

# Sparrow Search Algorithm with Adaptive Collaborative Updating

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**ABSTRACT.** *The research of swarm intelligence shows a rapid development trend. People have studied a lot of swarm intelligence algorithms inspired by biology. Sparrow search algorithm is concerned by many scholars because of its powerful search performance. However, in the case of high-dimension optimization problems, if only a single sparrow search algorithm is used to solve optimization problems, the optimal solution often cannot be found, and the phenomenon of dimension disaster will occur or it is easy to fall into the local optimal solution. In order to make sparrow search algorithm have better performance in solving high-dimensional complex problems, In this paper, we propose a new Sparrow Search Algorithm with Adaptive Collaborative Updating (SSA-ACU). The SSA-ACU algorithm initializes the population through the chaotic search strategy to make the population distribution more diverse. Then, adaptive parameters are introduced to update the position of the leader to provide a more reliable direction for particle search. Finally, in order to enhance the development ability of the population, the position of the adjoint is updated through the pseudo-affine transformation mechanism to change the mode of its position update. Then CEC2013 was used to test and compare SSA-ACU algorithm with other sparrow search algorithm variants and classical algorithms, and it was verified that SSA-ACU algorithm has good performance in convergence speed and optimization ability.*

**Keywords:** Sparrow search algorithm, Swarm intelligence, Adaptive collaborative updating

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1. **Introduction.** There are many different types of optimization problems in both scientific theory and engineering practice, and the solution of these problems has been the focus of research by researchers [1, 2, 3]. It has been proven that applying appropriate optimization methods to different fields can bring great benefits. Therefore, it is important to find reliable and easy-to-use optimization methods in various fields [4, 5, 6]. Ant colony algorithm (ACO) [7] introduces the concept of pheromone into the function by observing the way ants communicate with each other while foraging in nature. The ACO algorithm mainly allows ants to search for paths using probabilistic search, and the pheromone concentration is the main basis for determining the probability. Each ant carries a fixed amount of pheromone and will leave pheromone behind as it moves, and the pheromone left behind will evaporate partly over time. But because a part of the randomness is still retained, there is still a small fraction of ants that will choose a different route, and these ants are used to avoid the algorithm from falling into a local optimal solution. Genetic algorithm (GA), different from traditional search algorithms, is a selection optimization search mechanism based on Darwin's theory of evolution "natural selection" and "mutation may bring progress". It searches for the optimal solution of problems by simulating the idea of survival of the fittest in the process of biological evolution [8]. The particle swarm optimization algorithm (PSO) was inspired by the study of bird foraging behavior observed in the real world. The basic idea of the particle swarm algorithm is to describe the foraging process of a bird in terms of a particle [9]. The flight of the particle is equivalent to the flight process of a bird, and the speed of its flight is determined according to the historical optimal position of each particle and the final position of the whole population.

Sparrow Search Algorithm (SSA) is a new swarm intelligence algorithm proposed by Xue et al. [10]. The researchers studied the problems in different aspects and proposed a series of improvement methods for SSA. Wang et al. [11] introduced particle swarm optimization algorithm into SSA and introduced an enhanced sparrow search algorithm (PESSA) based on particle swarm optimization algorithm, which enhanced the random search mode of the leader. Each follower continued to learn the best behavior of the leader. Shi et al. [12] proposed firefly sparrow search algorithm (FSSA). Aiming at the shortcomings of SSA algorithm, firefly algorithm was introduced to improve the search efficiency of SSA through firefly disturbance strategy. Wu et al. [13] proposed the Chaotic mapping sparrow search algorithm (LCSSA), which used the Logistic chaotic mapping to improve the initial individuals of sparrow search and increase the population diversity. Ouyang et al. [14] introduced learning adaptive parameters in the leader stage and introduced learning SSA algorithm (LSSA). The algorithm promotes the diversity of the population in the search process by using the random reverse learning technique. The adaptive update strategy of sine and cosine parameters guides the leader's position update, which makes the leader search method more thorough. Zhang et al. [15] proposed ROSSA algorithm (random antilearning SSA), which changes algorithm parameters by linear decline method to balance the situation that the algorithm jumps out of the local optimal solution. Zhang et al. [16] proposed a discrete SSA algorithm (DSSA) with global perturbation technology. The algorithm selected the generated initial population through roulette, then added sequential decoding to adapt to the sparrow's position update, and used the Gaussian mutation operator and exchange operator of global perturbation technology to balance exploration and development capabilities. Kathirolu et al. [17] mixed difference algorithm and SSA to form DE-SSA algorithm, which combines advanced search ability of SSA and active potential of DE to expand the search space of the algorithm. Liu et al. [18] proposed an improved SSA called CASSA algorithm, which improved the population diversity of the initial value through chaotic search, added inertia weight to

change the convergence rate and exploration ability, recorded the advantages and disadvantages of inertia weight, and searched for the best value of inertia weight through an adaptive method. Finally, the Cauchy Gaussian mutation technique is used to jump out of the local optimal solution. Wang et al. [19] proposed an adaptive T-distribution based SSA algorithm (TSSA). The algorithm added an adaptive T-distribution mutation mechanism to SSA, which could change the search direction when the SSA algorithm fell into a local optimal solution and provide a reliable search direction to continue the search. Zhang et al. [20] introduced a sine and cosine algorithm into the SSA algorithm, called SCA-CSSA algorithm, which reconstructs the labor cooperation structure of sparrow. The new structure introduces the sine and cosine algorithm, which can search with high quality and avoid local optimization. Compared with the proposed sparrow search algorithms and their variants, a new sparrow search algorithm is proposed in this paper. The specific contributions of this paper are as follows:

(1) In order to solve the problem of premature convergence, the SSA-ACU algorithm is proposed. The SSA-ACU algorithm initializes the population through chaotic search strategy, then introduces adaptive parameters to update the position of the leader, and finally updates the position of the adjoint through the quasi-affine transformation mechanism.

(2) The new population location is obtained, and the corresponding fitness value and optimal solution are calculated. The performance of SSA-ACU algorithm is verified by comparing SSA-ACU algorithm with other sparrow search algorithm variants and classical algorithm using international standard test suite.

In the following chapter description, the main contents include the following. In Section 1, we introduce some classical swarm intelligence algorithms and some variants of SSA, and explain how many researchers improve SSA and the problems that still exist. In Section 2, the mathematical model of the original SSA is introduced in detail. The standard SSA is prone to precocious convergence in the search process. In Section 3, the original sparrow search algorithm is improved, and a new algorithm variant is proposed to improve SSA by chaotic initialization, introducing adaptive parameters and cooperative update strategy. In Section 4, the new algorithm is evaluated comprehensively. The fifth Section summarizes.

**2. Related Works.** The inspiration of SSA comes from the division of labor and cooperation among sparrow groups to complete foraging and anti-predation tasks, and the sparrow group is divided into three types: leader, companion and scout. In order to find the best food and successfully avoid the threat of predators, each of the three types of sparrows takes on different tasks. Its characteristics are as follows:

(1) The leader's task is to look for food sources in the field. After searching, the leader will feed back the source information to the companion to provide foraging areas and directions. The companion will constantly monitor the leader to increase its foraging rate.

(2) Scouts are randomly generated and occupy a small proportion in the population. Scouts are responsible for observing the surrounding environment. When they find that they are threatened by predators nearby, the scouts will remind the group of the surrounding danger.

(3) The leader and the follower occupy a large proportion in the group, and the identity between individuals is not fixed. In the search process, as long as the food information is searched, the individual can become the leader, and the identity of the leader and the follower can be switched between each other. However, the proportion of the leader in

the population is fixed. When an individual searches for food information and becomes a leader, one of the leaders in the population will turn into an accompanying one.

(4) The leader is the main role in the group, and the fitness value of the individual will affect the energy level of the leader. When the individual fitness value is excellent, the leader has a higher energy level; when the individual fitness value is poor, the leader has a lower energy level. Individuals with higher energy levels will be the leaders, while individuals with lower energy levels will continue to search for more energy elsewhere.

(5) When threatened by warning, sparrows in the edge of the safe area will move closer to the center of the safe area to escape the threat of predators, while sparrows in the center area will move randomly to ensure the safety of the area to a greater extent.

**2.1. The basic idea of the original sparrow search algorithm.** In the experimental simulation iteration process, the mathematical model is used to describe the foraging and anti-predation process of sparrows. The position of individual sparrows can be represented by a matrix, as shown in Equation (1). The individual information can be calculated by Equation (2), and the fitness value can be calculated by the individual value, as shown in Equation (3).

$$X = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,D} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,D} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N,1} & x_{N,1} & \cdots & x_{N,D} \end{bmatrix} \quad (1)$$

$$x_{i,j} = x_{j,\min} + r \cdot (x_{j,\max} - x_{j,\min}) \quad (2)$$

$$F_x = \begin{bmatrix} F_1 [X_{1,1}, X_{1,2}, \dots, X_{1,D}] \\ F_2 [X_{2,1}, X_{2,2}, \dots, X_{2,D}] \\ \vdots \\ F_N [X_{N,1}, X_{1,2}, \dots, X_{N,D}] \end{bmatrix} \quad (3)$$

In Equation (1),  $N$  is the number of sparrow population;  $D$  is the dimension of the solution to be optimized;  $i = 1, 2, \dots, N$ ,  $j = 1, 2, \dots, D$ ,  $r$  is a random number,  $r \in [0, 1]$ ;  $x_{i,j}$  is the individual information of the  $j$ th dimension of the sparrow  $i$ th iteration,  $x_{i,j} \in [x_{j,\min}, x_{j,\max}]$ .  $F_x$  is the objective function, and each row in the matrix represents the fitness value corresponding to each individual sparrow.

The leader has energy reserve in the population, and the leader with higher health value has stronger optimization ability. Therefore, there are two states of the leader in the population. One is that when there are no predators around, the explored area is larger so that the population can get more food, and when predators appear, it will move to the safe area. Therefore, the position of the leader is updated according to the rule, as shown in Equation (4).

$$X_{i,j}^{g+1} = \begin{cases} X_{i,j}^g \cdot \exp\left(\frac{-i}{\alpha \cdot G}\right), & \text{if } R < ST \\ X_{i,j}^g + Q \cdot L, & \text{if } R \geq ST \end{cases} \quad (4)$$

In the above equation,  $\alpha$  is an arbitrary number,  $\alpha \in [0, 1]$ ,  $X_{i,j}^g$  is the individual information of the  $j$ th dimension of the  $i$ th sparrow in the  $g$ th iteration,  $R$  is the alarm value.  $ST$  indicates the security threshold;  $R < ST$  means that the leader can explore the food in the field within the safety threshold,  $R \geq ST$  means that the scout has found the predator beyond the safety threshold, and all sparrows need to change their positions randomly.  $R \in [0, 1]$ ,  $ST \in [0.5, 1]$ .  $Q$  is a random number obeying the normal distribution.  $L$  stands for a  $1 \cdot D$  matrix where every entry is 1.

The accompanying person will monitor the leader all the time. When the leader finds good food, they will fight for it and leave the current position. The accompanying person's position is updated as shown in Equation (5).

$$X_{i,j}^{g+1} = \begin{cases} Q \cdot \exp\left(\frac{X_{worst}^g - X_{i,j}^g}{i^2}\right), & \text{if } i > N/2 \\ X_P^{g+1} + |X_P^g - X_P^{g+1}| \cdot A^+ \cdot L, & \text{otherwise} \end{cases} \quad (5)$$

In the above equation,  $X_{worst}^g$  and  $X_P^g$  respectively represent the worst and best position of the leader when the current iteration is  $g$  times.  $A$  is  $1 \cdot D$  matrix, whose elements are all composed of  $-1$  and  $1$ .  $A^+ = A^T (AA^T)^{-1}$  is the pseudo-inverse matrix of matrix  $A$ . When  $i > N/2$  means that the companion has a poor energy level, i.e. poor fitness, and needs to find food elsewhere. Otherwise, the sparrow will continue to forage in the field to replenish energy, that is, it will continue to iteratively solve around the optimal position.

When sparrows forage for food, a small number of sparrows will play the role of scouts. The positions of scouts are randomly generated, accounting for  $0.1 - 0.2$  of the total number of individuals. The positions of scouts are updated as shown in Equation (6).

$$X_{i,j}^{g+1} = \begin{cases} X_{best}^g + \beta \cdot |X_{i,j}^g - X_{best}^g|, & \text{if } f_i > f_g \\ X_{i,j}^g + K \cdot \frac{|X_{i,j}^g - X_{worst}^g|}{(f_i - f_w) + \varepsilon}, & \text{if } f_i = f_g \end{cases} \quad (6)$$

In Equation (6),  $X_{best}$  is the best position at the  $g$ th iteration, and  $\beta$ , as the step factor, has a normal distribution control parameter with an average value of  $0$  and a variance of  $1$ .  $K$  is any number in the range  $[-1, 1]$ .  $f_i$  is the fitness value calculated by the objective function,  $f_w$  and  $f_g$  are the global worst fitness value and global best fitness value respectively in  $g$ th iterations.  $\varepsilon$  is a minimal constant value to avoid errors in the division process.

When  $f_i > f_g$  indicates that individual sparrows are located at the boundary of the group, predators can easily attack these individual sparrows, and these individual sparrows need to move to the safe center area.  $X_{best}^g$  is the global optimal position, that is, the security center;  $f_i = f_g$  indicates that the sparrows in the safety center receive an alarm signal and need to update their current position to be closer to other sparrows and reduce the risk of predation.

**2.2. Basic flow of the original sparrow search algorithm.** The program block diagram of SSA is shown in Figure 1:

Step 1: Initialize parameters. The size of the population was set as  $N$ , the number of iterations is set to  $G$ , the upper bound  $x_{max}$  and lower bound  $x_{min}$  of the individual value, the alarm value as  $R$ , the number of leaders as  $PD$ , and the number of scouts as  $SD$ . Initialize the position of the individual and evaluate its fitness according to Equation (1) and Equation (2);

Step 2: The fitness values are sorted to obtain the best individual position and the worst individual position. The individual with better fitness value is the leader and the individual with worse fitness value is the companion;

Step 3: Use Equation (4), Equation (5) and Equation (6) to update the positions of the leader, companion and investigator respectively;

Step 4: Obtain the new position of the current population individual;

Step 5: The fitness value of the new population individual is compared with the fitness value of the old individual. If the new fitness value is better, the corresponding individual position and fitness value will be updated immediately; otherwise, the original individual position will be retained and the update and iteration will continue;

Step 6: If the number of iterations reaches the convergence condition, go to the next step; otherwise, repeat Step 3;

Step 7: Output the global optimum position  $X_{best}$  and the global optimum fitness value  $f_g$ .

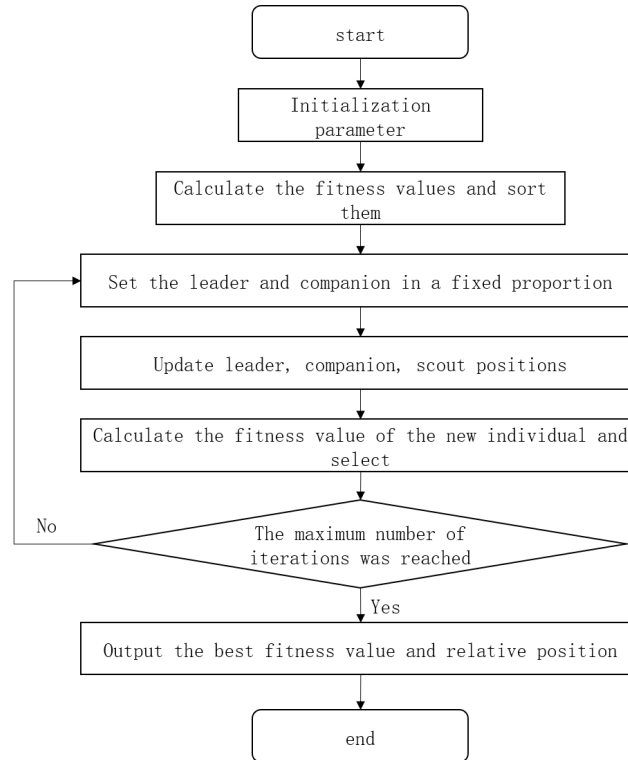


FIGURE 1. SSA program block diagram

**3. Improvement of SSA.** SSA algorithm, as a new group intelligent optimization algorithm with simple structure, few parameters and strong search ability, has been applied more and more in engineering optimization problems. The use of appropriate optimization methods to solve problems has always been the research direction in practical engineering. However, in the process of SSA search, the location of food is random. It is not known in advance, but the scattered search of the whole sparrow population finds food, so that the companion flies to the location where the food is abundant. For this process, the leader plays an elite guiding role, while the accompanying person is dependent on the leader. Once the sparrow finds the location where food is abundant, the rest of the accompanying people will directly wander in this location. If the leader stays in this location, the algorithm is often prone to fall into the local optimal solution. The location update strategy of the companion results in the imbalance between the global search ability and local search ability of the algorithm. Based on the above problems, the original SSA algorithm is improved, and a new adaptive collaborative sparrow search algorithm, namely SSA-ACU algorithm, is proposed in this paper. The improvements are as follows:

(1) Initialization mode of SSA-ACU. The initialization method of SSA algorithm uses random value generation, which not only reduces the diversity of the initial population, but also reduces the convergence speed, which makes its performance limited. In order to find the optimal candidate individuals more quickly and make the initial solutions more evenly distributed in the initial space, chaotic mapping strategy was introduced to initialize the population individuals [21].

(2) Adaptive parameters of SSA-ACU. When a scout finds a predator, the leader's safety factor exceeds the threshold, and the leader needs to randomly change its position. Timely update of the leader can provide a more reliable direction for the accompanying person's position update. However, random change of the leader's position in the iterative process will reduce the flexibility of the search. The global search capability of the algorithm is enhanced.

(3) Collaborative update strategy of SSA-ACU. Another reason why the algorithm is easy to fall into local optimality is that the location information of the accompanying person is closely related to the information of the leader. When the leader falls into local optimal search, the accompanying person also falls into local optimality, resulting in the failure to jump out of the current position. In order to balance the global search capability, QUasi Affine TRansformation mechanism in QUasi Affine TRansformation Evolution (QUATRE) is introduced [22]. The development ability of population is enhanced by quantum affine transformation.

**3.1. Initialization Mode of the SSA-ACU.** The initial values of the original SSA algorithm were not processed, making the resulting population values unrepresentative. Therefore, in this paper, chaos is characterized by regularity, sensitivity, randomness and universality [23], so that the initial value of SSA algorithm is processed by Logistics chaos mapping. Under the condition of not changing the randomness of initial population, the initial population has better ergodic property. The standard Logistics chaotic mapping function is shown in Equation (7).

$$\theta_{n+1} = 4\theta_n(1 - \theta_n) \quad \theta_n \in (0, 1), n = 1, 2, \dots, N \quad (7)$$

In swarm intelligence algorithms, Logistics chaotic mapping is often used as a processing method for population initialization [24], but the standard Logistics chaotic mapping still has the disadvantage of being spread and uneven. Therefore, this paper adopts an improved Logistics chaotic mapping to improve the uniform diversity of initial population values. A new chaotic sequence  $[\theta_n]$  is obtained, as shown in Equation (8).

$$\theta_{n+1} = \begin{cases} \left[ r\theta_n(1 - \theta_n) + \frac{(4-r)}{2}\theta_n \right] \bmod 1, & \theta_n < 0.5 \\ \left[ r\theta_n(1 - \theta_n) + \frac{(4-r)(1-\theta_n)}{2} \right] \bmod 1, & \theta_n \geq 0.5 \end{cases} \quad (8)$$

In the above equation,  $r$  is a random number,  $\bmod 1$  represents the remainder of the function divided by 1, and the feasible solution  $X_n$  for the initial position of individual sparrow obtained by linear transformation of chaotic sequence  $[\theta_n]$  can be obtained, as shown in Equation (9).

$$X_n = x_{\min} + (x_{\max} - x_{\min})\theta_n \quad (9)$$

Where  $N$  is the population number, and  $x_{\min}$  and  $x_{\max}$  are the minimum and maximum values of the variables to be solved. As shown in Figure 2 and Figure 3, the figure shows the scatter distribution of the Logistics chaotic map before and after the improvement under 5000 iterations. It can be seen from the figure that the standard Logistics chaotic map is more dense at both ends of 0 and 1, and the improved Logistics chaotic map can make the population resources in the search space more uniform.

**3.2. Adaptive parameters of SSA-ACU.** According to the leading position update Equation (4), leading the alarm value within the scope of the safety threshold, leader in its field for food, the scope of search for  $\exp(-i/\alpha \cdot G)$ , the parameter  $\alpha$  factor is randomly generated within the range (0, 1), its value is small, leading in the field of global

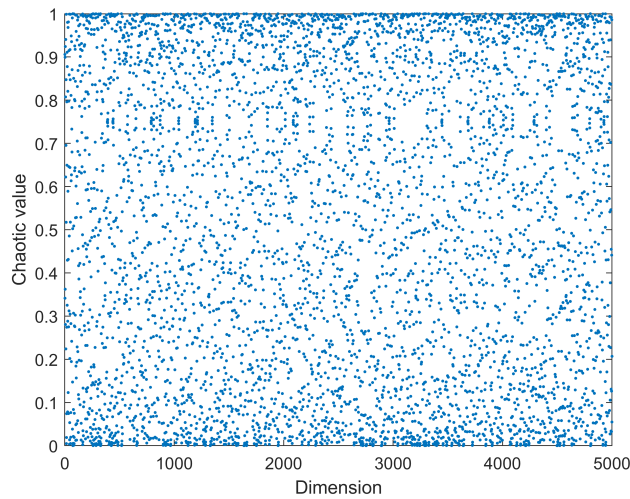


FIGURE 2. Standard Logistics chaos mapping

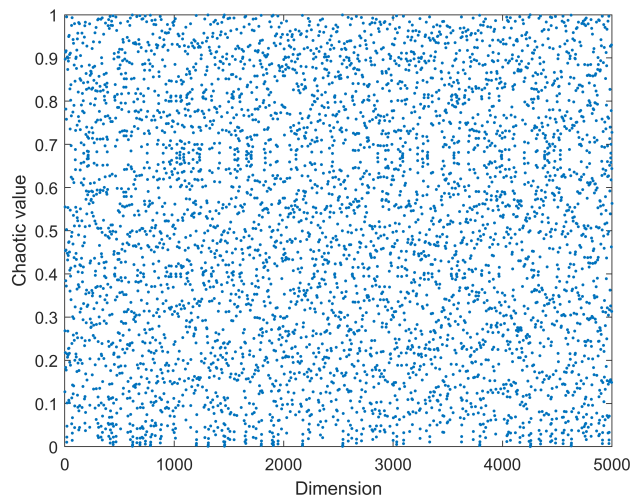


FIGURE 3. Improved Logistics chaos mapping

best position organic search, its value is bigger, the leader conducts a wide area search. It can be seen that in the late iteration period, with the decrease of sparrow population diversity, its search space also gradually decreases. At the same time, in the process of iteration, the adaptability of individual sparrows is also decreasing, which makes the algorithm easy to fall into the local optimal solution. Therefore, the generating mode of parameter  $\alpha$  factor has significant influence on the accuracy of convergence and iteration of SSA algorithm. Therefore, it is necessary to improve the adaptability of  $\alpha$  factor to avoid interleaving among individual sparrows in the search process, so as to improve the convergence accuracy of the algorithm. Based on the above analysis, this paper is inspired by the literature [25]. The adaptive scheme for  $\alpha$  factor is improved as shown in Equation (10) and Equation (11).

$$\alpha_i = \text{rand}c_i(\mu_\alpha, \sigma_\alpha) \quad (10)$$



$$\begin{cases} \mu_\alpha = \frac{\sum_{\alpha_i \in S_\alpha} w_{\alpha_i} \cdot \alpha_i^2}{\sum_{\alpha_i \in S_\alpha} w_{\alpha_i} \cdot \alpha_i} \\ w_{\alpha_i} = \frac{\Delta f_i}{\sum_{\alpha_i \in S_\alpha} \Delta f_i} \\ \Delta f_i = \frac{f_i - f_{\min}}{f_{\text{avg}} - f_{\min}} \end{cases} \quad (11)$$

In the above equation,  $\alpha$  factor is a random number obeying Cauchy distribution. At the beginning of iteration, the initial value of position parameter  $\mu_\alpha$  is set to 0.5, and the scale parameter  $\sigma_\alpha$  is set to 0.1. Each individual in the population has a corresponding  $\alpha$  factor, which avoids the interleaving of individual information in the evolutionary process.  $\mu_\alpha$  is updated through the weighted mean in the iterative process according to Equation (11),  $S_\alpha$  represents the set of  $\alpha_i$  factors that perform better in the search process.  $f_i$  is the current fitness value,  $f_{\min}$  is the minimum fitness value and  $f_{\text{avg}}$  is the average fitness value.  $\Delta f_i$  is to measure the pros and cons of  $\alpha$  factors, meaning that when the fitness value is less than the average value, it can be considered that the individual is currently trapped in the local optimization situation or far from the actual value, need to adapt to adjust.

**3.3. Collaborative Update Policy of SSA-ACU.** QUATRE algorithm is a coevolutionary algorithm first proposed by Professor Meng in 2016. Its algorithm has strong development ability and is widely favored by researchers. SSA algorithm has a strong ability in search performance, but due to a large search range, the convergence speed is slow, and it cannot provide a reliable direction within the search range, showing a weak trend in development ability. QUATR algorithm can make up for the defects of SSA algorithm. QUATRE algorithm is evolved from differential evolution algorithm. The variation strategies of QUATRE algorithm include "QUATRE/rand/1", "QUATRE/best/1" and so on. "QUATRE/rand/1" is to carry out random variation of individual variables, while "QUATRE/best/1" is to search and optimize individuals near the current optimal value, which is convenient for local development. Combined with the characteristics of QUATRE algorithm, the variant strategy of "QUATRE /best/1" is introduced into the accompanying position update strategy of SSA algorithm to generate new subgroups, which is convenient to jump out of the local optimal in the search process. Inspired by this strategy, a new adjoint location update mode in SSA algorithm is proposed, as shown in Equation (12) and Equation (13).

$$B_i = X_p^g + F \cdot (X_{r_1} - X_{r_2}) \quad (12)$$

$$X_{i,j}^{g+1} = \begin{cases} Q \cdot \exp\left(\frac{X_{\text{worst}}^g - X_{i,j}^g}{i^2}\right), & \text{if } i > N/2 \\ \bar{A}_i \cdot X_{i,j}^g + A_i \cdot B_i, & \text{otherwise} \end{cases} \quad (13)$$

Equation (12) and Equation (13) give a new position update formula. Where  $F$  is a random number,  $F \in (0, 1)$ .  $B_i$  is the difference form of the sparrow whose current position is updated to the  $i$ th,  $r_1$  and  $r_2$  are two different individuals randomly selected from an existing population,  $i \neq r_1 \neq r_2$ .  $A$  is matrix, and  $A_i$  represents the  $i$ th row information of  $A$  matrix.  $A$  matrix describes the transformation of a vector space through linear quasi affine transformation and shift to another vector space in geometric space. This method can solve the problem of unbalanced optimal solution of the algorithm in high dimensions. The evolution process is mainly guided by the collaborative search matrix  $A$ . The initial matrix is based on a lower triangular matrix, whose elements include 0 and 1. After generating the initial matrix, row transformation and column transformation are carried

out on the matrix. The second step is to perform the row operations on the row vectors of the matrix. The transformation matrix is the  $A$  matrix. The conversion process is shown in Equation (14) and Equation (15).

$$\begin{cases} Y = AX + B \\ Y = AX + \bar{A}B \end{cases} \quad (14)$$

$$\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & & \\ & & \cdots & \\ 1 & 1 & \cdots & 1 \end{bmatrix} \sim \begin{bmatrix} & 1 & & \\ & & 1 & \\ & & & \cdots \\ 1 & 1 & \cdots & 1 \end{bmatrix} \sim \begin{bmatrix} & & & 1 \\ & & \cdots & \\ 1 & & & 1 \\ & 1 & & 1 \end{bmatrix} = A \quad (15)$$

$Y = AX + B$  is the conversion function.  $Y = AX + \bar{A}B$  for affine transformation method, fully response the synergy of the particles in the process.  $\bar{A}$  is of incidence matrix of matrix  $A$ , said all the elements in the matrix  $A$  binary arithmetic. Represents the inverse operation of all elements of the matrix  $A$ . Within  $A$  matrix elements is zero, then  $\bar{A}$  matrix in the same position of the element into 1; Within  $A$  matrix elements is 1, then  $\bar{A}$  matrix elements within the same location of the transition of 0. Equation (15) is A quasi-affine transformation process, where  $A$  is a cooperative search matrix with an overall size of  $(N - PD) \cdot D$ .

**3.4. Mathematical calculation model of SSA-ACU.** According to initialization mode, adaptive parameter model and cooperative update strategy of SSA-ACU, the mathematical calculation model of SSA-ACU can be summarized as Algorithm 1.

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**Algorithm 1** SSA-ACU

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- 1: Initialization: population size  $N$ , dimension size  $D$ , maximum number of iteration  $G$ , individual maximum  $x_{max}$ , individual minimum  $x_{min}$ , number of leaders  $PD$ , number of scouts  $SD$ , safety threshold  $ST$ , Cauchy distribution parameters  $\mu_\alpha$ ,  $\sigma_\alpha$ , objective function  $F$ ;
- 2: The initial population was generated in the boundary range according to Equation( 8) and Equation( 9). The population size was  $(ND)$ , and the fitness values of all individuals were calculated according to the objective function  $F$ ;
- 3: The calculated fitness values are sorted to obtain the best fitness value and the worst fitness value, and the corresponding best individual value  $X_{best}^g$  and the worst individual value  $X_{worst}^g$  are labeled;
- 4: The lower triangular matrix  $A = [A_{ij}]_{((N-PD)*D)}$ , all elements above the diagonal are 0, and all elements below the diagonal are 1;
- 5: **for**  $g = 1$  to  $G$  **do**
- 6:     According to Eq (10),  $\alpha$  factor is obtained;
- 7:      $R = rand(1)$ ;
- 8:     **for**  $i = 1$  to  $PD$  **do**
- 9:         Use Equation( 4) to update the position of the leader;
- 10:     **end for**
- 11:     The random arrangement of the row elements of matrix  $A$ ;
- 12:     The random arrangement of the row vectors of matrix  $A$  (the row elements are unchanged);
- 13:     Generate synergy search matrix  $A$  and  $\bar{A}$ ;
- 14:     **for**  $i = (PD + 1)$  to  $N$  **do**
- 15:          $X_{r1} \neq X_i$ ,  $X_{r2} \neq X_i$  and  $X_{r1} \neq X_{r2}$  were randomly selected from the current adjoint.
- 16:          $B_i = X_p^g + F \cdot (X_{r1} - X_{r2})$ ;

17: Update the position of the companion according to Equation (13)  
 18: **end for**  
 19: **for**  $i = 1$  to  $SD$  **do**  
 20: Use Equation (6) to update the position of the investigator;  
 21: **end for**  
 22: The fitness value of the new individual was calculated according to the objective function  $F$ ;  
 23: Compare the fitness values, update the fitness values of better locations and reorder the fitness of all, obtain the current best fitness value and worst fitness value, and re-label the corresponding best individual value  $X_{best}^g$  and worst individual value  $X_{worst}^g$ ;  
 24: According to Equation( 11),  $\mu_\alpha$  is updated;  
 25: **end for**  
 26: Output: global best individual  $X_{best}^g$  and global fitness value  $F(X_{best}^g)$ .

The mathematical calculation program block diagram of SSA-ACU is shown in Figure 4:

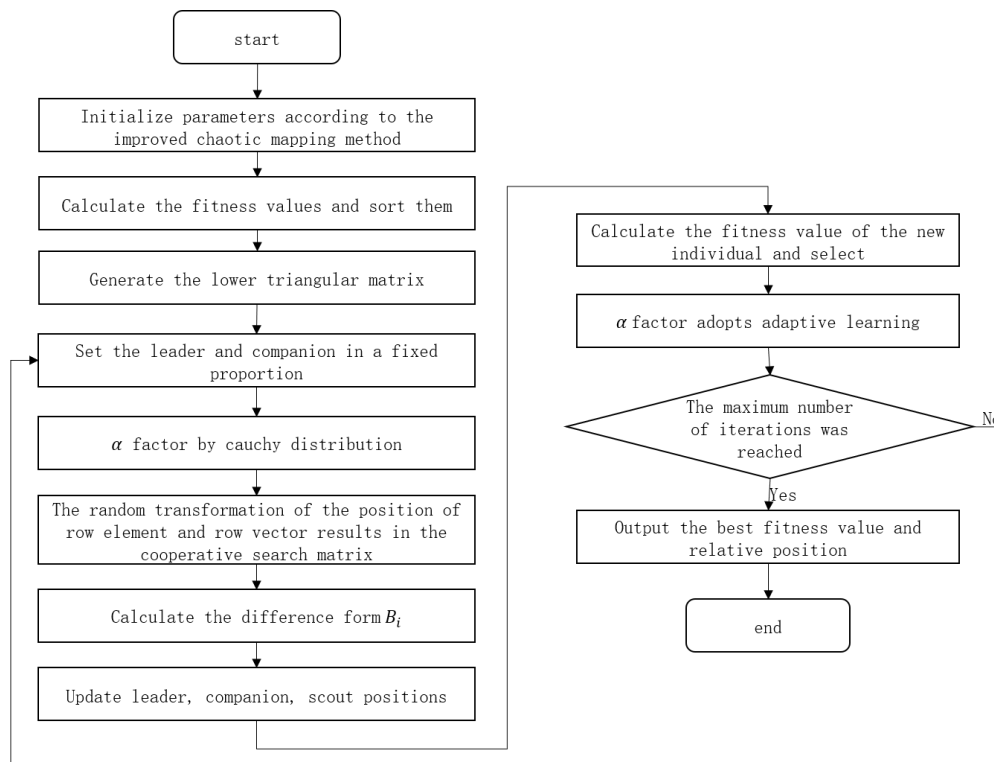


FIGURE 4. Schematic diagram of mathematical calculation model of SSA-ACU

**4. SSA-ACU Running Result Analysis.** In order to analyze the effectiveness and convergence of SSA-ACU on different types of functions, as well as its high efficiency and practicability in solving practical engineering problems. In this paper, the international general test set CEC2013 [26] is adopted to verify the test set. There are 28 test set functions in CEC2013, including three different types of functions: unimodal function  $f_1$ - $f_5$  and multimodal function  $f_6$ - $f_{20}$ . The combined function is  $f_{21}$ - $f_{28}$ . Each function defines a different function target value, and the purpose of the experiment is to find the target value of these functions using the algorithm.

By using the target value of the SSA-ACU correction function to analyze the performance of changes, the accuracy of changes determines the gap between the optimal

competitive value of the SSA-ACU algorithm and the calculated target value. When the error value is low, it means that the algorithm is closer to finding the optimal solution, and the algorithm has better collaborative performance under this function. The greater the error value is, the better the performance is. It means that there is a big gap between the fitness value calculated under the algorithm and the optimal solution, and the ability to find the target value of the task is worse.

In order to further show the advantages of SSA-ACU Algorithm, we combine SSA-ACU algorithm with the original Sparrow Search algorithm, Firefly Sparrow Search Algorithm (FSSA), Sparrow Algorithm Based on Logistic Chaos Mapping (LCSSA), The Whale Optimization Algorithm (WOA) [27], Particle Swarm Optimization (PSO) [28] and other famous swarm intelligence algorithms. When the parameter Settings of each algorithm are different, there is a great difference between different test functions. In order to balance the error of experimental results, the parameter Settings of each algorithm are designated as the original value without any change.

All algorithms were carried out on the simulation platform MATLAB 2021a, the hardware was configured as window 10 system, 16G installed memory, the population value  $N$  was set to 100, dimension  $D$  was set to 30. The maximum number of iterations  $G$  is set to 3000, the minimum  $x_{min}$  is set to -100 and the maximum  $x_{max}$  is set to 100. As each algorithm is allowed to have sometimes good results and sometimes poor results in the optimization process, in order to ensure that the experimental results are more convincing, the optimization results of 51 separate runs in the experiment process are selected as the data comparison. The experimental results can be seen from Table 1, the experimental result is the convergence accuracy between the fitness value and the target value of the global optimal solution obtained by each algorithm; mean represents the average value of the results of 51 runs; std represents the variance of the results of 51 runs; "+" means that the results of the SSA-ACU algorithm are better than those of other algorithms; "-" means that the comparison of experimental results of the SSA-ACU algorithm is worse than those of other algorithms; "=" means that the results of the SSA-ACU algorithm are little different from those of other algorithms.

It can be seen from Table 1 that when comparing the mean value of SSA-ACU with other algorithms, it has considerable advantages in unimodal function, multimodal function and combination function. The variance is slightly worse in  $f_{18}$  function and LCSSA algorithm and PSO algorithm, as well as in  $f_{20}$  function and LCSSA algorithm and WOA algorithm, that is, the data stability is less stable than the pair. However, the algorithm proposed in this paper is far better than other algorithms in the ability to find the global optimal solution. Among the 28 test functions, 25 reference functions of SSA-ACU are better than SSA, 24 reference functions are better than FSSA, 26 reference functions are better than LCSSA, 27 reference functions are better than WOA and 27 reference functions are better than PSO. For  $f_8$ , the results of SSA-ACU are similar to those of other algorithms.

TABLE 1. The convergence data (mean value and variance) of each algorithm are analyzed and compared

$f$	SSA-ACU Mean/std	SSA Mean/std	FSSA Mean/std	LCSSA Mean/std	WOA Mean/std	PSO Mean/std
$f_1$	1.83E+02/ 3.96E+02	4.15E+02/ 4.60E+02(+)	9.45E+02/ 7.01E+02(+)	4.46E+02/ 8.28E+02(+)	4.84E+02/ 5.27E+02(+)	3.08E+03/ 4.76E+02(+)
$f_2$	6.49E+06/ 3.96E+02	1.66E+07/ 2.38E+07(+)	2.12E+07/ 2.80E+07(+)	1.82E+07/ 2.23E+07(+)	5.48E+07/ 2.29E+07(+)	4.10E+07/ 1.45E+07(+)
$f_3$	3.24E+15/ 8.62E+17	1.37E+16/ 2.56E+19(+)	7.09E+16/ 3.54E+18(+)	4.42E+16/ 6.25E+18(+)	3.70E+17/ 5.81E+19(+)	2.14E+17/ 3.12E+18(+)
$f_4$	9.64E+03/ 9.75E+03	4.28E+04/ 1.39E+03(+)	6.37E+03/ 1.24E+03(-)	3.74E+04/ 1.64E+03(+)	6.00E+04/ 2.40E+04(+)	3.68E+04/ 2.67E+04(+)
$f_5$	5.93E+01/ 7.27E+17	1.17E+02/ 2.78E+18(+)	3.16E+02/ 1.13E+19(+)	1.39E+02/ 4.73E+18(+)	3.26E+02/ 6.62E+19(+)	8.91E+02/ 5.08E+19(+)
$f_6$	7.38E+01/ 1.06E+02	9.72E+01/ 1.17E+02(+)	1.75E+02/ 2.62E+02(+)	1.01E+02/ 2.62E+02(+)	1.90E+02/ 1.60E+02(+)	3.23E+02/ 1.72E+02(+)
$f_7$	1.48E+03/ 1.08E+05	3.54E+03/ 3.74E+05(+)	9.80E+04/ 9.58E+04(+)	3.40E+03/ 1.88E+05(+)	8.14E+03/ 3.62E+05(+)	7.76E+03/ 5.13E+05(+)
$f_8$	2.10E+01/ 1.66E-02	2.10E+01/ 2.11E-02(=)	2.10E+01/ 2.27E-02(=)	2.09E+01/ 3.30E-02(=)	2.10E+01/ 1.98E-02(=)	2.10E+01/ 1.88E-02(=)
$f_9$	2.83E+01/ 3.24E-02	3.77E+01/ 4.19E-01(+)	4.16E+01/ 6.74E-02(+)	3.69E+01/ 2.94E-01(+)	3.37E+01/ 1.42E+00(+)	3.00E+01/ 1.59E-01(+)
$f_{10}$	3.59E+01/ 5.81E+01	6.68E+01/ 6.06E+01(+)	1.60E+02/ 8.20E+01(+)	7.90E+01/ 1.51E+02(+)	1.78E+02/ 1.25E+02(+)	6.08E+02/ 9.37E+01(+)
$f_{11}$	1.02E+02/ 1.26E+01	2.50E+02/ 1.25E+01(+)	6.39E+02/ 3.44E+00(+)	2.59E+02/ 1.27E+01(+)	5.08E+02/ 1.63E+01(+)	3.24E+02/ 6.60E+00(+)
$f_{12}$	1.73E+02/ 5.73E+00	5.09E+02/ 1.80E+01(+)	6.89E+02/ 5.27E+00(+)	5.15E+02/ 2.39E+01(+)	5.27E+02/ 1.01E+01(+)	3.47E+02/ 4.06E+00(+)
$f_{13}$	2.53E+02/ 6.77E+00	4.56E+02/ 2.66E+01(+)	7.81E+02/ 6.11E+00(+)	4.72E+02/ 3.11E+01(+)	5.43E+02/ 2.08E+01(+)	4.07E+02/ 6.95E+00(+)
$f_{14}$	1.82E+03/ 6.28E+01	3.06E+03/ 7.40E+01(+)	4.63E+03/ 1.02E+02(+)	3.15E+03/ 6.42E+01(+)	4.85E+03/ 1.10E+02(+)	4.55E+03/ 8.80E+01(+)
$f_{15}$	4.25E+03/ 1.27E+02	4.87E+03/ 9.82E+01(+)	4.85E+03/ 7.42E+01(+)	5.08E+03/ 9.99E+01(+)	5.81E+03/ 1.73E+02(+)	5.03E+03/ 9.55E+01(+)
$f_{16}$	1.59E+00/ 2.10E-01	1.25E+00/ 1.25E-01(-)	1.25E+00/ 1.45E-01(-)	1.98E+00/ 2.18E-01(+)	1.88E+00/ 1.28E-01(+)	1.62E+00/ 9.39E-02(-)
$f_{17}$	1.61E+02/ 1.31E+01	6.65E+02/ 3.73E+01(+)	7.88E+02/ 3.57E+00(+)	6.38E+02/ 3.69E+01(+)	5.94E+02/ 1.59E+01(+)	4.67E+02/ 7.20E+00(+)
$f_{18}$	2.13E+02/ 7.91E+00	6.82E+02/ 9.17E+00(+)	7.76E+02/ 8.79E+00(+)	7.28E+02/ 2.18E+00(+)	5.96E+02/ 1.14E+01(+)	4.38E+02/ 6.86E+00(+)
$f_{19}$	3.98E+03/ 7.31E+04	2.08E+03/ 4.74E+03(-)	3.81E+03/ 4.51E+03(-)	2.22E+03/ 4.91E+03(-)	9.59E+03/ 1.28E+05(+)	5.83E+03/ 8.65E+04(+)
$f_{20}$	1.25E+01/ 1.60E-01	1.47E+01/ 2.14E-01(+)	1.49E+01/ 2.34E-01(+)	1.48E+01/ 7.10E-03(+)	1.48E+01/ 7.50E-02(+)	1.38E+01/ 1.89E-01(+)
$f_{21}$	3.21E+02/ 2.58E+01	4.27E+02/ 2.99E+01(+)	4.81E+02/ 3.80E+01(+)	4.07E+02/ 7.20E+01(+)	4.24E+02/ 4.86E+01(+)	1.32E+03/ 4.81E+01(+)
$f_{22}$	2.46E+03/ 7.89E+01	3.60E+03/ 1.81E+02(+)	5.92E+03/ 6.02E+01(+)	3.57E+03/ 1.33E+02(+)	6.39E+03/ 1.15E+02(+)	5.39E+03/ 8.17E+01(+)
$f_{23}$	4.85E+03/ 1.61E+02	5.79E+03/ 8.36E+01(+)	6.19E+03/ 6.97E+01(+)	6.17E+03/ 1.03E+02(+)	6.55E+03/ 1.40E+02(+)	5.73E+03/ 8.59E+01(+)
$f_{24}$	2.86E+02/ 8.44E-01	3.06E+02/ 1.25E+00(+)	3.35E+02/ 1.93E+00(+)	3.04E+02/ 2.25E+00(+)	3.11E+02/ 1.88E+00(+)	3.02E+02/ 2.02E+00(+)
$f_{25}$	3.02E+02/ 4.48E-01	3.19E+02/ 1.10E+00(+)	3.20E+02/ 1.82E+00(+)	3.17E+02/ 6.93E-01(+)	3.20E+02/ 1.85E+00(+)	3.09E+02/ 1.77E+00(+)

Figure 5, Figure 6, Figure 7 and Figure 8 show the convergence curves of SSA-ACU algorithm and other algorithms. It can be seen from the figure that the convergence accuracy of SSA-ACU algorithm is slightly better than that of other functions on the function of unimodal state. In the function of  $f_1 - f_5$ , although the convergence accuracy is not much different from that of other algorithms, the global optimal solution is found earlier than them, indicating that the convergence speed is better than other algorithms. The convergence curve of SSA-ACU algorithm is obviously better than that of other algorithms, and the convergence curve of SSA-ACu algorithm has a significant decline, especially for the functions  $f_9, f_{11}, f_{14}$  and  $f_{20}$ . Among the combinatorial functions, the convergence curves of  $f_{22}$  and  $f_{27}$  also have a large decline, and the convergence accuracy of other functions has a small increase, which indicates that the SSA-ACU algorithm can find the target value better when solving complex problems. Through the comparison of convergence curves, it can be found that SSA-ACU algorithm has better performance in dealing with high dimensional and multi-form problems.

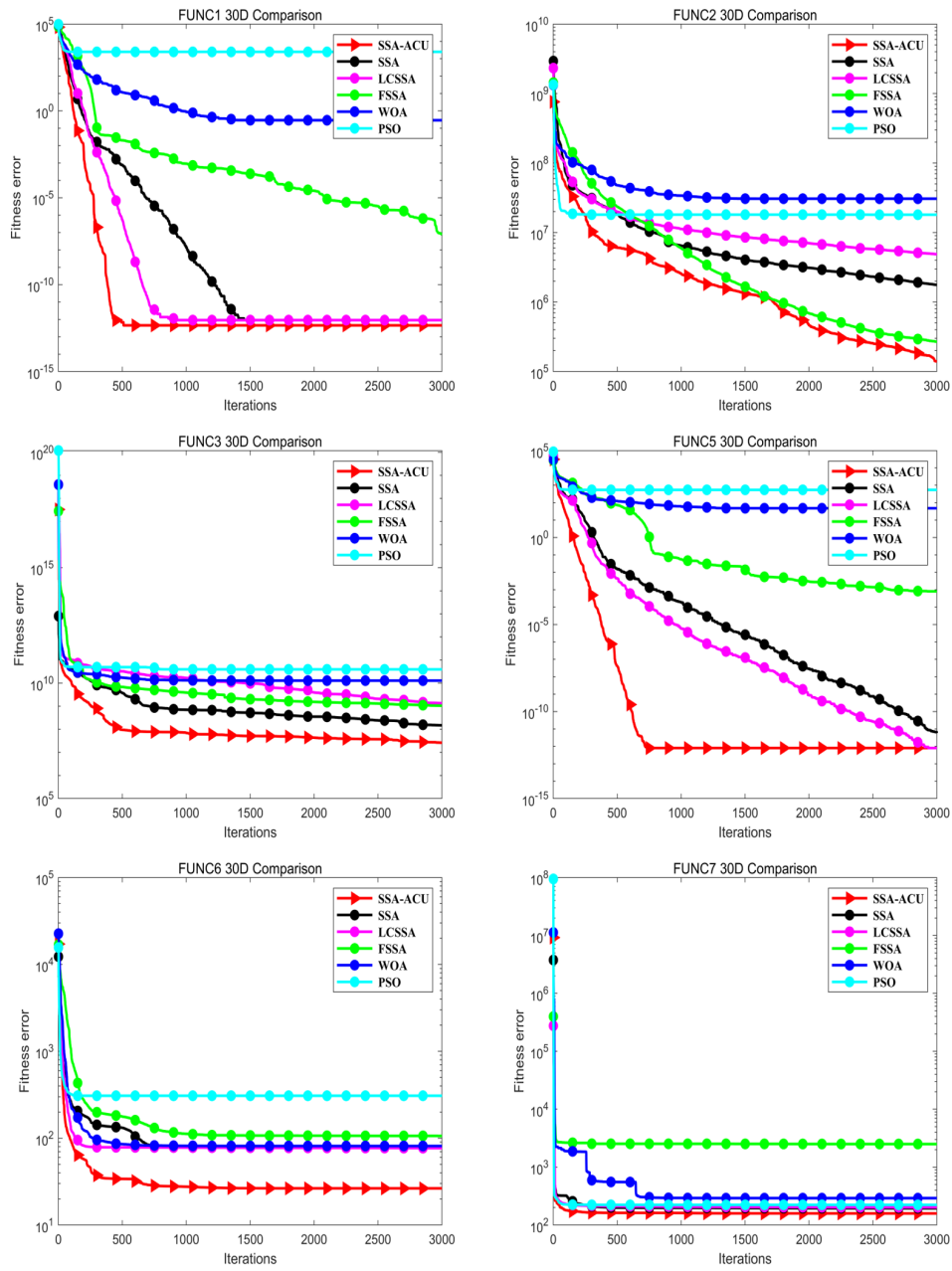
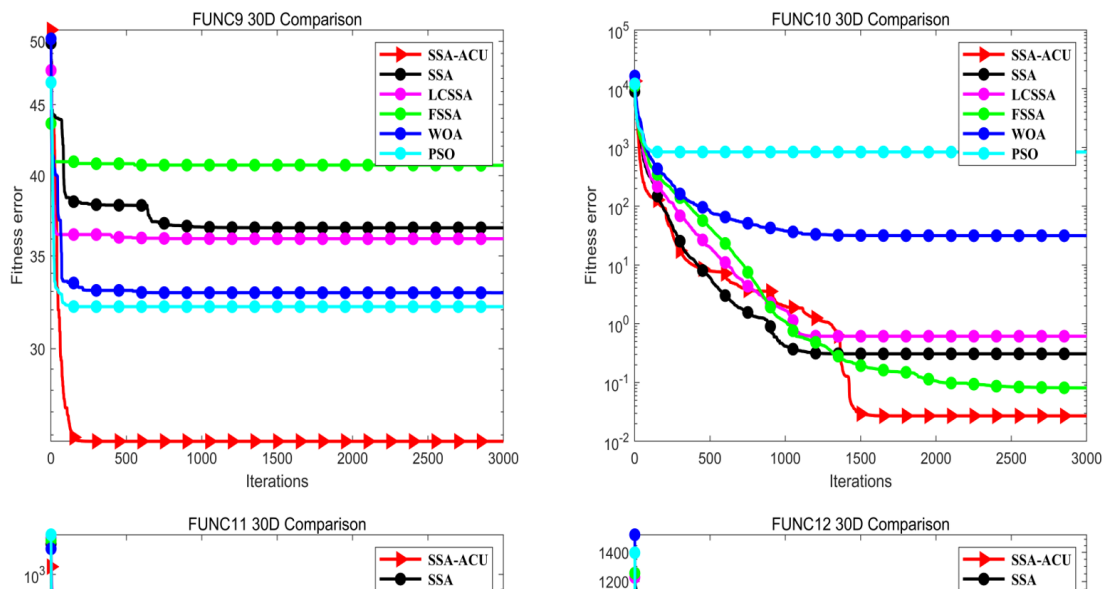


FIGURE 5. The convergence curves of SSA-ACU algorithm and other algorithms in CEC2013 test set with dimension of 30D (a)



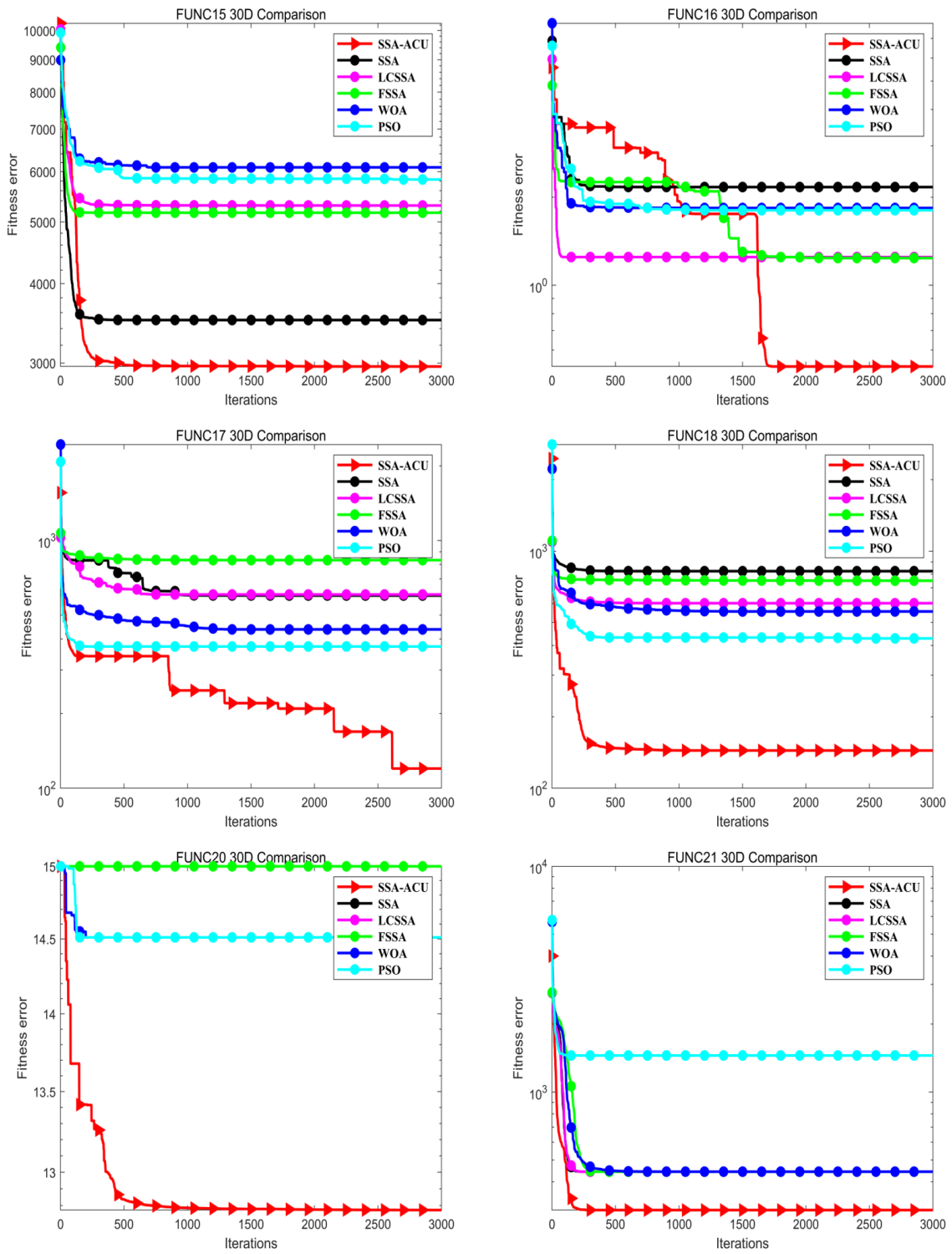


FIGURE 7. The convergence curves of SSA-ACU algorithm and other algorithms in CEC2013 test set with dimension of 30D (c)



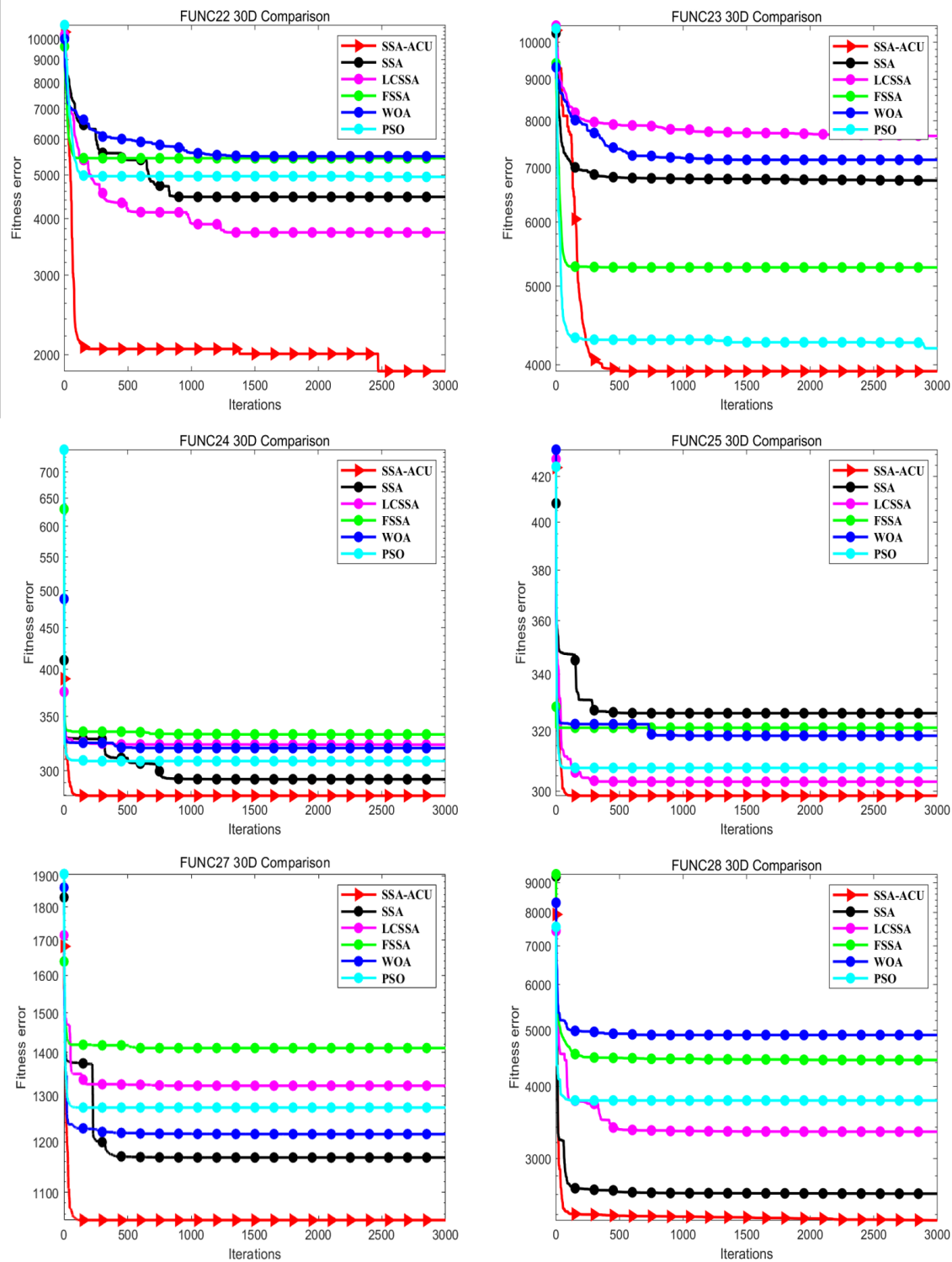


FIGURE 8. The convergence curves of SSA-ACU algorithm and other algorithms in CEC2013 test set with dimension of 30D (d)

5. **Conclusions.** This paper first introduces the characteristics of the standard SSA algorithm, through the division of labor of the leader, companion and scout to complete the sparrow’s search for food and reconnaissance behavior, and describes the basic principle and optimization process of the SSA algorithm in detail through the mathematical model and flow chart. The original sparrow algorithm is prone to fall into local optimality in

the search process. Based on this, this paper proposes an adaptive co-updating sparrow search algorithm, namely the SSA-ACU algorithm.

The SSA-ACU algorithm initializes the population by introducing an improved Logistics chaotic mapping to increase the ergodicity of the initial value, introduces an adaptive learning update mode for the leader's strategy, and uses a collaborative update strategy for the accompanying position update mode. At the end of the paper, the SSA-ACU algorithm is tested on CEC2013 test set function, and compared with the standard SSA algorithm, LCSSA algorithm, FSSA algorithm, WOA algorithm and PSO algorithm. The experimental results show that the new SSA-ACU algorithm is compared with other algorithms in terms of convergence speed and convergence accuracy. Both have strong competition.

The SSA-ACU algorithm presented in this paper enhances the development ability of SSA algorithm and makes it perform well in optimization problems. However, in CEC2013 function test set, the algorithm presented in this paper does not reflect well on some classical test functions and does not achieve the goal of optimization. So the next step is to further study and improve the sparrow search algorithm.

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