no. 1, pp. 1-13, 2021.
- [3] G. A. Pagani, M. Aiello, The Power Grid as a Complex Network: a Survey, *Physica A Statistical Mechanics & Its Applications*, vol. 392, no. 11, pp. 2688-2700, 2013.
- [4] P. D. A, M. P. B, A Fixed-Mass multifractal approach for unweighted complex networks, *Physica A: Statistical Mechanics and its Applications*, vol. 541, no. 8, pp. 1-15, 2019.
- [5] C. Gan, X. Yang, W. Liu, Propagation of computer virus both across the Internet and external computers: A complex-network approach, *Communications in Nonlinear Science & Numerical Simulation*, vol. 19, no. 8, pp. 2785-2792, 2014.
- [6] S. Ahajjam, M. E. Haddad, H. Badir, A new scalable leader-community detection approach for community detection in social networks, *Social Networks*, vol. 54, pp. 41-49, 2018.
- [7] B. Zhan, C. Zhang, Z. Xia, J. Wang, A fast parallel modularity optimization algorithm (FPMQA) for community detection in online social network, *Knowledge-Based Systems*, vol. 50, pp. 246-259, 2013.
- [8] M. Newman, Fast algorithm for detecting community structure in networks. *Phys. Rev. E Stat. Nonlin. Soft. Matter. Phys.* 69(6 Pt 2), 066133, *Physical Review E*, vol. 69, no. 6, pp. 1-16, 2004.
- [9] A. Clauset, Finding local community structure in networks, *Physical Review E Statal Nonlinear & Soft Matter Physics*, vol. 72, no. 2, pp. 1-7, 2005.
- [10] H. Kong, Q. Kang, W. Li, A hybrid iterated carousel greedy algorithm for community detection in complex networks, *Physica A: Statistical Mechanics and its Applications*, vol. 536, no. 15, pp. 2124-2136, 2019.

- [11] T. Wang, L. Yin, X. Wang, A community detection method based on local similarity and degree clustering information, *Physica A. Statistical mechanics and its applications*, vol. 490, no. 15, pp. 1344-1354, 2018.
- [12] F. Radicchi, C. Castellano, F. Cecconi, V. Loreto, Defining and identifying communities in networks, *Proceedings of the National Academy of Sciences of the United States of America*, vol. 101, no. 9, pp. 2658-2663, 2004.
- [13] U. Raghavan, R. Albert, S. Kumara, Near Linear Time Algorithm to Detect Community Structures in Large-Scale Networks, *Physical Review E*, vol. 76, no. 3, pp. 6106 -6117, 2007.
- [14] X. Q. Li, S. Zhou, J. F. Liu, Communities detection in social network based on local edge centrality, *Physica A: Statistical Mechanics and its Applications*, vol. 531, pp.1552-1533, 2019.
- [15] L. Han, X. D. Zhao, Z. L. Chen, Optimal allocation of defensive resources to defend urban power networks against different types of attackers, *International Journal of Critical Infrastructure Protection*, vol. 35, pp. 1-13, 2021.
- [16] J. X. Luo, Y. J. Du, Detecting Community Structure and Structural Hole Spanner simultaneously by using Graph Convolutional Network based Auto-Encoder, *Neurocomputing*, vol. 410, no. 14, pp. 138-150, 2020.
- [17] D. Y. Zhao, J. Li, Z. Y. Jiang, Effects of link perturbation on network modularity for community detections in complex network systems, *Modern Physics Letters B*, vol. 35, no. 13, pp. 1-10, 2021.
- [18] M. Girvan, M. E. J. Newman, Community Structure in Social and Biological Networks, *Proceedings of the National Academy of Sciences of the United States of America*, vol. 99, no. 12, pp. 7821-7826, 2002.
- [19] X. Chen, J. Li, Community detection in complex networks using edge-deleting with restrictions, *Physica A: Statistical Mechanics and its Applications*, vol. 519, pp. 181-194, 2019.
- [20] J. Matta, G. Ercal, J. Borwey, The vertex attack tolerance of complex networks, *Rairo Operations Research*, vol. 51, no. 4, pp. 1055-1076. 2017.