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Piecewise Mapping and Partitioned Search Multi-Objective Firefly Algorithm for Optimal Reservoir Scheduling

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Abstract. *Multi-objective firefly algorithm for random initialization and in the iterative process to guide a single strategy, very easy to fall into the local extreme value point, greatly weakening the algorithm optimization performance. For the purpose of address this, this paper proposes a refined search multi-objective firefly algorithm combining piecewise mapping: in order to improve the uneven distribution of the initial population, Piecewise mapping is used to improve the degree of dispersion of the population, resulting in a better traversal of the initial population; according to the local conditions and rational use of the advantages of different individuals, the introduction of the convergence evaluation index will be divided into the population in accordance with the differences in convergence Development area and exploration area, and in order to maximize the use of population information, optimize their respective learning strategies. The cosine perturbation strategy is introduced in the position updating stage, which utilizes the periodicity feature of the cosine function to motivate the firefly individuals to carry out refined exploitation in the finite neighborhoods of the superior individuals in order to improve the convergence accuracy. In the experimental part, the algorithm of this paper is compared with some multi-objective optimization algorithms on nine benchmark test functions, and a multi-objective reservoir optimization case is selected for optimization analysis. The results show that MOFA-P has obvious superiority over several other algorithms in improving the convergence and distribution of populations.*
Keywords: Firefly algorithm, Multi-objective optimization, Piecewise map, Regional division, Cosine disturbance, Optimized scheduling of reservoirs

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1. **Introduction.** Multi-objective Optimization Problem (MOP) [1–3], as a pervasive decision dilemma, is widely penetrated in daily life and various engineering applications. Its core feature is that it involves multiple independent and often conflicting objective functions, and the relationship between these sub-objectives is usually expressed as a trade-off: the optimization improvement of one objective is likely to be accompanied by the compromised performance degradation of other objectives. This characteristic makes it impossible to simply maximize or minimize a single objective when dealing with MOPs, but rather, it is necessary to seek an ideal set of solutions that can accommodate all objectives.

This ideal set of solutions is defined as the Pareto optimal solution set [4], and its members possess two key properties: mutual non-dominance and equal quality. Mutual non-dominance implies that for any two solutions within the set, there exists no solution that can simultaneously outperform the other in all objectives, while equal quality means that these solutions are at the same level of overall performance and cannot be further improved by unilaterally sacrificing one objective to another. In other words, the Pareto optimal solution set contains all possible solutions that cannot be further optimized to improve all objectives at the same time under the current constraints of the problem, thus providing decision-makers with a comprehensive set of alternatives to choose from based on actual preferences.

Multi-objective evolutionary algorithms (MOEA) [5], as an effective tool for solving MOPs, have a series of significant advantages. Firstly, MOEA is highly inclusive of the mathematical nature of the problem to be solved and does not need to assume the continuity and microscopic of the objective function or the linearity and convexity of the problem structure, thus it can flexibly cope with complex optimization scenarios, including discontinuous, multimodal, and non-convex. Secondly, MOEA constructs and iteratively updates a population containing multiple individuals (solutions) by simulating the mechanisms of inheritance, mutation, and selection in the natural evolutionary process, and can explore multiple regions of the solution space of the problem at the same time in a single run, thus efficiently approximating the Pareto frontiers. In addition, MOEA has the ability of self-adaptation, which can dynamically adjust its internal strategy to adapt to the problem characteristics and search progress during the search process, further improving the algorithm's global convergence ability and the coverage accuracy of the Pareto frontiers. Of its low requirements on the problem structure, the ability to generate the Pareto frontiers at one time, and the characteristics of population search and self-adaptation [6–11], MOEA has become an ideal choice to cope with all kinds of complex multi-objective optimization problems in real life and engineering practice. Such as Wang et al. solves the optimal reservoir scheduling problem by improving particle swarm optimization algorithm [12]; Ding et al. optimises the LSTM model for flood prediction by improved whale optimization algorithm [13]; Wu et al. optimises the neural network model to solve the high-precision image classification problem by Adaptive Fick's Law Algorithm [14], etc.

Faced with the complex problem of optimal scheduling of multi-objective reservoirs, different researchers have proposed innovative methods and strategies. Considering the many advantages that multi-objective evolutionary algorithms (MOEA) show in solving multi-objective optimization problems (MOPs), such as population search, adaptivity, and the ability to generate Pareto frontiers at one time, the academic community is generally inclined to apply evolutionary algorithms to meet the challenges of multi-objective reservoir optimization. In recent years, with the deepening of research, the field of multi-objective evolutionary algorithms has made significant progress, and many MOEA models with excellent performance have been born. Such as: Multi-Objective Particle Swarm Optimization (MOPSO) [15], Non-dominated Sorting Genetic Algorithm III (NSGA-III) [16], Tumbleweed Optimization Algorithm (TOA) [17], Phasmatodea Population Evolution Algorithm (PPE) [18], etc. Among them, in 2008, researcher Yang creatively proposed the Firefly Algorithm (FA) [19] for solving single-objective optimization problems by drawing on the flickering communication phenomenon of firefly groups in nature and achieved good results. Given the successful application of FA in the field of single-objective optimization, researchers further extended it to the Multi-objective Firefly Algorithm (MOFA) [20] to cope with multi-objective optimization problems. MOFA guides the flight trajectories of individual fireflies by comparing the Pareto dominance relationship between individual objective values, i.e., determining its search direction. The algorithm is favored for its streamlined parameter setting, clear structure, and easy implementation, but certain limitations are also exposed in practical applications, such as in the process of searching for the optimal easily leads to the population falling into the local optimal solution, which affects the diversity of the distribution of the population and the global convergence performance. Although there are numerous improvement works for the firefly algorithm, such as Hybrid Multi-Objective Firefly Algorithm (HMOFA) [21], multi-objective firefly algorithm based on compensation factor and elite learning (CFMOFA) [22], etc., regrettably, the applications and explorations of these improved versions on the multi-objective reservoir optimal scheduling problem are still rare and need to be further studied and verified.

Although the above algorithm improves the optimization performance of MOFA to some extent, the diversity of the population still needs to be improved, and the exploration and exploitation ability of individual fireflies still needs to be further strengthened. In view of the above, based on MOFA, this paper proposes a multi-objective firefly algorithm combining Piecewise mapping and partition development (MOFA-P). MOFA-P mainly optimizes MOFA in the following three aspects:

1) Piecewise mapping: In traditional MOFA, the generation of initial populations usually relies on randomization methods. Although this method is simple and easy to implement, due to the influence of random factors, the generated initial populations are often not uniformly distributed in the decision space, which may lead to some regions being too dense while other regions are relatively sparse, which is not conducive to the effective search of the populations in the global range. To overcome this shortcoming, this study introduces the Piecewise map strategy, which aims to significantly improve the dispersion of the population by systematically adjusting the distribution of individual populations in the decision space, thus enhancing the diversity and coverage of the population search. Specifically, Piecewise mapping is a method that divides the decision space into multiple subregions and designs specific mapping rules for each subregion. When generating the initial population, the population individuals are distributed into different subregions based on these mapping rules, ensuring that a certain number of individuals participate in the search in each subregion. This design helps to eliminate the phenomenon of local densities that may be caused by random initialization and promote the formation of a more uniform and extensive distribution pattern of the population in the decision space. In addition, by flexibly adjusting the mapping rules, the population can also be guided to give priority to specific subregions, adapting to the characteristics and needs of the optimization problem, and further improving the relevance and efficiency of the search.

2) Partitioned search. According to the convergence evaluation index, the population is divided into the development area and exploration area to perform their respective search tasks, and the two populations divide the work to improve the overall performance of MOFA. First, some members of the population, defined as "development zones", have the main task of fine-tuning the development of the known Pareto front. These individuals are strongly guided by the convergent global optimum particles and can quickly adjust their state to precisely approach the Pareto front. This mechanism ensures that the firefly individuals in the development zone can efficiently converge to the neighborhood of the existing optimal solutions, which significantly strengthens the overall convergence speed and accuracy of the MOFA algorithm, allowing the algorithm to approximate the Pareto-optimal set of solutions to the problem in a shorter period.

Meanwhile, the "exploration zone" consists of individuals that show high potential in the convergence evaluation, and their task is to continue exploring under-explored regions in the decision space to discover new potential Pareto frontiers. Given that these individuals already have some convergence advantage, we give them a dual guidance: on the one hand, we utilize the diverse global optimal individuals to stimulate their wide-area exploration in the decision space, encouraging them to go beyond the limitations of the currently known frontiers; on the other hand, we maintain the connection with the convergent global optimal individuals to ensure that they do not lose the track of the existing optimal solution set while exploring the new areas. In this way, individuals in the exploration area can search deeply in the fringes of the Pareto frontier and beyond, strongly enhancing the distributivity of the MOFA algorithm, i.e., its ability to find and cover a wide range of non-inferior solutions on a global scale. By dividing the population into development areas and exploration areas, and carefully designing the search strategies for each of them, MOFA can strike a good balance between convergence and distributivity, which ensures the fast approximation of the known Pareto front and maintains the active search for potential optimal solutions, thus significantly improving the overall performance and solution quality of the algorithm.

3) Cosine Perturbation. In the original MOFA algorithm, a randomized perturbation mechanism is used in the position update session, aiming to facilitate the exploration of the population in the decision space by introducing randomness. However, this randomized perturbation approach has certain limitations in actual operation: excessive randomness may lead to some regions being neglected during the search process, i.e., some potential high-quality solution regions may be under-examined due to the uncertainty of individual random movements. This undoubtedly reduces the search efficiency and solution quality of the algorithm, especially at the late stage of convergence, where excessive random perturbations may lead to the population repeatedly hovering around known solutions without being able to effectively mine the under-explored regions. To address this problem, we introduce the cosine perturbation strategy. This strategy skillfully utilizes the periodic and oscillatory properties of the cosine function to finely regulate the position update method of firefly individuals. Specifically, when an individual performs position updating, it no longer relies solely on random perturbation, but moves regularly and oriented in its neighborhood based on the cosine function. This movement pattern not only retains a moderate degree of explorability to ensure that individuals do not fall into local optimums prematurely, but also forms an organized and hierarchical local search around the more optimal individuals, thus realizing the fine-grained exploitation of a finite neighborhood.

The advantage of the cosine perturbation strategy is that it prompts firefly individuals to search cyclically in a specific region neighboring a better solution, adjusting the moving direction and step size according to the law of the cosine function at each iteration to ensure that this finite neighborhood is fully and orderly explored. In this way, even at boundary regions or complex terrains where random perturbations may lead to omissions, individuals can purposefully refine their search through cosine perturbations, which in turn increases the probability of converging to an exact Pareto frontier point. Therefore, compared with the original algorithm, MOFA-P with the cosine perturbation strategy exhibits stronger directionality and regularity during the position update process, which significantly improves the convergence accuracy and ensures that the algorithm can explore the decision space more comprehensively and meticulously during the iteration process, and ultimately obtains a high-quality solution set of Pareto fronts.

2. **Relevant basic knowledge.**

2.1. **multi-objective optimization problem (MOP).**

When dealing with continuous multi-objective optimization problems (MOPs), a mathematical model is usually constructed to describe and solve them accurately. This model is usually expressed in minimized form as follows:

$$\begin{cases} \min F\left(u\right) = \left(f_1\left(u\right), f_2\left(u\right), ..., f_m\left(u\right)\right)^T \\ u = \left(u_1, u_2, ..., u_n\right) \in \Omega \to R^n \\ F\left(u\right) = \left(f_1\left(u\right), f_2\left(u\right), ..., f_m\left(u\right)\right) \in \Omega \to R^m \end{cases} \tag{1}$$

Where, u is the decision vector in the -dimensional decision space and $F(u)$ is the goal vector in the -dimensional goal space.

2.2. **Multi-objective firefly algorithm (MOFA).** In the MOFA algorithm, the magnitude of the attraction between individual fireflies is closely related to the brightness level of the individuals themselves as well as the distance between them. Specifically, the attraction between individuals and their respective brightness levels show an obvious positive proportionality: the higher the brightness of a firefly, indicating that its fitness (i.e., the value of the objective function) in the current solution space is more optimal, so for other fireflies, the attractiveness of this high-brightness individual will be greater, and it is more likely to become the object of convergence and imitation of other individuals. The relationship between the attractiveness of individuals and their physical distance from each other is inversely proportional: the farther the distance between two fireflies, the weaker the attractiveness of each other; on the contrary, the closer the individuals, the stronger the attractiveness of each other. This inhibitory effect of distance on attraction reflects the need for fine-grained exploration of local neighborhoods during the search process, encouraging individual fireflies to preferentially approach their neighboring and better-adapted peers, thus realizing the efficient exploitation and optimization of the local solution space [23].The attraction of firefly j to i firefly is defined as follows:

$$\beta_{ij}\left(r_{ij}\right) = \beta_0 e^{-\gamma {r_{ij}}^2} \tag{2}$$

Where, β_0 is the maximum attraction, which corresponds to the attraction at $r = 0$, and is usually taken as $\beta_0 = 1$; γ is the light absorption coefficient, and is usually taken as $\gamma \in [0.01, 100]$; and r_{ij} is the Euclidean distance from firefly i to firefly j. The Euclidean distance from firefly j to firefly i is the Euclidean distance from firefly i to firefly j.

Using the concept of Pareto dominance to determine the attraction relationship between individual fireflies, if firefly i is used as the object of study, and if firefly $j \prec i$, indicating that j is the brighter firefly, then firefly i is attracted to the brighter j.

MOFA's firefly attraction moves in two ways:

If firefly i searches for the dominating solution firefly j , i.e., $j \prec i$, then the position of firefly i is updated to:

$$x_i(t+1) = x_i(t) + \beta_{ij}(r_{ij}) \cdot (x_j(t) - x_i(t)) + \alpha_t \cdot \varepsilon_i^t \tag{3}$$

Where, t is the current iteration number; x_i, x_j are the spatial positions of fireflies i and j, respectively; $\beta_{ij}\left(r_{ij}\right)$ is the attraction between fireflies i and j; α_t is a constant, generally taken as $\alpha \in [0, 1]$; ε_i^t is a vector of random numbers obtained from Gaussian distribution, uniform distribution or other distributions.

If firefly i does not find the dominant solution firefly j, then update the position of firefly i. The update formula for firefly i is:

$$x_i^{t+1} = g_*^t + \alpha_t \varepsilon_i^t \tag{4}$$

Where, g^* is the optimal value obtained by converting a plurality of objective functions into a single objective function in a random weighted sum.

2.3. **Fusion Ranking.** In this paper, a new strategy for subpopulation delineation based on differences in the degree of convergence of individual fireflies is proposed. To achieve this goal, the first task is to accurately quantify the degree of convergence of each firefly. To this end, we selected two recognized convergence evaluation metrics Average Ranking (AR) and Global Detriment (GD) and organically combined them to construct a new comprehensive measure of firefly convergence Fusion Ranking (FR), which is a new method for evaluating convergence of individual fireflies, and Fusion Ranking (FR), which is a new method for evaluating convergence of individual fireflies. Fusion Ranking (FR) [24–26].

The average ranking (AR) mainly reflects the relative position of a firefly in the current population concerning other individuals, i.e., its average ranking on each objective function. A higher average ranking implies that the firefly shows better performance on most of the objectives and has higher convergence. Conversely, a lower average ranking implies that the firefly has a significant gap with other individuals on at least some objectives and has relatively weak convergence.

Global Damage (GD), on the other hand, assesses the extent to which a firefly deviates from the current Pareto front from a global perspective. Specifically, it calculates how much the Pareto front will change if that firefly is removed while keeping the positions of other individuals constant. A smaller global damage value indicates that the firefly has a better fit to the existing Pareto frontier, i.e., its convergence is better; conversely, a larger global damage value implies that the firefly is likely to be located outside of the Pareto frontier and has poor convergence.

Combining the above two metrics, we constructed the fusion ranking (FR) as the final firefly convergence evaluation criterion. By reasonably integrating the local and global information provided by average ranking (AR) and global damage (GD), this index takes into account the relative merits of fireflies within the population as well as their match with the global Pareto front, thus providing a more comprehensive and accurate convergence assessment for each firefly. Based on the calculation results of the fusion ranking (FR), we classify the fireflies in the population into subpopulations with different convergence degrees, which in turn lays the foundation for subsequent differentiated search strategies and collaboration mechanisms.

Let the degree of convergence of the firefly i be $FR\left(x_i\right)$, and the result is given by the following Equation:

$$\begin{cases} AR\left(x_i\right) = \sum_{l=1}^{m} rank_l\left(x_i\right) \\ GD\left(x_i\right) = \sum_{i \neq j} \sum_{l=1}^{m} \max\left(f_l\left(x_i\right) - f_l\left(x_j\right), 0\right) \end{cases} \tag{5}$$

Where, $AR\left(x_i\right)$ is the average ranking of firefly i, $GD\left(x_i\right)$ is the global damage of firefly i, m is the number of objective functions, $rank_l\left(x_i\right)$ is the ranking of firefly i on the lth objective function, $f_l\left(x_i\right)$ is the adaptation value of firefly i on the lth objective function, and the values of other parameters can be referred to the literature [24].

In the multi-objective optimization problem, when using the AR (Average Ranking) index alone as the criterion for preferring firefly individuals, it is easy to lead to the concentration of the selected excellent individuals in the boundary region of the Pareto front, resulting in the over-concentration of the population, which is not conducive to the full exploration of the inner region of the front. Meanwhile, relying only on the GD (Global Detriment) metrics may lead to an underestimation of the value of important particles at the two ends of the Pareto front, which appear to be far away from the center region but may occupy an irreplaceable position in the actual decision-making. Given the limitations of the above two evaluation indexes when applied individually, the AR and GD indexes are organically combined to design a comprehensive evaluation approach to compensate for their respective shortcomings and achieve the optimization of the evaluation of individual convergence of fireflies. The resulting improved evaluation system can not only prevent the search inadequacy caused by excessive bias towards the edge of the Pareto front but also ensure that those key individuals located at the endpoints of the front are fully considered during the screening process, thus achieving a balanced and comprehensive optimization of the spatial distribution of the population search in the multi-objective optimization problem. The refined evaluation index is:

$$FR(x_i) = \Psi_1 AR(x_i) + \Psi_2 GD(x_i) \tag{6}$$

where, in this paper, we set Ψ_1=0.4, Ψ_2=0.6.

3. MOFA-P.

3.1. **Piecewise map.** The concept of chaos is usually used to describe the special form of motion in nature. In recent years, different scholars have tried to apply chaotic sequences to different phases of population iteration in different ways in swarm intelligence optimization algorithms, and according to the relevant research, it can be learned that chaotic sequences do play different degrees of optimization in the process of population evolution.

In the MOFA algorithm, the traditional approach is to generate the starting population set with the help of a randomization mechanism, but the inherent randomness factor of this approach inevitably brings about the problem of uneven distribution of the initial population. Due to the effect of random error, the individuals in the initial population may be more concentrated in some local regions of the decision space, while appearing sparse or even blank in other regions, which restricts the population's ability to search efficiently in the global range, thus weakening the algorithm's efficiency in fully utilizing the resources of the initial population to a certain extent [27]. Considering that the Pareto frontier distribution of a multi-objective optimization problem usually presents a complex shape covering a wide range of combinations of non-inferior solutions, it is crucial to take measures in the algorithm start-up phase, i.e., the initialization step, to ensure that the initial population can be dispersed as uniformly as possible within the entire feasible solution space. This uniformly distributed initial layout helps to allow the algorithm to reach and gradually approximate the true Pareto front more quickly in the subsequent optimization process, thus enhancing the effectiveness of the global optimization and the quality of the final solution set. In this way, the algorithm not only effectively avoids potential optimization blind spots caused by the uneven distribution of the initial population, but also ensures that various possible combinations of optimal solutions can be comprehensively and effectively explored in the early iteration stage.

The optimization result of the population in the firefly algorithm is limited to some extent by the initial distribution of the population, A Piecewise map as a kind of segmented linear mapping, which sets different transformation formulas for the initial values distributed in different positions. The specific mathematical expression is:

$$u_{n+1} = \begin{cases} \frac{u_n}{d}, 0 \leq u_n < d \\ \frac{u_n - d}{0.5 - d}, d \leq u_n < 0.5 \\ \frac{1 - d - u_n}{0.5 - d}, 0.5 \leq u_n < 1 - d \\ \frac{1 - u_n}{d}, 1 - d \leq u_n < 1 \end{cases} \tag{7}$$

Where, d is a control parameter used to determine the distribution range of the four segments so that there is no overlap between them. Experiments have proved that the best effect is when $d = 0.4$ or so, so in the experiments of this paper, d are taken to be set as a random value between $0.35 \sim 0.45$.

3.2. **Partition Search.** The population was divided into development and exploration areas based on the convergence index expressed in Equation (8):

$$x_i \in \begin{cases} \text{development zone} \quad , \quad FR(x_i) \geq 0.4N, \\ \text{exploration zone} \quad , \quad otherwise. \end{cases} \tag{8}$$

Where, $FR(x_i)$ denotes the value of the convergence degree of the firefly i; $MinFR$ denotes the minimum value of the convergence degree of the firefly.

The individual fireflies observed in the current development zone have relatively insufficient convergence performance in their optimization search process, and there exists a significant need for improvement to enhance the effectiveness of the development and optimization of firefly intelligence in this region. Given this, we propose to adopt the fireflies that show the most excellent convergence ability in the region as a reference and guide to guide the movement and evolution strategy of all particles in the region, aiming to significantly improve the speed of the algorithm to converge to the optimal solution and the accuracy of the localization, to vigorously promote the comprehensive performance of the entire algorithm system. If the firefly i is assigned to the development zone, then to realize this dynamic adjustment and optimization process, its position update formula is as follows:

$$x_i(t+1) = x_i(t) + \xi_1 \beta_{ic^*}(c^* - x_i(t)) + \xi_2 \beta_{ij}(x_j(t) - x_i(t)) + a \cdot \cos \tag{9}$$

$$x_i(t+1) = \zeta_1 c^* + \zeta_2 e^* + a \cdot \cos \tag{10}$$

Where, j is any firefly in the population, d^* denotes the firefly with the best diversity in the population, which is calculated based on the crowding distance, i.e., the firefly with the largest crowding distance; e^* is the archive elite solution, which is a randomly screened nondominated solution from the EA, $\xi_1, \xi_2, \zeta_1, \zeta_2 \in [0, 1]$, satisfies $\sum_{i=1}^{2} \xi_i = 1$, $\sum_{i=1}^{2} \zeta_i = 1$, and c^* denotes the firefly with the best convergence in the population, which is calculated based on the FR, i.e., the firefly with the smallest FR, and cos is the cosine perturbation term.

Fireflies in the exploration area showed a relative advantage in convergence performance, which implies that they have high focusing ability and good solution convergence tendency in the process of searching for globally optimal solutions. To further increase the likelihood that the entire population will effectively explore and approach the ideal solution, an effective strategy is to select not only the fireflies with the best convergence performance but also fireflies with a high diversity of particles, which will jointly participate in navigating and inspiring other particles in the exploration area. This dual-guidance mechanism aims to maximize the exploration range of fireflies and encourage them to dig deeper into spatial dimensions that may have been overlooked in the past and are rich in potentially good solutions. Assuming that the fireflies i are individuals in the exploration area, their learning behaviors and position update logic in the area are assigned with corresponding learning formulas, which define how the fireflies iteratively update themselves and explore the space based on the information of the convergence-optimal and diversity-optimized individuals, as defined below:

$$x_i(t+1) = x_i(t) + \lambda_1 \beta_{id^*}(d^* - x_i(t)) + \lambda_2 \beta_{ij}(x_j(t) - x_i(t)) + \lambda_3 \beta_{ic^*}(c^* - x_i(t)) + a \cdot \cos \tag{11}$$

$$x_i(t+1) = \varphi_1 d^* + \varphi_2 e^* + \varphi_3 c^* + a \cdot \cos \tag{12}$$

Where, $\lambda_1, \lambda_2, \lambda_3, \varphi_1, \varphi_2, \varphi_3 \in [0,1]$, satisfies $\sum_{i=1}^{3} \lambda_i = 1$, $\sum_{i=1}^{3} \varphi_i = 1$; c^* denotes the firefly with the best convergence in the population, That is, FR's smallest firefly.

Based on the predefined regional division scheme, we will implement a mechanism of synergy between regional populations, i.e., the firefly individuals in different regions will be subdivided into dominant and nondominant individuals according to their respective performance characteristics. On this basis, adhering to the principle of survival of the fittest and elimination of the fittest, we scientifically and reasonably screen out the fireflies that are suitable for their respective evolutionary paths, so that they can carry out active and positive interactions and collaboration within the region, which will be conducive to the fireflies in the regions to learn from each other's advantageous features, forming complementary effects, thus effectively enhancing the optimization efficiency of the whole algorithm in the solving process. In addition, this mechanism of inter-regional interaction and cooperation also helps to expand the breadth and depth of the firefly search problem space, so that each firefly can have the opportunity to access a broader and more diverse search field, thus enhancing the population's ability to search for the optimal solution on a global scale, and ultimately achieving significant optimization and enhancement of the algorithm's performance. After partitioning the search for optimization whether in the development area or exploration area firefly individuals are in the corresponding optimal solution to move under the guidance of the optimal solution, for these optimal solutions, in these solutions near the neighborhood of the small space to carry out the refinement of the development is necessary, can greatly enhance the probability of finding the best solution to improve the optimization results. The cosine perturbation strategy is introduced in the iterative process, which uses the periodic oscillation characteristics of the cosine function to drive the algorithm to search in the small neighborhoods near the elite firefly individuals.

3.3. Algorithm time complexity analysis.

Let the population have N individuals, M objectives, and the dimension of the test function is n. MOFA adopts the all-attractive model, i.e., the algorithm adopts a two-layer loop to traverse all the individuals, so its time complexity is $O(N^2)$. MOFA-P adds the pockets mapping, partition search, and cosine perturbation based on MOFA. The above three strategies are the improvement of the original firefly individual optimization strategy in the original MOFA algorithm, and the time complexity of MOFA-P and MOFA is kept in the same order of magnitude. Although MOFA-P increases part of the computational amount, from the experimental data, we can see that MOFA-P can converge to the Pareto front at a very fast speed, and it has more practical value compared with MOFA.

4. Experiments and Analysis of Results.

4.1. Test Function.

The experimental data in this paper mainly refer to the literature [28], and the test functions selected for the experiment are ZDT, Viennet, and DTLZ totaling nine test functions to verify the comprehensive performance of MOFA-P. Each of the selected test functions possesses unique properties and difficulties, aiming to comprehensively test the adaptability, convergence, and optimization-seeking ability of the MOFA-P algorithm when facing different types of multi-objective optimization problems. To visualize the properties of these test functions, the key features of each test function are listed and described in detail in Table 1.

When solving complex optimization problems, this paper adopts the evaluation criterion based on the Inverted Generational Distance (abbreviated as IGD) proposed in the

Table 1. Multi-objective test function

Problem	Objective	Constraints	Characteristics
ZDT1	2	$n = 30, 0 \leq x_i \leq 1, i = 1, ..., n$	Convex
ZDT2	2	$n = 30, 0 \leq x_i \leq 1, i = 1, ..., n$	Concave
ZDT3	2	$n = 30, 0 \leq x_i \leq 1, i = 1, ..., n$	Discontinuous type
ZDT4	2	$n = 10, 0 \leq x_1 \leq 1, \text{-}5 \leq x_i \leq 5, i = 2, ..., n$	Convex + Multimodal
ZDT6	2	$n = 10, 0 \leq x_i \leq 1, i = 1, ..., n$	Concave + Multimodal + partial
Viennet1	3	$-2 \leq x, y \leq 2$	Convex
Viennet3	3	$-3 \leq x, y \leq 3$	Hybrid + Degradation
DTLZ4	3	$n = 12, 0 \leq x_i \leq 1, i = 1, ..., n$	Concave + partial
DTLZ7	3	$n = 12, 0 \leq x_i \leq 1, i = 1, ..., n$	Discontinuity + Mixed + Multimodal

literature [29, 30] as a way to systematically analyze and compare the performance and advantageous differences of different algorithms. IGD, as an important evaluation index, covers two core As an important evaluation metric, IGD covers two core considerations: the convergence of the solution set and the distributional characteristics of the solution set. Firstly, IGD reveals the convergence level of the solution set by examining the distance between the solution set generated during the iteration process of the algorithm and the theoretical optimal solution set; secondly, it also explores the dispersion of the solution points within the solution set, i.e., the distributive property, to ensure that the solution set not only converges to the global optimal region but also has a good diversity within the region. The specific value calculated based on the IGD can intuitively reflect the high quality of the solution set obtained by the algorithm search. When the value of IGD is small, it means that the solution set formed by the corresponding algorithm in the process of solving is closer to the ideal solution set, and the distribution state of the solutions in the solution set is more reasonable and uniform, which ultimately manifests the higher quality of the solution set realized by the algorithm.

4.2. **Comparison with classical multi-objective optimization algorithms.** To verify the performance of MOFA-P, five classical multi-objective optimization algorithms are selected to compare with MOFA-P. The comparison algorithms include MOPSO [15], NSGA-III [16], Multiobjective Evolutionary Algorithm Based on Decomposition (MOEA/D) [31], Pareto Envelope-based Selection Algorithm II (PESA-II) [32] and MOFA [20]. Based on the set of multi-objective test functions established in Section 4.1, a total of six algorithms, including MOFA-P, were thoroughly and systematically tested and empirically analyzed using the objective and authoritative evaluation index of inverse generation distance (IGD). To ensure the fairness and validity of the comparison experiment, all the algorithms involved in the comparison adopt uniform experimental configuration conditions: the population size is set to 50 individuals, the maximum number of iterations allowed is uniformly limited to 300, and the capacity of the external archive is fixed to accommodate 200 solutions.

All algorithms are implemented in Matlab version 2021a programming environment and executed under the same hardware conditions to exclude the influence of environmental variables. To reduce the chance of errors caused by random initialization and other factors, we ran all the algorithms independently for a total of 30 times for each test problem and took the average performance as the final experimental results of each algorithm on the problem. At the same time, it is ensured that all parameter configurations of each algorithm strictly follow the settings recommended or proved to be effective in the relevant literature, and these specific parameter configurations are detailed in Table 2.

The Mean and Std of the IGD performance evaluation metrics of MOFA-P and the five classical multi-objective optimization algorithms on the nine test problems are given in

Table 2. Parameter settings of algorithms

Algorithm	Parameter setting	Reference
MOPSO	$w = 0.4, c_1, c_2 = Rand[0, 1]$	Zhang et al. [15]
NSGA-III	$pCrossover = 0.5,$ $nCrossover = 2 * round\,[pCrossover * nPop/2]$	Deb and Jain [16]
MOEA/D	$\gamma = 0.5$	Zhang and Li [31]
PESA-II	$pCrossover = 0.5, \beta = 1, \gamma = 2,$ $nCrossover = 2 * round\,[pCrossover * nPop/2]$	Lin et al. [32]
MOFA	$\alpha = 0.2, \beta_0 = 1, \gamma = 1$	Yang [20]
MOFA-P	$\alpha = 0.2, \beta_0 = 1, \gamma = 1$	

Table 3. The last three rows of the table provide further macro-level data on the overall performance of the algorithms. Specifically, the following items are included: the cumulative number of optimal IGD values obtained by each algorithm on all nine test problems (Total), which intuitively reflects the relative advantages of each algorithm in the overall optimization task; with the help of Friedman's test, the rank-averaged optimization results of all the algorithms are calculated, which is helpful to identify whether there is a significant difference between algorithms from a statistical point of view; based on the above statistical analysis, the average ranking of each algorithm in the overall optimization performance is given, which intuitively presents its performance in this comparison experiment. Based on the above statistical analysis, the average ranking of each algorithm in the overall optimization performance is given, which visually presents its relative advantages and disadvantages in this comparison experiment. It is worth noting that the table highlights the optimal results of the MOFA-P algorithm in each test by shading. It should be noted that, except for the experimental data of the MOFA-P algorithm, which was obtained directly by this study, the rest of the comparison algorithms and their corresponding result data are cited from the literature [28] to ensure the comparability of the experiments and the continuity of the study.

Referring to the data provided in Table 3, an in-depth analysis from the perspective of the overall performance of the algorithms reveals that MOFA-P has successfully achieved the best optimization results on 6 out of the 9 test problems examined, demonstrating strong optimization strength. In contrast, the PESA-II algorithm achieved the best optimization results 2 times on the same problem set, while NSGA-III reached the optimum on one test problem. By taking a comprehensive look at the performance of these 6 comparison algorithms on the 9 test instances, it is easy to see that MOFA-P consistently maintains an unbeatable lead when faced with a 2-objective optimization problem, finding the optimal solution to the problem whenever possible. Further counting the number of times each algorithm achieves the optimal solution on all the test problems, it can be clearly concluded that MOFA-P significantly outperforms the other compared algorithms in terms of comprehensive performance. The emergence of this significant advantage strongly verifies the significant optimization effect of the adopted improvement strategies on the optimization search process of the MOFA-P algorithm, indicating that these strategies effectively enhance the search efficiency and accuracy of the algorithm, and enable it to show higher competitiveness in solving multi-objective optimization problems.

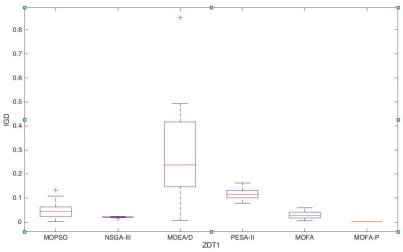
By analyzing the data obtained from the Friedman test, it can be observed that the rank average corresponding to the MOFA-P algorithm is the lowest among the six compared algorithms. This phenomenon has an important implication that directly indicates that the MOFA-P algorithm exhibits the most superior overall performance among all the

TABLE 3. Experimental results of MOFA-P with five classical MOEAs on IGDs

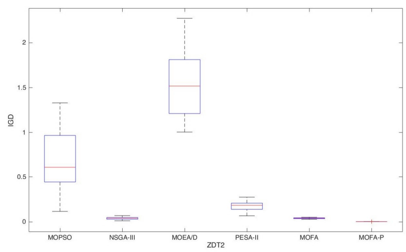
Problems	Results	MOPSO	NSGA-III	MOEA/D	PESA-II	MOFA	MOFA-P
ZDT1	Mean	1.76E-02	2.03E-02	3.34E-01	1.19E-01	2.06E-02	2.07E-03
	Std	3.85E-02	1.54E-03	1.92E-01	2.93E-02	2.58E-02	2.44E-05
ZDT2	Mean	5.86E-01	4.20E-02	1.60E+00	1.78E-01	4.04E-02	3.32E-03
	Std	6.90E-01	2.27E-02	5.60E-01	6.77E-02	1.19E-02	1.41E-03
ZDT3	Mean	3.55E-02	2.23E-02	5.63E-01	1.10E-01	2.00E-02	3.35E-03
	Std	4.89E-02	8.85E-03	4.05E-01	2.74E-01	4.29E-03	5.06E-04
ZDT4	Mean	2.30E+00	1.52E+00	4.92E+00	2.07E+00	3.10E-02	2.26E-03
	Std	1.28E+00	2.17E-01	4.07E+00	1.33E+00	4.97E-02	1.56E-04
ZDT6	Mean	9.21E-02	1.51E-01	3.73E+00	1.18E-02	3.98E-01	8.32E-03
	Std	4.62E-01	2.06E-01	1.31E+00	1.71E-02	1.18E-01	3.36E-03
Viennet1	Mean	1.12E-01	1.19E-01	4.12E-01	1.04E-01	1.38E-01	6.60E-02
	Std	3.15E-04	8.86E-03	6.68E-02	4.49E-03	8.85E-03	2.50E-03
Viennet3	Mean	5.61E-02	2.31E+00	2.27E+00	4.68E-02	1.68E+00	6.52E-02
	Std	1.34E-02	8.31E-02	4.58E-01	5.71E-03	7.03E-01	3.07E-02
DTLZ4	Mean	1.54E-01	2.93E-01	3.73E-01	7.31E-02	8.36E-01	3.75E-01
	Std	1.66E-01	3.28E-01	2.91E-01	7.76E-03	1.17E-01	7.66E-02
DTLZ7	Mean	5.41E-02	5.33E-02	6.46E-01	6.88E-02	7.43E-02	5.90E-02
	Std	3.12E-03	2.83E-03	2.87E-01	5.91E-03	6.64E-03	4.21E-03
Total		0	1	0	2	0	6
Ranking		3.11	3.33	5.67	3.11	3.89	1.89
Final rank		2	3	5	2	4	1

algorithms compared in this multi-objective optimization algorithm performance evaluation. The fact that this rank-mean is the smallest essentially quantifies that the MOFA-P algorithm achieves better IGD evaluation results more frequently than the other five classical algorithms on a range of test problems, thus establishing its statistical leadership in overall optimization effectiveness. Further, the results of Friedman's test not only confirm MOFA-P's superiority in overall performance, but also strongly confirm that this superiority is not an accidental phenomenon, but rather stems from the effectiveness of the optimization mechanisms and strategies inherent in the MOFA-P algorithm. This testing process, through rigorous statistical inference, eliminates chance bias that may be due to specific test problems or random factors, and thus reliably reveals MOFA-P's continuous and stable excellent performance in multi-objective optimization tasks. It fully reflects that the MOFA-P algorithm has significant advantages in optimization effectiveness, and its design and implementation effectively enhance the optimization ability and solution set quality when dealing with complex multi-objective problems.

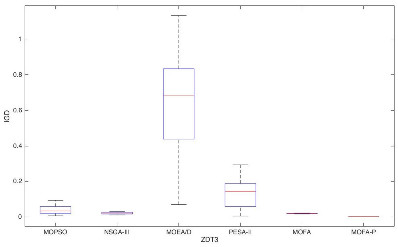
To present the overall performance of the MOFA-P method discussed in this paper compared with the traditional comparison algorithms, a box plot integrating the IGD metrics data from nine different test functions is drawn. As shown in Figure 1, the distribution and statistical characteristics of the solution quality of each type of algorithm in the face of multivariate optimization problems are shown centrally. By comparing the median, quartile, and outliers in the boxplots, the competitiveness of the MOFA-P algorithm under the multidimensional performance evaluation criteria, and its relative advantages or improvements over other classical methods can be revealed at a glance, thus providing a structured framework for researchers and readers. researchers and readers with a structured and quantitative perspective to scrutinize and evaluate the comprehensive efficacy of these algorithms.



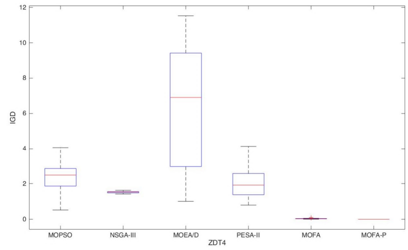
(a) ZDT1



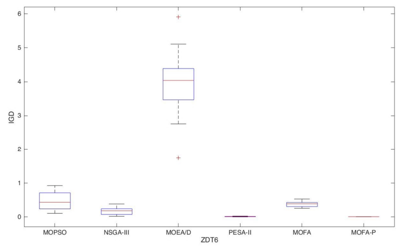
(b) ZDT2



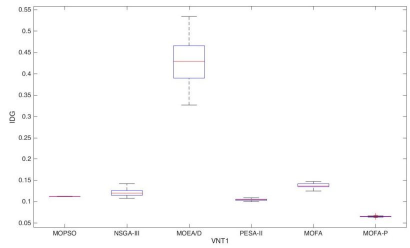
(c) ZDT3



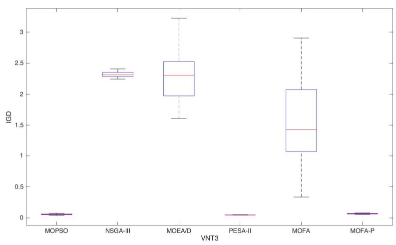
(d) ZDT4



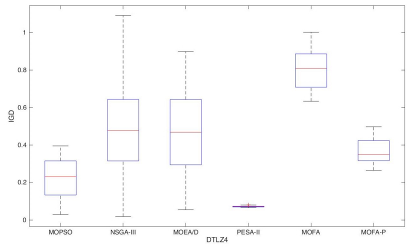
(e) ZDT6



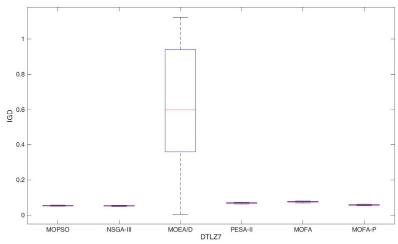
(f) Viennet1



(g) Viennet3



(h) DTLZ4



(i) DTLZ7

FIGURE 1. Boxplots of the six MOEAs on the nine test functions

Table 4. Algorithm parameter settings

Algorithm	Parameter setting	Reference
SMSEMOA	μ=0.5, σ=0.8	Zitzler and Künzli [33]
TVMOPSO	$c_{1i} = 2.5, c_{1f} = 0.5, c_{2i} = 0.5, c_{2f} = 2.5$ $w_1 = 0.7, w_2 = 0.4, b = 5$	Tripathi et al. [34]
DMSPSO	$Pc_i = 1, Pc\_mean = 0.1$	Liang et al. [35]
NSLS	$\mu = 0.5, \sigma = 0.1$	Chen et al. [36]
MOEA/IGD-NS	$p_c = 1, p_m = 1/n$, $\eta_c = 20, \eta_m = 20$	Tian et al. [37]
CFMOFA	$\alpha = 0.2, \beta_0 = 1, \gamma = 1$	Lv et al. [22]
HVFA-M	$\alpha = 0.2, \beta_0 = 1, \gamma = 1$	Zhao et al. [28]
MOFA-P	$\alpha = 0.2, \beta_0 = 1, \gamma = 1$	-

In Figure 1 IGD indicator boxplot it is clear to see the comparison of convergence accuracy and convergence stability of each comparison algorithm, in 2-objective optimization problems ZDT1-ZDT4 and ZDT6 it can be seen that the convergence accuracy as well as the convergence stability of MOFA-P is better than each comparison algorithm, and the convergence and stability of MOEA/D is inferior compared to the other comparison algorithms. On the 3-objective test problem, MOFA-P has better convergence and stability than the comparison algorithms in VNT1, PESA-II has the best convergence and stability in VNT3 and DTLZ4, and NSGA-III has the best convergence and stability in DTLZ7. It is worth mentioning that, although the algorithm MOFA-P in this paper does not achieve the optimal convergence accuracy and convergence speed in VNT3, DTLZ4 and DTLZ7, compared with the original algorithm MOFA, the convergence accuracy and convergence speed of MOFA-P proposed in this paper have been improved to different degrees.

4.3. **Comparison with recent multi-objective optimization algorithms.** In order to further test the performance of MOFA-P, seven recent multi-objective optimization algorithms are selected in this section: including: SMSEMOA [33], TVMOPSO [34], DMSPSO [35], NSLS [36], MOEA/IGD-NS [37], CFMOFA [22], and HVFA-M [28] for comparison with MOFA-P. The parameter settings for each algorithm are taken from their respective references, and the detailed values are shown in Table 4. Five representative bi-objective test functions, i.e., ZDT1, ZDT2, ZDT3, ZDT4, and ZDT6, and one tri-objective test function, DTLZ4, are selected for the experiments to comprehensively examine the adaptive ability of each algorithm under different complexity and dimensionality conditions. The data for the comparison experiments are taken from the literature [28] to ensure the consistency of the benchmarks reviewed. To ensure fairness, we strictly follow the settings in the literature [28], and configure all the algorithms involved in the comparison with the same parameters: the population size is unified at 100 individuals, the maximum number of iterations is set to 250 for the bi-objective problem while it is extended to 500 for the tri-objective problem, and at the same time, the capacity of the external archive is fixed at 100 solutions. All comparison algorithms were written in Matlab version 2021a and executed in the same hardware and software environment to eliminate the influence of environmental factors on the experimental results. The results are shown in Table 5.

The results in Table 5 show that on the six test functions, MOFA-P obtained the most number of optimal values, five times, followed by TVMOPSO on test function ZDT6 all of which obtained optimal values, and SMSEMOA, DMSPSO, NSLS, MOEA/IGD-NS, CFMOFA, and HVFA-M did not obtain the optimal values on the test results. It

Table 5. Experimental results of MOFA-P with seven recent MOEAs on IGDs

Problem		SMSEMOA	TVMOPSO	DMSPSO	NSLS	MOEA/IGD-NS	CFMOFA	HVFA-M	MOFA-P
ZDT1	Mean	1.21E-01	1.45E-02	1.69E-01	6.95E+00	1.61E-02	9.31E-03	9.20E-03	4.14E-03
ZDT2	Mean	5.62E-01	5.00E-02	2.20E-01	8.79E+00	3.02E-02	1.06E-02	1.03E-02	4.73E-03
ZDT3	Mean	1.02E-01	5.73E-02	1.98E-01	7.35E+00	6.64E-02	1.24E-02	9.81E-03	5.56E-03
ZDT4	Mean	1.22E+00	4.29E-01	1.40E+00	1.38E+00	1.65E-02	9.37E-03	1.01E-02	4.22E-03
ZDT6	Mean	8.96E-01	3.60E-03	5.75E-02	8.08E-03	4.46E-03	1.06E-02	7.44E-02	6.51E-03
DTLZ4	Mean	3.31E-01	4.15E-01	4.53E-01	2.11E-01	1.00E-01	8.58E-01	3.88E-01	7.90E-02
Total		0	1	0	0	0	0	0	5
Ranking		6.17	4.17	6.83	6.33	3.67	4.00	3.50	1.33
Final rank		6	5	8	7	3	4	2	1

is worth noting that on the test function ZDT6 although MOFA-P did not obtain the optimal value, but for the sub-optimal and optimal value data remain in the same order of magnitude.

According to the results of Friedman's test, it can be seen that MOFA-P has the best optimization results, followed by HVFA-M and MOEA/IGD-NS, and the worst DMSPSO. Taken together, MOFA-P shows an undisputed advantage in overall performance, and its excellent performance in optimization tasks is not only reflected in the excellent results of a single test function or specific conditions but also in its ability to consistently and stably outperform the other comparative algorithms in a variety of test scenarios. This conclusion strongly verifies the comprehensive performance advantage of MOFA-P in the field of multi-objective optimization, which not only confirms the validity of its design principle and implementation mechanism at the technical level but also provides a solid theoretical basis for prioritizing the MOFA-P algorithm at practical applications. Therefore, MOFA-P establishes its leading position among current multi-objective optimization algorithms with its outstanding performance, both from the perspective of comparing single optimization results and evaluating the overall performance.

Summarizing the experimental studies described in the previous section, a comparative analysis of the MOFA-P algorithm with a series of classical and state-of-the-art multi-objective optimization algorithms reveals that the Piecewise mapping and partitioned search concepts adopted by the algorithm are highly effective in solving several key challenges prevalent in the optimization process. Specifically, these innovative designs successfully counteract the tendency of traditional algorithms to suffer from strategy homogenization during the evolutionary solution process, i.e., over-reliance on a certain fixed search pattern, which leads to limited exploration in complex multi-dimensional solution spaces and easy fall into local optima.

By introducing the Piecewise mapping mechanism, MOFA-P realizes the refined division and dynamic adjustment of the search space, which enables the algorithm to adopt more adaptive optimization strategies in different regions, effectively enhancing the diversity and coverage of the global search. At the same time, the integration of partitioned search ensures that the algorithm, when facing complex optimization problems such as multi-peak and non-linear problems, can independently carry out local optimization in each partition in a targeted manner, while maintaining the continuous pursuit of the global optimal solution, thus avoiding the phenomenon that the population is over-aggregated in a certain local region, and is unable to effectively spread to the global optimal region.

Experimental results show that thanks to the above strategy, MOFA-P significantly improves the distribution characteristics of the population, resulting in a more uniform and wider distribution of individuals in the solution space, which covers more potential high-quality solution regions, thus enhancing the algorithm's ability to resist premature convergence. In addition, MOFA-P ensures good convergence while maintaining excellent exploration capability, and can efficiently approximate the global optimum or the Pareto frontier with limited computational resources. This optimization performance of both distributional and convergence has won MOFA-P a competitive advantage in comparison with many classical and recent algorithms, which fully confirms the effectiveness and advancement of the idea of Piecewise mapping and partitioned search in improving the performance of multi-objective optimization algorithms.

5. Optimal Reservoir Scheduling Problem.

5.1. Multi-objective optimization model for optimal reservoir scheduling.

In the multi-objective scheduling model of reservoirs, the maximum amount of power generation is used as the power generation benefit target, and the Amended Annual Proportional Flow Deviation function (AAPFD), as a measure of the ecological benefit target, the smaller the value of this index indicates that the impact of flow changes on the river ecosystems after the reservoir scheduling is less, and the river ecological environment is better [38]. The model is described as follows:

$$\begin{aligned} F_1 &= \max(\sum_{t=1}^{T} kq_t H_t \Delta_t) \\ F_2 &= \min\left[\sum_{t=1}^{T}\left(\frac{U_t - U_t^n}{U_t^n}\right)^2\right]^{0.5} \end{aligned} \tag{13}$$

Where, F_1 is the total power generation; Δt is the length of time slot t; T is the number of time slots; k is the reservoir output coefficient; q_t is the power generation flow at time slot t in the reservoir; and H_t is the power generation head at time slot t in the reservoir. The F_2 ecological AAPFD value, U_t is the outflow from the hydroelectric power plant at time slot t, and U_t^n is the ecologically appropriate flow rate of the downstream stream channel at time slot t.

The constraints are as follows:

$$\begin{aligned} &Subject\ to: \\ &V_{t+1} = V_t + (I_t - q_t)\,\Delta t \\ &Z_{\min,t} \le Z_t \le Z_{\max,t} \\ &q_{\min,t} \le q_t \le q_{\max,t} \\ &N_{\min,t} \le N_t \le N_{\max,t} \end{aligned} \tag{14}$$

Where, V_t and V_{t+1} are the opening and closing capacities of the reservoir at time period t; I_t and q_t are the inlet and outlet flows of the reservoir at time period t. $Z_{\min,t}$ and $Z_{\max,t}$ are the minimum and maximum water levels of the reservoir at time period t. $q_{\min,t}$ and $q_{\max,t}$ are the minimum and maximum discharge flows of the reservoir at time period t. $N_{\min,t}$ and $N_{\max,t}$ are the minimum and maximum discharge flows at time period t.

5.2. Optimization results analysis Optimization results analysis.

MOFA-P and MOFA are selected for comparison, the algorithm population size, as well as the number of evaluations, refer to section 4.2, and the HV index is selected for comprehensive experimental evaluation, the larger the value of the HV index indicates that the better the results are, and the optimization results of the two algorithms' HV indexes are shown in Table 6. Five reservoir optimal scheduling schemes in the Pareto optimal solution set of MOFA-P are selected for comparative analysis, where the first one has the largest total generation, the second one has the smallest AAPFD, and the remaining three are randomly sampled on the Pareto fronts. The results are shown in Table 7. In this chapter, the improved firefly algorithm and the original algorithm are analyzed and solved in the

TABLE 6. Experimental results on an optimal reservoir scheduling model

Problem	result	MOFA	MOFA-P
Reservoir Optimal	Mean	1.32E-01	3.56E-01
Dispatch Model	Std	2.08E-02	6.11E-03

TABLE 7. Optimized Reservoir Scheduling Program

program	total generating capacity	AAPFD
1	922.7	0.745
2	918.3	0.697
3	920.9	0.715
4	919.9	0.701
5	921.9	0.739

reservoir optimal scheduling problem, and the results show that the optimization results of this paper's algorithm are improved compared with the original algorithm in dealing with the reservoir optimal scheduling problem and the stability of the optimization is also improved significantly. The results of HV indexes in Table 6 show that MOFA-P has a better optimization ability than MOFA under the same conditions, and can generate a better Pareto optimal frontier when solving the same model. Table 7 shows that the results of the five scheduling schemes given by MOFA-P are relatively smooth, proving that the scheduling schemes have strong feasibility.

6. **Summary.** Aiming at the problem that the multi-objective firefly algorithm has a single learning strategy during the evolution process and is prone to fall into premature convergence, which leads to the poor comprehensive performance of the algorithm, this paper proposes a multi-objective firefly algorithm that combines Piecewise mapping and partition search. The Piecewise mapping to improve the uneven distribution of the initial population to improve the dispersion of the population, to produce a better traversal of the initial population; through the introduction of the convergence evaluation index will be divided into two sub-areas by the differences in convergence, namely: the development area and the exploration area, through the partitioning of the learning so that the fireflies maximize the use of the useful information among them; and finally, in the position updating stage to introduce the cosine perturbation strategy, which utilizes the periodic characteristics of the cosine function to induce individual fireflies to refine their exploitation in the finite neighborhood of the better individual to improve the convergence accuracy. The experimental results show that MOFA-P has obvious superiority over several other algorithms in improving the convergence and distribution of the population, and finally, the application of this paper's algorithm to the optimal scheduling problem of reservoirs also gives better results. In the future, we will mainly focus on solving the multimodal multi-objective optimization problem for the optimization process.

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