Research on Resource Allocation in Vocational Education Based on Improved Whale Optimization Algorithm

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ABSTRACT. *Vocational education resources often face limited budgets and teachers. Meanwhile, how to ensure that students from different regions and backgrounds have access to high-quality vocational education resources and reduce the inequality in the distribution of education resources has been a key issue in vocational education resource allocation. In order to solve the above problems, this work designs a vocational education resource allocation model involving multiple objectives and constraints. Whale Optimization Algorithm (WOA), as an efficient global optimisation tool, is able to deal with these complex multi-objective optimisation problems. Firstly, the necessity of educational resource allocation is analysed and relevant indicators for teachers, classes and teacher-class combinations are defined. Then, in order to improve the local optimisation ability of WOA in model solving, Sine chaos theory and adaptive weighting mechanism are introduced for changing the position of the best whale, which improves the convergence accuracy. Secondly, In order to improve the global optimisation-seeking ability of WOA in model solving, the mutation strategy based on differential evolutionary algorithm is integrated into WOA. The WOA is prevented from falling into local optimum prematurely by the variation factor. Finally, the improved whale optimisation algorithm (CADNWOA) is compared with four other heuristic optimisation algorithms on five benchmark functions, which proves that the performance of CADNWOA is higher than the other optimisation algorithms. A case study was conducted on the faculty situation of a vocational school as an example. The proposed multi-objective vocational education resource allocation model can help educational administrators make optimal allocation decisions with limited resources to ensure the most effective use of resources.*
Keywords: Multi-objective multi-constraint; Resource allocation; Whale optimisation algorithm; Chaos theory; Differential evolution; Adaptive weighting

1. **Introduction.** The need for research on the allocation of resources to vocational education lies in its important role in upgrading the quality of the country's overall labour force and level of economic development. With the acceleration of economic globalisation and technological innovation, there is a growing demand for skilled talents, and vocational education, as a key way to cultivate such talents, has a direct relationship with the quality and efficiency of education in terms of the rationality of its resource allocation. Optimising resource allocation can ensure the most effective use of educational resources, meet the

needs of different regions and different types of vocational colleges and universities [1, 2], and thus improve the fairness and inclusiveness of education. In addition, good resource allocation can help stimulate the innovative vitality of vocational education, promote the docking of educational content and market demand, and provide a more accurate supply of human resources for society [3].

The current status quo of vocational education resource allocation, on the other hand, presents certain imbalance [4, 5]. On the one hand, there is a significant difference in educational resources between developed and underdeveloped regions, which leads to uneven distribution of educational resources and unfair educational opportunities [6]; on the other hand, with the reform of the education system and changes in market demand, the traditional resource allocation model has been difficult to adapt to the new educational development needs [7]. In addition, the distribution of educational resources among different majors and programmes is also unreasonable, with some popular majors having an excess of resources while some shortage majors have insufficient resources [8]. The existence of these problems not only affects the overall effectiveness of vocational education, but also restricts the quality and structure of talent training. Therefore, it is of great significance to study in depth the allocation of resources for vocational education and explore a more scientific and reasonable allocation mode and management mechanism to promote the sustainable development of vocational education.

Vocational education resources often face limited budgets and teachers. Meanwhile, how to ensure that students from different regions and backgrounds have access to high-quality vocational education resources and reduce the inequality in the distribution of educational resources has been a key issue in the allocation of vocational education resources [9]. Whale Optimization Algorithm (WOA), as a highly efficient global optimization tool, is able to deal with these complex multi-objective optimization problems [10, 11]. The Whale Optimisation Algorithm, as an intelligent optimization tool, can help decision makers to quickly find the optimal or near-optimal solution in complex educational resource allocation problems and improve the efficiency and effectiveness of decision making. Therefore, the research objective of this work is to improve the use efficiency of vocational education resources and education quality through efficient WOA optimisation educational resource allocation strategies, and at the same time, to solve the inequality problem in resource allocation, in order to promote educational fairness, and promote the balanced development of vocational education.

1.1. **Related work.** Currently, research on the efficiency of educational resource allocation mainly focuses on efficiency evaluation. Researchers usually adopt methods such as data envelopment analysis (DEA) to quantitatively analyse the efficiency of educational resources [12]. For example, some studies have assessed the efficiency of educational activities in higher education institutions in different countries and regions through DEA models, as well as analysed the input-output efficiency of higher education in different regions based on different models (e.g., Window-Malmquist index method [13], etc.).

In terms of model construction, the researcher tries to construct a higher education resource allocation efficiency evaluation system that includes non-desired outputs, and adopts new methods such as the super-efficient SBM-ML method [14] for efficiency measurement. Through empirical analyses, the researcher tested the mechanism of the digital economy's impact on the efficiency of educational resource allocation and put forward corresponding policy recommendations. To ensure the reliability of the findings, the researchers conducted a series of empirical tests and robustness analyses. For example, endogeneity test, instrumental variable method [15], and GMM method [16] were used to

verify the facilitating effect of digital economy on the improvement of higher education resource allocation efficiency. Caiyan [17] studied the impact of different ways of allocating educational resources on educational equity, which was conducted by comparing the resource allocation of different schools and the level of educational equity. The results of the study show that the rational allocation of educational resources can improve educational equity and plays an important role in improving educational equity. Guo [18] explored how to assess the allocation efficiency of educational resources using Data Envelopment Analysis (DEA) and the Tobit model, a non-parametric approach to efficiency evaluation that is able to assess the relative efficiency of a decision unit (e.g., a school) given its inputs and outputs, and the Tobit model, a statistical model used to deal with finite-valued or truncated data, which may be used here to deal with the distribution of efficiency scores. Chu et al. [19] proposed a particle swarm optimisation (PSO) algorithm based model for allocating educational resources. The model considered multiple constraints such as budget constraints, student demand and school capacity. The results of the study showed that the model can effectively allocate resources and improve the quality of education. Sunday et al. [20] proposed a model for improving educational resource allocation using the taboo search algorithm. The model considers various constraints such as resource limitations, student demand and educational quality. The results of the study show that the model can find better solutions than traditional methods while avoiding falling into local optimums.

Nasiri and Khiyabani [21] provided an in-depth analysis of WOA, including the core principles and practical applications of the algorithm.WOA is a population intelligent optimization algorithm based on the feeding behaviour of whale populations, especially the bubble net feeding strategy of humpback whales. The algorithm has been widely used in engineering due to its simplicity, fewer operating parameters, fast convergence rate, and ability to strike a good balance between exploration and development phases. Kaur and Arora [22] explored the development of WOA in different engineering fields in terms of applications, improvements and hybridisation. The article also mentions improved versions of WOA and hybrid algorithms that are designed to address the limitations of WOA in some cases, such as slow convergence, low optimisation accuracy and tendency to fall into local optima.

1.2. **Motivation and contribution.** The main advantages of WOA over other heuristic optimization algorithms are its natural behaviour-based design, simplicity and ease of implementation, fast convergence, good balance between exploration and exploitation, adaptability, less parameter tuning and robustness. These features make WOA an attractive option for solving various optimisation problems [23, 24]. However, when dealing with multi-objective and multi-constraint educational resource optimisation problems, traditional WOA may fall into local optimums without escaping, thus failing to find a globally optimal solution. Therefore, a chaotic adaptive differential nonlinear WOA (CADNWOA) is proposed in this paper. The main innovations and contributions of this work include:

(1) For the multi-objective optimisation problem of educational resource allocation, this paper reasonably defines the relevant indicators of teachers, classes and teacher-class combinations, including teachers' teaching ability, comprehensive quality of classes, the degree of superiority of teacher-class resources, and the degree of difference in teaching matching.

(2) In order to improve the local optimisation ability of WOA in model solving, Sine chaos theory and adaptive weighting mechanism are introduced for changing the position of the best whale, which improves the convergence accuracy. Secondly, in order to improve the global optimisation-seeking ability of WOA in model solving, the mutation strategy

based on differential evolutionary algorithm is integrated into WOA. The WOA is avoided to fall into local optimum prematurely by the variation factor.

(3) A multi-objective vocational education resource allocation model based on CAD-NWOA was constructed and a comprehensive objective function was designed.

2. Overview of educational resourcing models and related technologies.

2.1. **The necessity of educational resource allocation.** The core purpose of the teacher allocation model is to achieve the equitable distribution of educational resources and the overall improvement of teaching quality. In reality, due to the influence of history, geography, economy and other factors, there is often an imbalance in the distribution of teachers between schools and within schools, which not only affects the realisation of educational fairness, but also restricts the overall improvement of educational quality. By constructing a scientific teacher allocation model, teacher resources can be rationally deployed to ensure that every student can enjoy relatively balanced educational resources, thus promoting educational equity and improving educational quality. The teacher deployment model also focuses on the match between the personal development of teachers and the needs of students. Teachers are key factors in the education process, and their teaching ability, professional knowledge and enthusiasm for education have a direct impact on students' learning outcomes. Through the teacher allocation model, the abilities and characteristics of teachers, as well as the needs of students and class characteristics, can be analysed to achieve a reasonable match between teachers and classes, so that teachers can give full play to their maximum educational potential, and at the same time meet the personalised learning needs of students, and promote the common development of teachers and students.

2.2. **Multi-objective optimisation problem.** The multi-objective optimisation problem of the teacher allocation model is a complex decision-making process that aims to improve the quality of education and achieve educational equity through rational allocation of teacher resources. This problem involves multiple conflicting objectives, such as maximising teaching effectiveness, minimising imbalance in resource allocation, considering teachers' personal development and meeting students' individual needs. In practical applications, it is necessary to consider the teaching ability of teachers, the needs of students in the class, and the resource constraints of the school, and find an optimal solution that balances the interests of all parties through mathematical modelling and algorithmic optimisation. Solving this problem usually requires the use of multi-objective optimisation techniques, such as weighted summation and Pareto optimisation [25, 26], as well as the use of heuristic algorithms such as particle swarm optimisation and genetic algorithms to search the solution space, thus obtaining a set of solutions, which are able to achieve the optimal trade-offs between the different objectives and provide a scientific basis for educational decision-making. In this paper, the main object of study in the problem of allocation of educational resources is the class composed of teachers as well as students, so the relevant indicators for defining teachers, classes, and teacher-class combinations are shown in Table 1.

2.3. **Whale Optimisation Algorithm.** Whale Optimization Algorithm (WOA) is a population intelligence optimization algorithm that simulates the feeding behaviour of whales and was proposed by Professor Seyedali Mirjalili in 2016. The algorithm is inspired by the feeding habits of humpback whales, especially the social behaviours and intelligent strategies they exhibit while feeding, such as encircling prey, spiral feeding, etc. The basic idea of the WOA algorithm is to simulate the behavioural patterns of a group of whales

Table 1. Indicators in the vocational education resource allocation model

Norm	Notation	Descriptions
Teachers' pedagogical competence	T^i	Measuring Teachers' i Pedagogical Competence
General quality of the class	C^j	Measuring the overall quality of class j
Teacher-Classroom Resource Strengths and Weaknesses	z_{ij}	A measure of the overall strengths and weaknesses of the i-th teacher as a resource when combined with classes.
Instructional Matching Difference Degree	p_{ij}	Measuring the degree of variation when the i-th teacher is assigned to teach the i-th class.

while searching for food, as shown in Figure 1. In the algorithm, each potential solution is regarded as a whale and they swim in the search space to find the optimal solution. The algorithm starts with a random distribution of whale schools in the search space and then updates their positions through an iterative process.
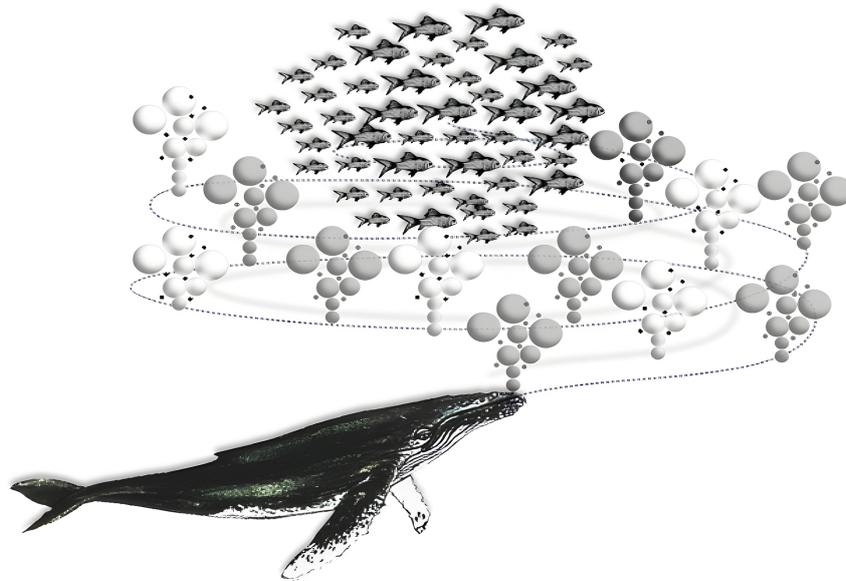

Figure 1. Modelling bubble net predation behaviour

Assuming that the population size of whales is N and the spatial dimension is D, the position of the i-th whale at the t-th iteration can be expressed as

$$X_i^t = \left(x_{i1}^t, x_{i2}^t, \ldots, x_{id}^t, \ldots, x_{iD}^t \right),$$

where x_{id}^t denotes the position of the i-th whale in the d-th dimension at the t-th iteration. The updating strategy consists of three main predatory behaviours.

(1) Swimming to be fed. The WOA algorithm simulates the random searching behaviour of whales in the absence of obvious prey signals to maintain the diversity and exploratory nature of the algorithm. The WOA algorithm selects a random whale individual as the navigation target for feeding activities during this phase of the iterative process. The position update method is as follows.

$$X_i^{t+1} = X_{\text{rand}}^t - A^t \cdot D^t \tag{1}$$

$$D_i^t = \left| C^t \cdot X_{\text{rand}}^t - X_i^t \right| \tag{2}$$

$$A^t = 2a^t \cdot r^t - a^t \tag{3}$$

$$C^t = 2r^t \tag{4}$$

where X_i^t is the position of the i-th whale at the t-th iteration; X_{rand}^t is the position of a random individual in the population at the t-th iteration; A^t and C^t are vectors of coefficients; a^t is the convergence factor that decreases linearly from 2 to 0; and r^t is the random vector in $[0, 1]$.

(2) Surrounding contraction. Simulate the behaviour of the whale group around the prey, and concentrate the search power by contracting the search range, as shown in Figure 2. The WOA algorithm defines the individual with the best performance of the fitness function value as the optimal individual. In this stage, while the whales are searching for prey, other individuals try to encircle the optimal individual, and the position is updated as follows.

$$X_i^{t+1} = X_{\text{best}}^t - A^t \cdot \left| C^t \cdot X_{\text{best}}^t - X_i^t \right| \tag{5}$$

where X_{best}^t is the individual with the best fitness performance in the current population; A^t and C'^t are vectors of coefficients in the iterative process.
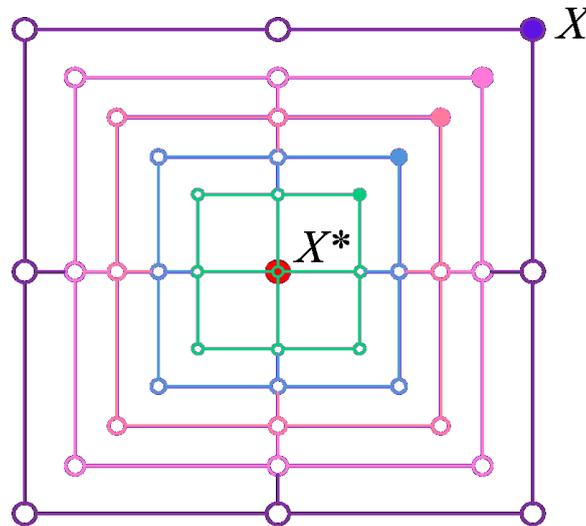

Figure 2. Surrounding contraction behaviour

(3) Spiral predation. Simulating the behaviour of whales approaching prey along a spiral path enhances the local search capability of the algorithm, as shown in Figure 3. In this stage, the whale swims along the spiral path in a way to the optimal individual for encirclement, the position update method is shown as follow.

$$X_i^{t+1} = D_i' \cdot e^{bl} \cdot \cos(2\pi l) + X_t^* \tag{6}$$

$$D_i' = \left| X_t^* - X_i^t \right| \tag{7}$$

where D_i' is the absolute value of the distance between the whale's X_i^t coordinate information and the optimal whale's X_t^* coordinate information; b is a constant, usually set to 1; and l is a random number between $[-1, 1]$.
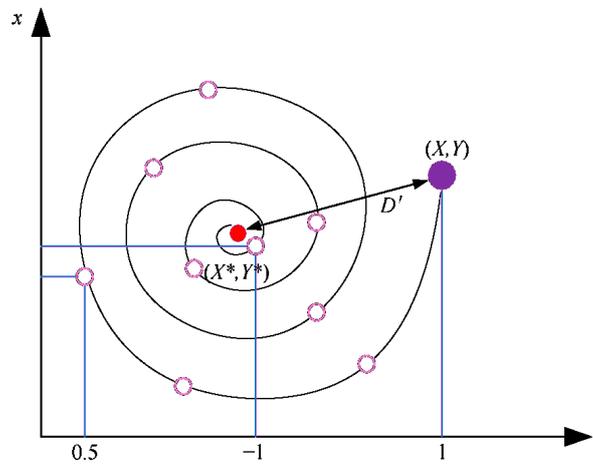

Figure 3. Spiral predation mechanism

Whales not only swim in a reduced circle towards their prey during feeding, but also swim along a spiral. To mimic this behaviour, a random variable $p(0 \leq p \leq 1)$ is introduced. If $0 \leq p < 0.5$ and the coefficient vector $|A| \geq 1$, the whale updates its position by swimming and feeding. If $0 \leq p < 0.5$ and the coefficient vector $|A| < 1$, the whale updates its position using encirclement contraction. If $0.5 \leq p < 1$, the whale optimisation algorithm enters the spiral predation phase and updates its position. In summary, the mathematical model of the WOA algorithm is as follows.

$$X_i^{t+1} = \begin{cases} X_i^t - A^t \cdot D^t & 0 \leq p < 0.5 \\ D_t' \cdot e^{bl} \cdot \cos(2\pi l) + X_t^* & 0.5 \leq p < 1 \end{cases} \qquad (8)$$

The key parameters of the WOA algorithm include the convergence factor, which controls the speed at which the whale population converges to the optimal solution. As the number of iterations increases, the convergence factor gradually decreases, which makes the algorithm have a better global search ability in the early stage and focus more on local search in the later stage [27]. The WOA algorithm has been successfully applied in many optimisation problems, such as function optimisation, neural network training, engineering design and other fields, due to its simple, efficient search strategy and better convergence performance. Compared with other optimisation algorithms, WOA can effectively avoid falling into local optimal solutions while maintaining computational efficiency, and thus shows good performance in solving complex optimisation problems.

3. Chaotic adaptive differential nonlinear WOA algorithm.

3.1. The introduction of Sine chaos theory.
The population initialisation of the traditional WOA algorithm adopts the mode of pseudo-randomisation of individual positions, this pseudo-random mode can make the whole population distributed over the whole solution space, but it may not guarantee the uniform distribution of the population.

In the WOA, the position of the initialised whale stock is calculated as follows:

$$X_i = lb + r \times (ub - lb) \qquad (9)$$

where X_i is the position of individual i; lb and ub are the lower and upper bounds of the search space; and r is a random number between $[0, 1]$.

In order to increase the population diversity and lay the foundation for the global search of the WOA algorithm, Sine chaos theory was introduced at the initial stage, as follows:

$$x_{n+1} = \sin\left(\frac{2}{x_n}\right), \quad n = 0, 1, \ldots, N \quad (-1 \leq x_n \leq 1, x_n \neq 0) \tag{10}$$

The initial value x_n is not 0, and does not generate immovable points and zeros in $[-1, 1]$.

3.2. **Adaptive weights.** In the traditional WOA algorithm, the whale that has the best fitness function value in the herd is considered as the best whale. In the process of local optimisation, the weights on the positions of the optimal whales are fixed, which leads to the fact that in the later stages of optimisation, the WOA algorithm can only wander around the optimal solution but cannot find the optimal solution well. Therefore, the ideal inertia weighting strategy should be: to set a larger weight value in the initial stage of iteration to ensure that the WOA algorithm has better global optimisation performance, and to set a lower weight value in the end stage of iteration to ensure that the WOA algorithm has better local optimisation performance. Accordingly, this paper proposes an adaptive weighting formula as follows:

$$\omega = \frac{e - e^{\cos\left(\frac{\pi}{2} - \frac{\pi}{2} \cdot \frac{t}{T}\right)}}{e - 1} \tag{11}$$

where t is the current iteration number, T is the maximum iteration number.

As shown in Figure 4, the adaptive weights maintain a larger weight value at the beginning of the iteration, which enhances the global optimisation ability of the whale. In the late iteration to maintain a smaller weight value, adaptive weights to enhance the whale's local search ability, can avoid the WOA algorithm prematurely into the local optimum.
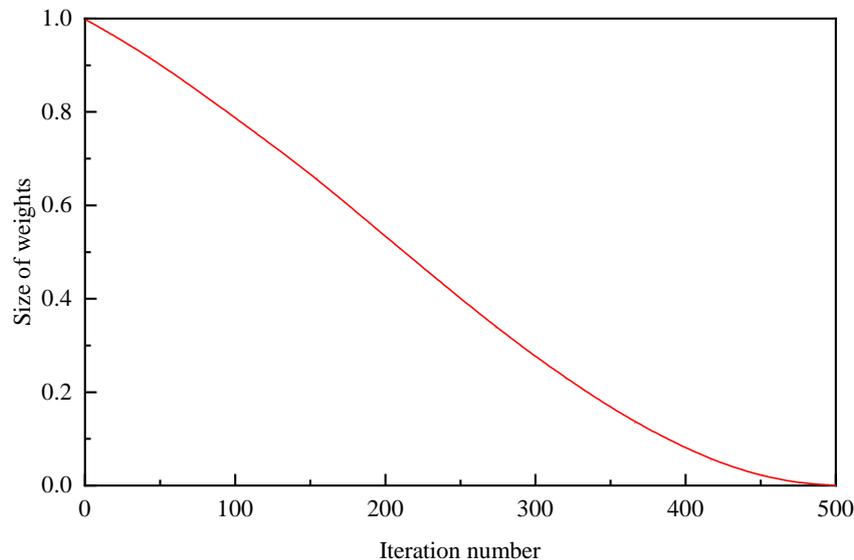

Figure 4. Adaptive weighting

The improved WOA algorithm uses Equation (12) to update whale positions when whales undergo encircling constriction. The whale optimisation algorithm uses Equation (13) to update the whale position when the whale performs spiral predation.

$$X_i^{t+1} = \omega \cdot X_{\text{best}}^t - A^t \cdot \left| C^t \cdot X_{\text{best}}^t - X_i^t \right| \tag{12}$$

$$X_i^{t+1} = D_t' \cdot e^{bl} \cdot \cos(2\pi l) + \omega \cdot X_t^* \tag{13}$$

3.3. **Variational crossover for fusion difference evolutionary algorithms.** . When the random variable $p < 0.5$ and the coefficient vector $|A| > 1$ in the WOA algorithm, the global search is performed using wandering foraging. During the global search, the WOA algorithm needs to randomly select a whale as a reference whale, and the other whales approach the random whale to update their positions. Since the WOA algorithm selects the reference whale too randomly during the initial global search, it is not easy to determine the range of the global optimal solution. Therefore, in order to improve the global optimality seeking ability of the WOA algorithm, the differential evolution (DE) algorithm [28] can be used for mutation. In the early global optimisation of the WOA algorithm, the DE algorithm can help the whale to expand the optimisation space in order to better find the optimal solution by means of cross-variation selection. In the early stage of iteration, DE algorithm should use mutation factors with large values when performing mutation operations on whales in order to enhance the diversity of the population. At the later stage of iteration, the DE algorithm should use a small value of mutation factor to improve the local optimisation ability of whales. Accordingly, this paper proposes an improved mutation factor based on DE algorithm to perform mutation operation on whales, and the new mutation factor is shown as follow:

$$F = \begin{cases} 2 * F_0 \left(2 - \sin\left(\frac{(2-|A|) \cdot \pi}{2} \right) \right) - 0.5 & |A| > 1 \\ 0 & |A| \leq 1 \end{cases} \tag{14}$$

where F_0 is the initial variance factor, taking the value of 0.5; $|A|$ is the coefficient vector of the whale optimisation algorithm.

In the early stage of iteration, when the vector coefficient $|A| > 1$, a larger mutation factor is used to mutate the whales, to improve the diversity of whale populations, and to enhance the algorithm's ability of global optimisation. In the late iteration, when the vector coefficient $|A| \leq 1$, a smaller mutation factor is used to mutate the whales, which reduces the randomness of the mutation and facilitates the local optimisation. The improved mutation strategy is shown below;

$$V_i(g + 1) = X_{r_1}(g) + F \times (X_{r_2}(g) - X_{r_3}(g)) \tag{15}$$

3.4. **Non-linear convergence factor.** The convergence factor a of the standard WOA is linear during the iteration process, which does not correspond to the nonlinear search mode of the WOA algorithm, resulting in a poor regulation of the local search capability and the local exploitation capability, and it is very easy to fall into the local optimum. Therefore, we design a nonlinear strategy to collect the roses, the improved a becomes slow in the early stage, which can improve the global search ability, and in the late stage of the algorithm, a decreases rapidly, which can improve the local search ability. The improvement of a after the introduction of ω is as follows.

$$a = \frac{2}{\lg 2} \times \lg \left(2 - \left(\frac{t}{T} \right)^2 \right) \tag{16}$$

The pseudo-code of the Chaotic Adaptive Differential Nonlinear WOA Algorithm (CAD-NWOA) proposed in this work is shown in Algorithm 1.

4. **CADNWOA-based multi-objective vocational education resource allocation model.**

Algorithm 1 Chaotic adaptive differential nonlinear WOA

Input: dimension of the problem D, number of whale populations N, maximum number of iterations T, parameters (a, b, c) in Chaos Theory, variance factor p in DEA, length of variance l in DEA, coefficient vectors $(A, r1, r2)$ in WOA, location of current optimal solution X_{best}

Output: the location of the optimal solution X

1: Initialise the whale population position matrix $Position[N \times D]$ and velocity matrix $Velocity[N \times D]$
2: Calculate initial fitness $Fitness[N]$ for each whale
3: Find and record the global optimal solution X_{best} and the corresponding fitness $Fitness_{\text{best}}$
4: **for** $t = 1$ to T **do**
5:     **for** each whale $i = 1$ to N **do**
6:         Calculate chaotic sequences to update position and velocity
7:         Updating Whale Position and Velocity Using Sine Chaos Theory
8:     **end for**
9:     **for** each whale $i = 1$ to N **do**
10:         Updating the whale's position using an adaptive weighting strategy
11:     **end for**
12:     **for** each whale $i = 1$ to N **do**
13:         Mutation operations using an improved differential evolutionary algorithm
14:     **end for**
15:     Update the nonlinear convergence factor a
16:     Updating the position and speed of each whale
17:     Update the global optimal solution X_{best} and the corresponding fitness $Fitness_{\text{best}}$
18: **end for**
19: Output X_{best} as the optimal solution

4.1. **Model objectives and constraints.** The proposed multi-objective vocational education resource allocation approach is illustrated with a simplified case of a teacher allocation model. Suppose there is a school with 5 maths teachers and 6 classes. Each teacher needs to be assigned to teach 2 classes and only 1 teacher can be in charge of the Maths curriculum in each class.

The objectives of the vocational education resource allocation model designed in this work are twofold: (1) Fairness objective: to ensure fairness in the distribution of teacher resources and to avoid the concentration of too many excellent teacher resources in certain classes. (2) Rationality goal: to improve teaching quality by reasonably matching teachers according to their teaching ability and the needs of their classes. Vocational education resource allocation model constraints include: (1) Each teacher must be assigned to 2 classes. (2) Each class can only be handled by 1 teacher.

Teacher-class resource superiority Z_{ij}:

$$Z_{ij} = \gamma_1 \cdot T_i + \gamma_2 \cdot C_j \tag{17}$$

where γ_1 and γ_2 are teacher and class resource weights; T_i is the teacher's teaching ability rating; and C_j is the class composite rating.

Instructional Matching Variance P_{ij}:

$$P_{ij} = \frac{1}{5} \sum_{k=1}^{5} \left| \frac{T_i - T_{\min}}{T_{\max} - T_{\min}} - \frac{C_j - C_{\min}}{C_{\max} - C_{\min}} \right| \tag{18}$$

where $T_{\min}$ is the minimum value of the teacher's rating, $T_{\max}$ is the minimum value of the teacher's rating, $C_{\min}$ is the minimum value of the class rating, and $C_{\max}$ is the maximum value of the class rating.

4.2. **Objective function and model solution.** As mentioned above, the proposed vocational education resource allocation has two objective functions, which are shown below:

(1) Fairness objective function F_1:

$$F_1 = \frac{1}{M} \sum_{j=1}^{M} \left(Z_{ij} - \bar{Z} \right)^2 \tag{19}$$

where M is the total number of classes and $\bar{Z}$ is the average merit of all teacher-class combination resources.

(2) Rationality objective function F_2:

$$F_2 = \frac{1}{N} \sum_{i=1}^{N} P_{ij} \tag{20}$$

where N is the total number of teachers.

Thus, the final integrated objective function F is given as follows.

$$F = \lambda_1 \cdot F_1 + \lambda_2 \cdot F_2 \tag{21}$$

where λ_1 and λ_2 are the weighting coefficients used to balance the fairness and reasonableness objectives.

Finally, the proposed CADNWOA algorithm is used to solve the above model. The CADNWOA algorithm will search for the optimal teacher assignment scheme X such that the composite objective function F is minimised while satisfying all constraints.

5. Experimental results and analyses.

5.1. **Validation of benchmark functions.** In this paper, the five benchmark functions corresponding to Table 2 are used to validate the effectiveness of the proposed CADNWOA algorithm.

Table 2. 5 benchmark functions

No.	Function	Dimension	Range of values	Minimum value
1	$F_1(x) = \sum_{i=1}^{n} x_i^2$	30	$[-100, 100]$	0
2	$F_2(x) = \sum_{i=1}^{n} |x_i| + \prod_{i=1}^{n} |x_i|$	30	$[-10, 10]$	0
3	$F_3(x) = \sum_{i=1}^{n} \left(\sum_{j=1}^{i} x_j \right)^2$	30	$[-100, 100]$	0
4	$F_4(x) = \max_i |x_i|, 1 \leq i \leq n$	30	$[-100, 100]$	0
5	$F_5(x) = \sum_{i=1}^{n-1} \left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \right]$	30	$[-30, 30]$	0

In order to test the performance of the improved WOA algorithm in this paper, CADNWOA, the traditional Whale Optimisation Algorithm (WOA), Salp Swarm Algorithm (SSA) [29], Seagull Optimisation Algorithm (SOA) [30], and Grey Wolf Optimisation Algorithm (GWO) [31] are compared. Table 3 labels the initialisation settings of the parameters of each algorithm in detail.

In order to have a uniform judgement standard, this paper sets the initial value of the population size of all algorithms to 30, and the number of iterations to 500. The number of runs is set to 30 to find out the mean and standard deviation of each algorithm to the

Table 3. Parameters of the experimental comparison algorithm

Algorithm	Parameters
CADNWOA	$r_1 : 0 \sim 1, r_2 : 0 \sim 1, b = 1, p : 0 \sim 1, F_0 = 0.5$
WOA	$r_1 : 0 \sim 1, r_2 : 0 \sim 1, b = 1, p : 0 \sim 1$
SSA	$c_2 : 0 \sim 1, c_3 : 0 \sim 1$
SOA	$f_c = 2, r_d : 0 \sim 1, u = 1, v = 1, \theta : 0 \sim 2\pi$
GWO	$r_1 : 0 \sim 1, r_2 : 0 \sim 1$

benchmark function, and the experimental results are shown in Table 4. It can be seen that the mean value of the improved WOA algorithm (CADNWOA) in this paper is lower than all other algorithms. The smaller mean value represents a higher performance of the algorithm in finding the optimal, thus indicating that the CADNWOA algorithm has a great advantage in optimisation performance in solving the optimisation problem.

Table 4. Experimental comparison results

Function	Indicator	CADNWOA	WOA	SSA	SOA	GWO
FI	Average	0	6.31E-75	2.22E-07	1.2245	1.07E-27
FI	Variance	0	6.41E-148	1.80E-13	8.2471	7.81E-54
F2	Average	0	3.57E-53	9.70E-03	2.52E-02	1.96E-32
F2	Variance	0	9.40E-105	1.03E-04	5.85E-03	3.53E-63
F3	Average	0	2.93E+02	3.54E-06	3.71E+03	4.03E-25
F3	Variance	0	2.18E+05	2.23E-10	5.02E+06	1.02E-48
F4	Average	5.46E-32	7.5957	4.41E-05	8.6313	2.73E-18
F4	Variance	1.04E-206	7.75E+01	9.08E-09	79.1808	5.42E-35
F5	Average	6.6482	6.9009	1.08E+02	1.42E+02	6.8002
F5	Variance	8.51E-02	1.0869	2.95E+04	3.66E+05	5.06E-01

5.2. **Vocational education case study.** A case study of the proposed multi-objective vocational education resourcing methodology was carried out using the teacher profile of a vocational school as an example. Five maths teachers and six classes were randomly selected from this school. Each teacher needs to be assigned to teach 2 classes and only 1 teacher can be in charge of the maths programme in each class. Teachers' teaching ability scores T_i and classes' composite scores C_j are shown in Table 5.

Table 5. Scoring Matrix for Cases

Teachers/classes	C_1	C_2	C_3	C_4	C_5	C_6
T_1	82	77	72	87	92	80
T_2	74	70	75	82	87	77
T_3	90	92	87	94	90	88
T_4	67	62	69	72	77	70
T_5	80	84	81	88	82	79

By running the CADNWOA algorithm, we can obtain an optimal teacher allocation scheme that assigns teachers to classes such that the values of F_1 and F_2 are minimised. By comparing the F values of different allocation schemes, we can assess which scheme

performs best in terms of fairness and rationality. Ultimately, the teacher allocation scheme with the smallest F value (fitness value of 0.8329) is selected as the best allocation scheme. Compared with the original allocation scheme, the best allocation scheme has a 43.6% reduction in the degree of variation P of teaching matching, which validates the effectiveness of the proposed method.

6. **Conclusion.** In this paper, a Chaotic Adaptive Differential Nonlinear WOA (CAD-NWOA) is proposed. Firstly, the relevant indicators of teachers, classes, and teacher-class combinations are reasonably defined, including teacher teaching ability, class comprehensive quality, teacher-class resource superiority, and teaching matching difference. Then, Sine chaos theory and adaptive weighting mechanism are introduced for changing the position of the best whale, which improves the convergence accuracy. Secondly, the variation strategy based on differential evolutionary algorithm is integrated into WOA. The WOA is avoided to fall into local optimum prematurely by the variation factor. Finally, a multi-objective vocational education resource allocation model based on CADNWOA is constructed and a comprehensive objective function is designed. Experiments show that compared with the original allocation scheme, the teaching matching discrepancy of the optimal allocation scheme is reduced by 43.6%, which verifies the effectiveness of the proposed method.

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