Improvement of Swarm Intelligence Algorithm and Its Application in Logistics Network Routing

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Abstract. The Vehicle Routing Problem (VRP) is a key aspect of logistics network routing, and an excellent routing optimization strategy can effectively improve the service experience of users and reduce transportation costs. With the explosive growth of e-commerce, the logistics network routing system is faced with a huge number of demand points and customer-specified receiving time slots, which poses a huge challenge for routing optimization of large-scale delivery. To address the problems of high total cost and low effective vehicle utilisation in the logistics network routing system, this work introduces a new swarm intelligence method, the pigeon-inspired optimization (PIO), and improves it. Two improvement strategies are proposed to address the strengths and weaknesses of the PIO algorithm. Firstly, by combining the high swarm dispersion of the quantum evolutionary algorithm and the fast convergence of the PIO, the PIO method is upgraded by mixing the algorithms to achieve the effect of complementing each other’s strengths and improving the global exploration ability of the PIO algorithm; secondly, a Gaussian variation operator is added to the PIO algorithm to enhance its local exploitation capability and prevent premature in order to retain the variety in future iterations. Each individual contains information on both client points and routes. The effectiveness of the improved pigeon flock intelligence optimisation algorithm is verified through test function simulations. The effectiveness and rationality of the improved PIO algorithm is verified in a case study based on the Solmon arithmetic example, which has some engineering application value.

Keywords: Routing optimization; Swarm intelligence algorithms; Pigeon-inspired optimization; Time windows
1. **Introduction.** The Vehicle Routing Problem (VRP) \[1,2,3\] provides important fundamental theoretical support for practical applications in logistics and transportation, public transportation, etc. The basic problem of the VRP is how to plan the routing of vehicles given a set of customer demands and available vehicles. The basic problem is how to plan the routing of vehicles given a set of customer demands and available vehicles so that each customer’s demand is met and the total cost of the routing (usually the distance or travel time of the vehicle) is minimised.

After many years of development, VRP has become a very active research direction in the field of combinatorial optimisation \[4,5\]. Nowadays, VRP research involves a variety of different problem types and algorithmic approaches. For example, for basic VRP problems, there are already very mature algorithms such as heuristic search algorithms based on forbidden search, genetic algorithms, simulated annealing, etc., as well as exact algorithms such as branch delimitations and branching limits \[6,7\]. Besides, there are many extended VRP problems, such as VRP with time windows, multi-vehicle VRP, VRP with capacity constraints, etc., which have also attracted a large number of researchers to conduct in-depth research. In addition, VRP-related research involves some cutting-edge technologies such as artificial intelligence, machine learning, and metaheuristics \[8,9\]. Artificial intelligence and machine learning methods, such as deep reinforcement learning, have gained wide application in VRP problems, and can improve solution efficiency and solution quality by learning the process and patterns of optimisation algorithms. Metaheuristics methods, on the other hand, are a class of general ideas based on optimisation algorithms, such as multi-objective optimisation and fitness function design, which can achieve good performance on various VRP problems \[10,11\]. In conclusion, VRP is a very important problem in logistics network routing research, and its research status is very rich and diverse, and more researchers will definitely devote themselves to the in-depth research and innovation of VRP problems in the future.

The ultimate goal of a logistics network routing system is to minimise the loss of goods during vehicle transport and to minimise transport costs, thereby maximising the benefits of distribution \[12\]. How to minimise the total cost of logistics and maximise the benefits depends on ensuring that the right quality and right quantity of produce is delivered to the customer at the right price, at the right time and using the right route (Right quality and Right quantity of produce is delivered to the customer at the Right price, at the Right time and using the Right route). This is the so-called 5R principle \[13,14\]. In this paper, we argue that if a logistics network routing system is able to execute the transportation of goods according to the 5R principle, then in principle the total cost of logistics and distribution can be minimised and the benefits maximised.

Group intelligence algorithms are a class of optimisation algorithms inspired by the behaviour of biological groups, which have good performance in solving optimisation problems by modelling the collaboration and competition between individuals in a group. Currently, swarm intelligence algorithms have been used in a wide range of fields, such as engineering, medicine and finance \[15,16\]. The Pigeon-Inspired Optimisation (PIO) algorithm is an emerging swarm intelligence algorithm \[17\], which is based on the understanding and application of pigeon and human intelligence. Pigeons exhibit strong perception, memory and the ability to cooperate effectively when searching for food and planning their course, and these characteristics have been applied to the PIO algorithm, which not only retains the advantages of traditional genetic algorithms and particle swarm optimisation algorithms, but also improves on the disadvantages of these algorithms. In fact, the PIO algorithm has been widely used in many complex optimisation problems and has achieved good results.
The aim of this work is to optimise the VRP problem in logistics network routing using the PIO algorithm, so as to satisfy the 5R principle and minimise the total cost of logistics distribution. Therefore, a new swarm intelligence method, the pigeon-inspired optimization (PIO), is introduced and improved to address the problems of high total cost and low effective vehicle utilisation. Each individual contains both customer point and routing information. The effectiveness of the improved pigeon-inspired optimisation algorithm was verified through test function simulations. The effectiveness and rationality of the improved PIO is verified in a case study based on the Solmon arithmetic example, which has some engineering application value.

1.1. Related Work. The VRP optimisation problem is one of the very classical combinatorial optimisation problems. It is a mathematical model to determine the least costly vehicle routing when the geographical coordinates and the customer demand at each point are known. Typically, each vehicle is required to visit each customer only once, and the capacity limit is the same for each vehicle. For VRP optimisation, in addition to the constraints inherent in the problem, many types of goods are also perishable and difficult to preserve, such as agricultural products. For these problems, a large range of logistics network routing optimisation also requires a combination of time, space and other influencing factors.

Since the VRP is an NP-hard (non-deterministic polynomial) problem, efficient and accurate algorithms for solving it are almost non-existent, and the most reliable way of solving it is now heuristic algorithms. Baker and Ayechew [18] proposed an improved genetic algorithm and solved the VRP problem, which also yielded more realistic results. Yusuf et al. [19] proposed an approach to improve the traditional genetic algorithm and applied it to optimize the vehicle routing problem so that the convergence of the optimal solution is accelerated but does not fall into local convergence, ultimately making the optimal solution better than the result of the ordinary algorithm. Fu et al. [20] proposed a hybrid simulated annealing forbidden algorithm and investigated different search strategies that resulted in a significant reduction of more than 50% in computational time for solving the vehicle routing problem. Theurich et al. [21] argue that the vehicle routing problem with capacity constraints remains central to logistics planning, and they propose two new integer planning models using the tree structure of the problem. Nguyen et al. [22] et al. proposed two new vehicle routing problems with distance constraints and capacity constraints and investigated the potential benefits of flexible allocation of start and end depots and found that flexible allocation can reduce the cost by 49.1%. Zhang et al. [23] proposed a green vehicle routing problem (G-VRP) and formulated a heuristic algorithm, and the experimental data obtained illustrated the good results of this algorithm. Olgun et al. [24] et al. proposed a multi-trip green vehicle routing problem and proposed a heuristic algorithm by combining a genetic algorithm with a local search process.

Although heuristic algorithms and swarm intelligence algorithms have different ideological foundations and implementations, the two share similar ideas in solving optimisation problems. Also, the two types of algorithms can be used in combination to obtain better performance in practical problems. Among the many optimisation algorithms, the Pigeon Inspired Optimisation (PIO) algorithm, a new type of heuristic swarm intelligence optimisation algorithm [25], has been shown to have good optimisation capabilities. inspired by the principle of autonomous guidance of pigeons, the map compass operator and landmark operator are used by the PIO to conduct optimization searches and address optimization issues. Similar to other swarm intelligence algorithms, the PIO algorithm uses the sharing of information in the swarm and the competition between individuals to find the optimum, and has a fast convergence capability.
two-stage optimisation approach, using the map compass operator and the landmark operator respectively, which makes the algorithm have a strong local exploration capability and higher optimisation accuracy, and can solve complex multi-dimensional optimisation problems.

In recent years, pigeon flock intelligence algorithms have been widely used in engineering, validating the algorithm’s superior optimisation capabilities. Zhao and Zhou [26] applied the PIO algorithm to glide track trace optimisation. In addition, PIO algorithms have also been applied to control system design, for example, Dou and Duan [27] used the PIO algorithm for model prediction controller parameter tuning. Although PIO algorithms can have fast convergence, they tend to be less accurate for complex optimisation problems and tend to fall into local optima overall. In global optimisation problems, especially for multi-peak problems, the PIO algorithm tends to converge prematurely and thus fall into the local optimum trap when it fails to generate new children. Low variability and low variety are the major reasons of prematurity.

1.2. Motivation and contribution. To address the problems of high total cost and low effective vehicle utilisation in logistics network routing systems, this work introduces an improved PIO algorithm to achieve VRP optimisation in logistics network routing systems.

The main innovations and contributions of this study are shown below:

1) A VRP-based routing model for logistics networks is established, providing a specific optimisation object for subsequent optimisation applications of the PIO algorithm. The proposed model includes a description of the problem, the basic constraints of the model, the definition of the symbols of the variables involved in the model, etc. This work also adds the element of a time window to the basic constraints, making this model more relevant to reality and thus making the resulting conclusions more realistic.

2) In order to improve the variability and variety of the standard PIO, two improvement strategies are proposed in this work. First, to combine the high swarm dispersion of the quantum evolutionary algorithm and the fast convergence of the pigeon flock optimization algorithm, the algorithm is improved by mixing the algorithms to achieve the effect of complementing each other’s strengths and improving the global exploration ability of the algorithm; second, to maintain the diversity of the algorithm at a later stage, a Gaussian variation operator is introduced into the pigeon flock intelligence algorithm to improve the local exploitation ability and avoid prematurity.

2. VRP-based routing model for logistics networks.

2.1. Basic description of VRP. The logistics distribution routing problem can be reduced to a vehicle routing problem with a time window.

There is only one distribution centre in the distribution system and the coordinates are known. A schematic diagram of the distribution route for the VRP problem is shown in Figure 1. The main components of the VRP problem are the transport vehicle, the transported goods, the distribution centre, the customer, the constraints, the transport network and the objective function. The distribution centre is the location of the main work in the logistics transport process. VRP should satisfy the following basic constraints: a) satisfy all customer restrictions on the quantity, quality and variety of goods; b) satisfy customer restrictions on the arrival time of goods; c) satisfy that the actual load of the vehicle cannot exceed the rated load limit of the vehicle.
2.2. Problem description. When building a logistics distribution model, time costs need to be added to the traditional VRP model, with waiting costs added to the total costs if the transport vehicle arrives early within the time requested by the customer, or penalty costs added to the total costs if the transport vehicle arrives late within the time requested by the customer. For the above reasons, this paper needs to include a time window in the model to refine it. The following is a specific description of the time window.

(1) Hard time window.

The hard time window requires the vehicle to reach at the demand point and deliver the goods to the customer within the time frame required by the customer. According to Figure 2, the customer has the right to refuse the delivery if the vehicle does not finish the delivery procedure in the amount of time requested by the customer and this causes the customer to incur a loss. This circumstance is described by the following penalty function.

\[
\sigma_{ik} = \begin{cases} 
MI, & T_{ik} < T_1(i) \text{ or } T_{ik} > T_2(i) \\
0, & T_1(i) \leq T_{ik} \leq T_2(i)
\end{cases}
\]

where \(\sigma_{ik}\) denotes the loss of vehicle \(k\) due to the time window constraint when serving customer \(i\). \(MI\) denotes a very large number, \(T_{ik}\) denotes the point in time when vehicle \(k\) arrives at customer \(i\), and \([T_1(i), T_2(i)]\) denotes the time window at demand point \(i\).

(2) Soft time windows.

Compared to a hard time window, a soft time window is less time demanding and less demanding. As shown in Figure 3, the soft time window allows transport vehicles to arrive outside the time window, although there is a waiting cost for early arrival and a penalty cost for late arrival, as shown in the cost function below.

\[
\sigma_{ik} = \begin{cases} 
\eta_1 \sum_{i=1}^n s_{ik} [T_1(i) - T_{ik}], & T_{ik} < T_1(i) \\
0, & T_1(i) \leq T_{ik} \leq T_2(i) \\
\eta_2 \sum_{i=1}^n s_{ik} [T_{ik} - T_2(i)], & T_{ik} > T_2(i)
\end{cases}
\]

where \(\eta_1\) denotes the value of unit waiting cost to be paid if the vehicle arrives early, \(\eta_2\) denotes the value of unit penalty cost to be paid if the vehicle arrives late. \(s_{ik} = 0, 1\), if
Figure 2. Hard time window

$s_{ik}$ is 1, it means that vehicle $k$ provides delivery transport service to customer $i$; if $s_{ik}$ is 0, it means that vehicle $k$ does not provide delivery service to customer $i$.

Figure 3. Soft time window

As it is more in line with a more realistic situation, this work investigates the VRP problem with a soft time window. There is only one distribution centre in the distribution area and the coordinate locations are known. The distribution centre has a number of delivery vehicles with the same rated load. There are several customers within the distribution area of the vehicles and each customer has a different quantity of goods required. Each vehicle is loaded with a certain weight of goods and delivers the goods within a soft time window requested by each customer. Ultimately the vehicle has to return to the distribution location. The task is to optimise the routing and sequencing of deliveries in a scientific way to minimise the costs incurred.

2.3. Definition of the basic constraints and variable signs of the model. The factors affecting the quality and total cost of products should be fully considered before building the model, and a series of assumptions should be made on the model in order to
simplify the number of constraints in the model and thus improve the speed of solution operations.

The assumptions of the logistics network routing model are shown below.

1) The model has and has only one distribution or warehousing centre and the distribution centre is adequately stocked.
2) The coordinates of the distribution points and the location of each customer point are known.
3) The needs of each customer point and its required soft time window are known.
4) The amount of goods loaded on individual vehicles departing from the logistics centre must not exceed the rated capacity of the vehicle.
5) Each customer point has and can have only one vehicle to meet its needs.
6) The load of the transport vehicle must be less than the maximum load limit of the vehicle itself.
7) Each vehicle will depart from the distribution centre and will return to the distribution centre after transporting the goods.

The more involved variables and symbols in the process of building the model are shown in Table 1.

<table>
<thead>
<tr>
<th>Variable symbols</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P = {i</td>
<td>i = 1, 2 \ldots n}$</td>
</tr>
<tr>
<td>$K = {k</td>
<td>k = 0, 1, 2 \ldots m}$</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of clients</td>
</tr>
<tr>
<td>$m$</td>
<td>Number of transport vehicles</td>
</tr>
<tr>
<td>$C_i$</td>
<td>Transport costs during transportation</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Unit distance cost of distribution vehicles</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Unit activation cost of distribution vehicles</td>
</tr>
<tr>
<td>$M$</td>
<td>Maximum load capacity of distribution vehicles</td>
</tr>
<tr>
<td>$d_i$</td>
<td>Volume of goods required by customer i</td>
</tr>
<tr>
<td>$c_{ij}$</td>
<td>Distance from customer i to customer j</td>
</tr>
<tr>
<td>$T_{ik}$</td>
<td>Point in time when vehicle k arrives at customer i</td>
</tr>
<tr>
<td>$t_i$</td>
<td>Service hours at client i</td>
</tr>
</tbody>
</table>

2.4. Modeling. The objective function of the logistics network routing model requires the lowest total transportation cost. The total cost includes the transportation cost of the vehicles and the time cost. The transportation cost includes the activation cost of all vehicles and the travel cost, while the time cost includes the waiting cost for early arrival of the vehicles and the penalty cost for late arrival of the vehicles.

The objective function of the proposed model is as follows:

$$\min C = \alpha \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=0}^{m} c_{ij} x_{ijk} + \beta \sum_{k=0}^{m} \sum_{j=0}^{n} x_{ijk} + \sum_{k=0}^{n} \sigma_{ik}$$ (3)

The objective function requires the lowest total cost, which includes the cost of transporting the vehicle as well as the cost of time. $x_{ijk} = 1$ for vehicle $k$ serving customer $i$ and customer $j$ and $x_{ijk} = 0$ for vehicle $k$ not serving customer $i$ and customer $j$. 
The model requires that the number of vehicles departing from a distribution centre must not exceed the number of vehicles owned by that centre.

$$\sum_{k=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq m, \quad i = 1$$ \hspace{1cm} (4)$$

The total demand on the transport route must not exceed the maximum load of the vehicle.

$$\sum_{i=1}^{n} d_{is} s_{ik} \leq M, \quad k = 0, 1, ..., m$$ \hspace{1cm} (5)$$

Equation (6) and Equation (7) represent constraints on routing:

$$\sum_{i=1, i\neq j}^{n} x_{ijk} = s_{jk}, \quad j = 1, 2, ..., n, \quad \forall k$$ \hspace{1cm} (6)$$

$$\sum_{j=1, i\neq j}^{n} x_{ijk} = s_{ik}, \quad i = 1, 2, ..., n, \quad \forall k$$ \hspace{1cm} (7)$$

The model requires that only one vehicle is serviced per customer and that each customer is served.

$$\sum_{k=1}^{m} s_{ik} = 1, \quad i = 1, 2, ..., n$$ \hspace{1cm} (8)$$

$$\sum_{i=1}^{n} \sum_{k=1}^{m} s_{ik} = n, \quad x_{ijk} = \{0, 1\}, \quad \forall i, j \in P, \quad s_{ik} = \{0, 1\}, \quad \forall i \in P, \quad \forall k \in K$$ \hspace{1cm} (9)$$


3.1. Standard PIO algorithm. Pigeons are among the most common birds in the entire globe, yet they have an odd kind of autoguiding. They have been used in the past to send letters. By investigating the ability of pigeons to sense magnetic fields, researchers have found that this excellent autoguiding ability of pigeons relies mainly on tiny magnetically induced particles on their beaks, and that these iron crystal particles have the property of pointing north [28].

According to research, pigeons could have a unique mechanism that uses the trigeminal nerve to send geomagnetic signals detected by magnetosensitive granules in the beak to the head. By separating the various solar altitude data from their beginning position and present position, they may simultaneously give navigational information.

In the standard pigeon flock intelligence algorithm, $N$ represent the number of swarms, $n$ represent the dimension. By generating beginning values for optimization iterations at random, every individual in the swarm symbolizes one potential solution with a certain fitness value

$$X(k) = [x_1(k), x_2(k), \ldots, x_n(k)], \quad (k = 1, 2, \ldots, N)$$ \hspace{1cm} (10)$$

$$V(k) = [v_1(k), v_2(k), \ldots, v_n(k)], \quad (k = 1, 2, \ldots, N)$$ \hspace{1cm} (11)$$

where $X(k)$ is the location of pigeon $k$ and $V(k)$ is the velocity of pigeon $k$. The refresh procedure for location and velocity is the basis of the PIO method. The PIO method has a two-stage optimisation framework, with the map compass operator used in the first stage to refresh location and velocity while the landmark operator in the second.

(1) Map compass calculator.

Suppose $X_g$ is the global optimal solution in the current swarm[29]. Pigeons create mental maps by detecting the geomagnetic field while in flight, and they utilize data
about the solar height as a compass to change the direction of their flight. Their reliance on geomagnetic and solar information decreases as they approach their target position.

\[ V^{(t)}(k) = V^{(t-1)}(k) \cdot e^{-Rt} + \text{rand} \cdot (X_g - X^{(t-1)}(k)) \quad (12) \]

\[ X^{(t)}(k) = X^{(t-1)}(k) + V^{(t)}(k) \quad (13) \]

where \( R \) is the map compass factor.

(2) Landmark operator.

As the pigeons approach the target location, they will rely more on information about the typical landmarks in the vicinity. In the landmark operator, the better adapted part of the individuals is treated as pigeons familiar with the landmarks and the remaining pigeons are assumed to follow the better individuals. In each iteration, the flocks are ranked according to the better or worse fitness, and the less well adapted half of the flock is omitted, leaving the better individuals [30].

\[ C(t) = \frac{\sum X^{(t)}(k) \cdot \text{fitness}(X^{(t)}(k))}{\sum \text{fitness}(X^{(t)}(k))} \quad (14) \]

\[ X^{(t)}(k) = X^{(t-1)}(k) + \text{rand} \cdot (C^{(t)} - X^{(t-1)}(k)) \quad (15) \]

where \( C(t) \) is centre of the swarm, \( N_P \) is the current swarm size.

3.2. Quantum PIO improvement algorithm. The Quantum Evolutionary Algorithm (QEA) is a commonly used intelligent optimisation algorithm. The algorithm introduces the concept of random encoding for quantum computing and is able to represent multiple solutions to a problem with a smaller swarm size, making the algorithm characterised by high swarm dispersion, strong global search capability and fast convergence.

The Quantum Evolutionary Algorithm differs from other intelligent algorithms in that there are no factors such as selection and crossover, making it easy to integrate with other optimisation algorithms. Therefore, this work combines the Quantum Evolutionary Algorithm and the PIO algorithm to propose the Quantum-Pigeonholing Inspired Optimisation (QPIO) algorithm.

The quantum evolutionary algorithm uses quantum bit encoding, and the quantum bit chromosome can be represented as a string of \( m \) quantum bits.

\[ q = \begin{bmatrix} \alpha_1 & \ldots & \alpha_i & \ldots & \alpha_m \\ \beta_1 & \ldots & \beta_i & \ldots & \beta_m \end{bmatrix} \quad (16) \]

where \(|\alpha_i|^2 + |\beta_i|^2 = 1\), \( i = 1, \ldots, m \), \( m \) is the chromosome length. \(|\alpha_i|^2\) and \(|\beta_i|^2\) denote the magnitude of the probability that the state of the quantum bit is at 0 and 1. The linear superposition of possible solutions can be achieved by combining different quantum bits in the chromosome, thus the diversity of quantum evolutionary algorithms is superior to that of classical evolutionary algorithms.

Traditional mutation operations occur randomly and without direction, and therefore converge slowly. In quantum evolutionary algorithms, quantum bit states can be seen as mutation operators, and quantum bit states \([\alpha_i, \beta_i]^T\) are updated through a revolving gate.

\[ \begin{bmatrix} \alpha_i' \\ \beta_i' \end{bmatrix} = \begin{bmatrix} \cos \theta_i & \sin \theta_i \\ -\sin \theta_i & \cos \theta_i \end{bmatrix} \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} \quad (17) \]

The positive and negative of variation angle \( \theta_i \) can be obtained from Table 2.

In Table 2, \( x_i \) is the \( i \)-th location of the present chromosome, \( \text{best}_i \) is the \( i \)-th location of the present optimal chromosome, \( \Delta \theta_i \) is the size of the rotation angle (which controls the
Table 2. Selection of variation angle $\theta_i$.

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>$best_i$</th>
<th>$\Delta \theta_i$</th>
<th>$s(\alpha_i, \beta_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\alpha_i \beta_i &gt; 0$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0.01$\pi$</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0.01$\pi$</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0.01$\pi$</td>
<td>+1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0.01$\pi$</td>
<td>+1</td>
</tr>
</tbody>
</table>

speed of convergence), and $s(\alpha_i, \beta_i)$ is the direction of the rotation angle (which ensures convergence).

A schematic diagram of the quantum revolving gate polar coordinates is shown in Figure 4. Taking advantage of the good swarm dispersion of the quantum evolutionary algorithm,

![Figure 4](image.png)

**Figure 4. Schematic of a quantum revolving gate in polar coordinates**

the quantum mutation operation is introduced into the map compass operator update phase of the PIO algorithm in order to avoid premature convergence of the algorithm into a local optimum. Similar to the standard PIO algorithm, the QPIO algorithm also consists of two components: the map compass operator and the landmark operator.

1. **Map compass calculator.**

   In quantum space, the probability density function of a particle appearing at a specific place is expressed by the Schrödinger equation solution in the QPIO technique, which
then utilizes a Monte Carlo stochastic simulator to determine the position state.

\[
X_i(t) = \mathbf{P}_i \mathbf{P}_g(t) \pm \frac{L}{2} \ln \left( \frac{1}{u} \right) \tag{18}
\]

\[
\mathbf{P}_i \mathbf{P}_g(t) = f(t) \times \mathbf{P}_i(t) + (1 - f(t)) \times \mathbf{P}_g(t) \tag{19}
\]

\[
L = 2\omega(t) |\mathbf{m}_{best}(t) - X_i(t)| \tag{20}
\]

where \( u, f \) is a random number that follows a uniform distribution on \([0,1]\), \( \mathbf{P}_i(t) \) is the historical best position of the particle at the \( t \)th iteration, \( \mathbf{P}_g(t) \) is the global best position of the swarm at the \( t \)th iteration, \( \omega(t) \) is the inertia weight, and \( \mathbf{m}_{best}(t) \) is the average best position of all individual particles in the swarm at the \( t \)-th iteration.

\[
\omega(t) = \omega_{max} - (\omega_{max} - \omega_{min}) \times \frac{t}{t_{max}} \tag{21}
\]

\[
\mathbf{m}_{best}(t) = \frac{1}{N_p} \sum_{i=1}^{N_p} \mathbf{P}_i(t) \tag{22}
\]

The QPIO algorithm position update rules are

\[
X_i(t + 1) = \begin{cases} 
\mathbf{P}_i \mathbf{P}_g(t) + \omega(t) \times |\mathbf{m}_{best}(t) - X_i(t)| \times \ln \frac{1}{f(t)}, & f(t) \geq 0.5 \\
\mathbf{P}_i \mathbf{P}_g(t) - \omega(t) \times |\mathbf{m}_{best}(t) - X_i(t)| \times \ln \frac{1}{u(t)}, & f(t) < 0.5
\end{cases} \tag{23}
\]

(2) Landmark operator.

The landmark operator of the QPIO algorithm is consistent with the standard PIO algorithm, where the swarm size is halved at each iteration, the swarm centre position is obtained according to Equation (14), and the position is updated using Equation (15).

3.3. Gaussian variational PIO improvement algorithm. Like most classical swarm intelligence algorithms, the PIO algorithm converges on the positions of individuals in the swarm late in the iteration and is prone to fall into local optima. Drawing on the ideas of genetic algorithms, a variation operation is added to the QPIO algorithm to ensure the diversity of the algorithm, namely the Gaussian variation QPIO (GM-QPIO) algorithm.

In a swarm, "diversity" refers to the degree of variance among individuals as measured by their Euclidean distance from one another. There are three forms of swarm diversity: diversity of design variables \( D_{v,t} \), diversity of individuals \( D_{p,t} \) and diversity of swarms \( D_{s,t} \). Individual diversity and swarm diversity are usually considered primarily, and the diversity of individuals in a swarm is calculated for each iteration as follows:

\[
D_{p,t} = \left[ \sum_{j=1}^{d} (x_{i,j}(t) - \bar{x}_j(t))^2 \right]^{\frac{1}{2}}, \quad p = 1, 2, \ldots, s; \quad t = 1, 2, \ldots, T \tag{24}
\]

\[
\bar{x}_j(t) = \frac{1}{s} \sum_{i=1}^{s} x_{i,j}(t) \tag{25}
\]

The location of each individual in respect to the average individual may be used to determine the swarm diversity.

\[
D_{s,t} = \frac{1}{s} \sum_{i=1}^{s} \left[ \sum_{j=1}^{d} (x_{i,j}(t) - \bar{x}_j(t))^2 \right]^{\frac{1}{2}} \tag{26}
\]

This work uses a Gaussian variation operator to improve the standard pigeon flock intelligence algorithm by avoiding information becoming identical across individuals through the variation operation in order to increase the swarm diversity in the later stages of
the algorithm and to avoid falling into local optima. In order to produce mutation, an algebraic operation was applied to the produced random quantity and the gene value of the previous chromosome. The random number was created using a Gaussian function $G_{rand}$ within a self-defined region. The fluctuation value is then subject to saturation limit based on the self-defined value region.

Based on the average position information and the present position information, the group diversity is determined. If the population diversity $d$ falls below a predetermined threshold $d_{low}$, the mutation operator is engaged after updating the individual’s speed and position in accordance with Equations (12) to (15). Variation is carried out according to the following rules.

$$if \ d < d_{low} \ then \ x_{i,j}(t) = x_{i,j}(t - 1) + \xi \cdot G_{rand}$$

(27)

Where $\xi$ is the weight.

3.4. Implementation steps of the GM-QPIO based logistics network routing model. This work improves the standard PIO algorithm and applies it to the VRP problem in the logistics network routing model with the following execution steps:

1. Quantum bit coding was chosen as the coding method for the parameters of the PIO algorithm according to the requirements of the logistics network routing model.
2. Generate an initial swarm of possible solution sets by combining the priority levels of customer services and other influencing factors.
3. Depending on the objective function of the problem, choose an adaptation function that corresponds to it.
4. After the fitness of each individual has been calculated, the selection operation is then performed;
5. After the selection operation is completed, the crossover operation is performed on the individuals with adaptive probability. In this paper, we consider that the crossover of close relatives will have serious impact on the result and process of the algorithm, so in this paper, before the crossover operation, we first judge whether the two sides of the crossover are close relatives or not, if not, then the crossover operation will be performed.
6. Gaussian variation was performed on individuals after the crossover was completed in order to facilitate the increase in swarm diversity.
7. Iterate and mutate continuously until the set maximum number of iterations is reached or the stopping condition is met, or vice versa, and return to the third step to continue iteration.

4. Simulation results and analysis.

4.1. Validity analysis of the GM-QPIO algorithm. In order to verify the effectiveness of the proposed GM-QPIO, comparative simulation experiments were conducted using benchmark optimisation test functions to compare and analyse the standard PIO algorithm, GM-QPIO and the commonly used particle swarm (PSO) algorithm. For fairness of the comparison and to exclude the influence of individual special results, all experimental data are averaged over 40 independent runs of each algorithm.

Two common benchmark optimisation test functions were used in the simulation experiments: Ackley function and Rastrigin function. The Ackley function is a non-linear multi-peak function with the following functional expression.

$$if (X) = -20 \exp \left(-0.2 \sqrt{\sum_{i=1}^{n} x_i^2 / n}\right) - \exp \left(\sum_{i=1}^{n} \cos (2\pi x_i) / n\right) + 20 + e, |x_i| \leq 32$$

(28)
The Rastrigin function is a non-linear multi-peak function with the following functional expression.

\[
if(X) = \sum_{i=1}^{D-1} \left[ x_i^2 - 10 \cos(2\pi x_i) + 10 \right]
\]  

(29)

Schematic diagrams of the two benchmark function are shown in Figure 5 and Figure 6 respectively.

![Figure 5. Ackley functions](image)

![Figure 6. Rastrigin functions](image)

The swarm size \(M = 20\), the variable dimension \(D = 3\) and the number of iterations \(N = 50\). The geomagnetic factor was set to \(r = 0.5\) for both the PIO algorithm and the GM-QPIO algorithm. The PSO algorithm parameters were set to \([w, c_1, c_2] = [0.6, 2, 2]\). The
optimal adaptation values for the various algorithms are given in Table 3. The convergence curves for Ackley and for Rastrigin are shown in Figures 7 and 8 respectively.

Table 3. Optimal adaptation values for the three algorithms.

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Average adaptation value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ackley functions</td>
</tr>
<tr>
<td>PIO</td>
<td>0.0014</td>
</tr>
<tr>
<td>PSO</td>
<td>0.00018</td>
</tr>
<tr>
<td>GM-QPIO</td>
<td>3.61E-8</td>
</tr>
</tbody>
</table>

Figure 7. Ackley’s convergence curve

The convergence speed of the PIO algorithm and the GM-QPIO algorithm is significantly better than that of the PSO algorithm, which verifies the fast convergence capability of the PIO algorithm. According to the simulation results, the convergence speed and optimisation accuracy of the GM-QPIO algorithm are higher than those of the standard PIO algorithm. This is because the GM-QPIO algorithm can guarantee the diversity of the swarm to a certain extent during the iterative process. At the same time, due to the increased randomness in the late stages of the whole algorithm iteration after the introduction of quantum bits, the GM-QPIO algorithm can effectively improve the late local optimisation-seeking ability of the algorithm, which verifies its feasibility and effectiveness.

4.2. Solmon’s example-based case analysis. The data source used for the real case analysis is the Solomon algorithm proposed for the study of VRP problems with time window constraints.
The Solomon example consists of three parts, the C, R and RC datasets, the only difference between the three datasets is that the coordinates of the client points and the time window parameters are set differently. The C dataset sets the coordinates of the client points in a structured way, the R dataset sets the coordinates of the client points in a uniformly distributed way, and the RC dataset sets the coordinates of the client points by combining the characteristics of both. The RC type combines the characteristics of both.

Simulation tests were carried out in MATLAB R2016a software using the data from the R130 part of the algorithm of the GM-QPIO algorithm. The simulation results for the R130 arithmetic example using the GM-QPIO algorithm are shown in Table 4. The optimal solution is 15214.7, with a total of eight vehicles involved in the transport, implying a total of eight optimal distribution routes. It was verified that all routes satisfy the constraints and limitations of the logistics distribution model and are valid results.

The standard PIO algorithm and the GM-QPIO algorithm were used to simulate and compare some of the data from different datasets in the Solomon database respectively, and the results are shown in Table 5. Compared to standard PIO, the GM-QPIO algorithm proposed in this work shows better optimisation results in the VRP problem of the logistics network routing model. the GM-QPIO algorithm reduces the optimal cost value and accelerates convergence without falling into a local optimum solution.

5. Conclusion. This work investigates the logistics network routing optimization problem based on the GM-QPIO algorithm. The integration of the VRP problem with real-life practical problems is one of the key reasons that promote the VRP problem to be widely studied and discussed. Firstly, this work constructs a VRP-based routing model for logistics networks. Secondly, the standard PIO algorithm is improved in two aspects in order
Table 4. Results of the R130 partial arithmetic example.

<table>
<thead>
<tr>
<th>Calculation results</th>
<th>Optimum solution</th>
<th>Number of vehicles</th>
<th>Algebra for reaching the optimal solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Routing number</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>15214.7</td>
<td>8</td>
<td>321</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
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<td>5</td>
<td></td>
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</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5. Comparison of different data sets.

<table>
<thead>
<tr>
<th>Example</th>
<th>Standard PIO</th>
<th>GM-QPIO</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>convergent algebra</td>
<td>Optimal cost</td>
<td>convergent algebra</td>
</tr>
<tr>
<td>R130</td>
<td>369</td>
<td>16132.3</td>
<td>321</td>
</tr>
<tr>
<td>R131</td>
<td>356</td>
<td>12152.4</td>
<td>303</td>
</tr>
<tr>
<td>CI30</td>
<td>347</td>
<td>17570.3</td>
<td>314</td>
</tr>
<tr>
<td>CI31</td>
<td>347</td>
<td>13082.5</td>
<td>301</td>
</tr>
<tr>
<td>RC130</td>
<td>351</td>
<td>14623.4</td>
<td>332</td>
</tr>
<tr>
<td>RC131</td>
<td>342</td>
<td>13445.9</td>
<td>327</td>
</tr>
</tbody>
</table>

To improve the global exploration capability and diversity of the PIO algorithm. The simulation results of the data of the R130 partial arithmetic example show that the optimal solution obtained by the GM-QPIO algorithm at 321 iterations is 15214.7. This indicates that using the GM-QPIO algorithm to solve the VRP optimisation in the logistics network routing model, the convergence speed of the optimal solution can be effectively improved by using the GM-QPIO algorithm to solve the VRP optimization problem in the logistics network routing model, and better results can be obtained compared with the standard PIO algorithm. The generalisation and complexity of the GM-QPIO algorithm will be investigated in order to enhance its overall applicability and use less computer resources.

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