Collaborative Optimization of Heavy and Empty Car Flows in Heavy-Haul Railways Considering Reverse Cargo Transportation

Meng Wang[1,2]

[1]School of Transportation and Logistics, Southwest Jiaotong University, Chengdu 610031, China
[2]China Energy Railway Equipment Co., Ltd., China Energy Investment Corporation Co., Ltd., Beijing 100089, China
15010179689@163.com

Tao He[1]

[1]School of Transportation and Logistics, Southwest Jiaotong University, Chengdu 610031, China
hetao@my.swjtu.edu.cn

Hong-Xia Lv[1,3,4], Miao-Miao Lv[1,3,4]

[1]School of Transportation and Logistics, Southwest Jiaotong University, Chengdu 610031, China
[3]National United Engineering Laboratory of Integrated and Intelligent Transportation, Southwest Jiaotong University, Chengdu 610031, China
[4]National Engineering Laboratory of Integrated Transportation Big Data Application Technology, Southwest Jiaotong University, Chengdu 610031, China
hongxialu@163.com, lvmiaomiao@swjtu.edu.cn

Guang-Zhi Lu[5,*]

[5]Guangzhou Canwin Computer Technology Co., Ltd., Guangzhou 510000, China
edward@cloudmtr.com

*Corresponding author: Guang-Zhi Lu

Abstract. *To enhance market competitiveness and operational profitability, "CHINA ENERGY INVESTMENT CORPORATION CO., LTD. (CHN ENERGY)" has actively expanded its external market. Based on the original one-way transportation structure primarily focused on coal transport, the company has gradually developed a bi-directional transportation demand centered around coal goods. This shift presents new challenges in coordinating the current heavy-haul railway flows of loaded and empty cars. This paper examines the differences in the forward and reverse transportation demands for coal and non-coal goods. It comprehensively considers multiple factors such as combined decomposition operations, train type matching, and empty car distribution, with the goal of minimizing in-transit time, empty car delays and detention costs. The paper proposes a collaborative optimization model for heavy and empty car flows that accounts for reverse cargo transportation. Case studies on the Shen-Shuo line and the Shuo-Huang line demonstrate that, after considering reverse transportation demands, the transportation time increases slightly by 3.64%. However, by optimizing the combination and decomposition plans and adjusting the loading quantity decision, a rational succession of heavy and empty car flows was achieved. This resulted in a 16.70% increase in the completion rate of reverse transportation demands, effectively tapping into the potential of reverse transportation resources. The proposed solution validates the model's effectiveness in improving transportation efficiency, resource utilization, and economic benefits, offering a feasible solution to the issue of imbalanced bi-directional heavy-haul railway transportation.*
Keywords: Freight train operation plan; Empty car distribution plan; Reverse transportation; Heavy-haul railway

1. **Introduction.** In recent years, the rapid development of multimodal transportation has intensified competition for railway freight services. Meanwhile, competition in the cargo transportation market between "CHINA ENERGY INVESTMENT CORPORATION CO., LTD. (CHN ENERGY)" and China State Railway Group (CRRC) has become increasingly fierce. To respond, CHN ENERGY has expanded into external markets while satisfying its internal coal transportation demand, gradually undertaking the transportation of scattered non-coal and high-value-added goods. Relying on heavy-haul railways that connect coal resource-rich regions with downstream markets and major users, CHN ENERGY has evolved from a coal-oriented one-way system to a bi-directional transportation structure with coal transportation as its core. However, with the expansion of market scale and the diversification of demand, improving operational profitability and competitiveness has become a key challenge.

Extensive research has been conducted on railway freight transportation optimization. For conventional railway freight, Bożejko et al. [1] proposed an improved Dijkstra algorithm for efficient path and schedule determination, while Zhu [2] developed a three-layer spatiotemporal network for tactical planning. For high-speed freight, Jiang et al. [3] integrated dynamic spatiotemporal networks with genetic algorithms to address fluctuating demand. At a macro level, Borndörfer et al. [4] established a MINLP–MILP framework for freight path optimization, and Li et al. [5] employed column generation to improve the parallel operation of passenger and freight trains. Yu et al. [6] focused on cold-chain logistics, using an NSGA-II algorithm to jointly optimize container allocation, routing, and temperature layers, achieving significant reductions in cost and emissions.

Regarding heavy-haul railway freight train scheduling, Zhao et al. [7] proposed a minimum-cost maximum-flow model to minimize costs and maximize line capacity. Han et al. [8] and Jing [9] optimized train combination schemes under multiple station configurations, while Jing further addressed path selection and combination decomposition. Fu [10]

incorporated collection and distribution flow characteristics and developed a scheduling model based on multi-commodity flow theory.

Research on empty car distribution has addressed various operational perspectives. Yang [11], Duan et al. [12], and Li [13] developed models considering loading demand, coal enterprise operations, and destination concentration. Additionally, Wang [14], Xu et al. [15], and He [16] proposed collaborative optimization models integrating loaded and empty car flows, solving them with genetic algorithms, NSGA-II, and hybrid heuristic methods to enhance distribution efficiency.

Despite these advances, existing studies do not fully meet CHN ENERGY's practical operational needs. Most heavy-haul freight research focuses on one-way coal transport, neglecting the bi-directional demand arising from CHN ENERGY's evolving transport structure, thereby limiting the applicability of traditional empty car distribution methods. Moreover, balancing the differentiated demands of non-coal cargo while prioritizing coal transport, optimizing empty car allocation, and improving resource utilization remains a critical challenge. Therefore, this study proposes a collaborative optimization model for loaded and empty car flows that incorporates reverse cargo transportation, using CHN ENERGY's heavy-haul railway system as a case study, and validates its effectiveness through real-world applications.

2. **Problem Analysis.** In heavy-haul railway transportation, some rail lines experience the coexistence of single and double tracks, as well as intertwined loaded and empty car flows, resulting in capacity constraints in certain sections. To improve transportation efficiency, some heavy-haul trains complete loading at loading stations and are then sent to combination stations for train combination operations before heading to unloading stations. Other combined trains undergo decomposition at decomposition stations and are subsequently dispatched to different unloading points. The remaining heavy-haul trains travel directly from loading stations to unloading stations, where they unload and convert to empty cars for return. Although loaded and empty car flows are organized differently, they follow similar allocation principles, relying on flexible combination and decomposition operations to reduce the number of trains and increase single-train hauling capacity, thereby enhancing overall line capacity. Therefore, optimizing heavy-haul freight train operation plans requires the coordinated consideration of combination and decomposition processes, section capacity, and loading and unloading efficiency.

Against this backdrop, this paper focuses on the heavy-haul railway system of CHN ENERGY, analyzing the diversity of its transportation demands and the associated organizational challenges. Coal transportation remains the primary task of CHN ENERGY, while surplus transportation capacity is allocated to non-coal freight. Coal transportation is characterized by bulk volume, stable flows, and predominantly one-way demand, with relatively simple reporting requirements. In contrast, non-coal transportation involves more detailed demand information, such as specific origin–destination pairs, which necessitates differentiated organizational strategies for loaded car flows.

Additionally, when coal transportation is regarded as the forward direction, CHN ENERGY's heavy-haul railway also undertakes a certain amount of reverse cargo transportation. The origins and destinations of reverse cargo flows directly determine the return paths of empty cars. Moreover, reverse-loaded trains must complete unloading before empty cars can be released for forward operations, which may lead to temporal imbalances between empty car supply and demand, thereby increasing the complexity of empty car distribution and loaded car flow organization.

In summary, the operational characteristics of CHN ENERGY's heavy-haul railway impose new requirements on transportation organization. First, non-coal transportation

must be optimized while prioritizing coal transport, making full use of the flexibility of combination and decomposition operations under capacity constraints; second, the increased complexity of empty car distribution caused by reverse transportation must be effectively managed. Based on these considerations, this paper develops a collaborative optimization model for loaded and empty car flows, incorporating differentiated demands for coal and non-coal goods, train type matching, line and station capacity constraints, and reverse transportation demand, with the aim of improving the efficiency and effectiveness of heavy-haul railway operations.

3. Collaborative Optimization Model for Heavy and Empty Car Flows in Heavy-Haul Railways Considering Reverse Cargo Transportation.

3.1. Assumptions and Symbol Explanations.

3.1.1. *Condition Assumptions.* The condition assumptions of this model are as follows:

(1) The loading and unloading stations for the forward and reverse transportation demand of non-coal goods are known.
(2) Since heavy-haul railways typically use fixed carriages, both heavy-haul freight train operation and empty car distribution are organized on a train-based basis. In this paper, a 5,000-ton unit train is adopted as the basic unit, and 10,000-ton and 20,000-ton trains are formed by combining multiple 5,000-ton unit trains.
(3) Each cargo transportation demand requires one 5,000-ton unit empty car to be loaded.
(4) Combined heavy-haul trains can either perform only combination operations, only decomposition operations, or both.
(5) There is no difference in the section operation speed between combined heavy-haul trains and direct heavy-haul trains with different load capacities.
(6) The type of empty car, supply stations, and supply time periods are known.
(7) The impacts of locomotive and train operation failures are not considered.

3.1.2. *Symbol Explanation.* **Set Definitions:**

- I: Set of forward loading stations, which can also serve as reverse unloading stations or empty car demand stations. i is the index of I.
- J: Set of forward unloading stations, which can also serve as reverse loading stations or empty car supply stations. j is the index of J.
- Z: Set of combination stations, $Z = Z^+ \cup Z^-$ where Z^+ and Z^- represent the forward and reverse combination stations, respectively. z is the index of Z.
- F: Set of decomposition stations, $F = F^+ \cup F^-$ where F^+ and F^- represent the forward and reverse decomposition stations, respectively. f is the index of F.
- G: Set of combination and decomposition plans, $G = G^+ \cup G^-$ where G^+ and G^- represent the forward and reverse combination and decomposition plans, respectively. g is the index of G.
- N: Set of states representing the load capacities of trains dispatched from the origin station, where $N = \{1, 2, 3, 4\}$, with each number representing the 5,000-ton, 10,000-ton, 15,000-ton, and 20,000-ton trains, respectively. n is the index of N.
- M: Set of states representing the load capacities of trains arriving at the destination station, where $M = \{1, 2, 3, 4\}$, with each number representing the 5,000-ton, 10,000-ton, 15,000-ton, and 20,000-ton trains, respectively. m is the index of M.
- T: Set of decision periods, $T = T^+ \cup T^-$ where T^+ and T^- represent the forward and reverse decision periods, respectively. t is the index of T.

- X: Set of vehicle types, where $X = \{1, 2, 3\}$ with each number representing the C60, C70, and C80 vehicle types; x is the index of X.
- P: Set of coal types, where $P = \{1, 2, 3\}$ with each number representing raw coal, clean coal, and blended coal; p is the index of P.
- A: Set of pending coal cargo transportation demands, where $a \in A$ represents one of the coal cargo transportation demands.
- B: Set of pending forward non-coal cargo transportation demands, where $b \in B$ represents one of the non-coal cargo transportation demands.
- C: Set of unit empty cars to be dispatched from the empty car supply station, where $c \in C$ represents a unit empty car.
- D: Set of pending reverse cargo transportation demands, where $d \in D$ represents one of the reverse cargo transportation demands.
- L: Set of sections, where $l \in L$ represents one of the sections.

Decision variables:

- $y_{(i,a,j)}^{(g,n,m,x,p)}$: 0-1 decision variable, which indicates whether the transportation demand a for p-type coal in time period t is carried by a train with vehicle type x through the plan g, from origin station i to destination station j.
- $y_{(i,b,j)}^{(g,n,m,x)}$: 0-1 decision variable, which indicates whether the non-coal cargo transportation demand b in time period t is carried by a train of vehicle type x, through the plan g, from origin station i to destination station j.
- $y_{(j,c,i)}^{(g,n,m,x)}$: 0-1 decision variable, which indicates whether a unit empty car c of vehicle type x passes through the plan g in time period t, from empty car supply station j to empty car demand station i.
- $y_{(j,c,i,d)}^{(g,n,m,x)}$: 0-1 decision variable, which indicates whether a unit empty car c of vehicle type x carries reverse cargo transportation demand d in time period t, through the plan g, from the loading station j to the destination station i.
- $S_{(i,t,x)}$: Non-negative integer variable, representing the number of unit empty cars of vehicle type x that the empty car distribution plan can provide to loading station i in time period t.
- $O_{(i,t,x)}$: Non-negative integer variable, representing the number of detained unit empty cars of vehicle type x at loading station i in time period t.
- $L_{(i,t,x)}$: Non-negative integer variable, representing the number of missing unit empty cars of vehicle type x at loading station i in time period t.

Other parameters are defined as follows:

- t_g: The average combination and decomposition time of the plan g.
- $t_{(i,n)}/t_{(j,n)}$: The average loading time for a starting train with a load capacity of n at the forward and reverse loading stations i/j.
- $t_{(i,j)}^{+}/t_{(j,i)}^{-}$: The section travel time from the origin station to the destination station in both forward and reverse directions.
- $\theta_{(g,n)}$: The 0-1 parameter, indicating whether the combined heavy-haul train of plan g is formed by trains with a load capacity of n.
- $\theta_{(g,m)}$: The 0-1 parameter, indicating whether the combined heavy-haul train of plan g generates a train with a load capacity of m after decomposition.
- $\theta_{(g,n_1,n_2)}$: The 0-1 parameter, indicating whether the combined heavy-haul train of plan g is formed simultaneously by trains with load capacities of n_1 and n_2.
- $\theta_{(g,m_1,m_2)}$: The 0-1 parameter, indicating whether the combined heavy-haul train of plan g generates trains with load capacities of m_1 and m_2 after decomposition.

- $\psi_{(g,n)}$: Indicates how many trains with a load capacity of n are combined to form a combined heavy-haul train of plan g.
- $\psi_{(g,m)}$: Indicates how many trains with a load capacity of m are generated after the decomposition of a combined heavy-haul train of plan g.
- K_g: Indicates how many 5,000-ton unit trains make up a combined heavy-haul train of plan g.
- C_i^t/C_j^t: The maximum loading capacity that loading station i/j can complete in time period t in both forward and reverse directions.
- $\psi_{g,l}^1$: Indicates whether section l is located before the combination station specified in plan g.
- $\psi_{g,l}^2$: Indicates whether section l is located between the combination station and the decomposition station specified in plan g.
- $\psi_{g,l}^3$: Indicates whether section l is located after the decomposition station specified in plan g.
- $v_{i,l}^+/v_{j,l}^-$: Indicates whether the forward and reverse loading station (empty car supply station) i/j is located before section l.
- $v_{j,l}^+/v_{i,l}^-$: Indicates whether the forward and reverse unloading station (empty car demand station) i/j is located after section l.
- $L_{(l,t)}^+/L_{(l,t)}^-$: The maximum line throughput capacity for section l during time period t in the forward and reverse directions, respectively.
- C_t^z/C_t^f: Maximum combination capacity and maximum decomposition capacity, respectively, for combination and decomposition station z/f during time period t.
- $\omega_{(g,z)}/\omega_{(g,f)}$: Indicates whether the combination station z and the decomposition station f are the combination and decomposition stations specified by the plan g.
- $X_{(j,p)}^{need}$: The demand for type p coal at coal demand station j.
- $\gamma_{(j,n,g,i,t_1,t_2)}^-$: The 0-1 parameter, indicating whether a unit empty car, dispatched in the planning period t_1 of the empty car distribution plan, with a load capacity of n, originating from empty car supply station j, can reach the empty car demand station i in the planning period t_2 after undergoing the combination and decomposition operations of plan g, as specified in the heavy train operation plan.
- $\gamma_{(j,n,g,i,t_1,t_2)}^{re}$: The 0-1 parameter, indicating whether a unit empty car, dispatched in the planning period t_1 of the empty car distribution plan, after being loaded with a n-ton train at loading station j, undergoes the combination and decomposition operations of plan g and the unloading operation, can reach the unloading station i in the planning period t_2, as specified in the heavy train operation plan.

3.2. Model Construction.

3.2.1. *Model Objectives.* In this work, the model objectives are as follows:
(1) Minimize cargo in-transit time.
Although "CHN ENERGY" has some non-coal cargo transportation demand, the heavy-haul railway was fundamentally constructed to serve coal demand. Therefore, in the organization of heavy car flows, coal demand should be given priority, and non-coal and reverse cargo transportation should be considered only after coal demand has been satisfied. Accordingly, this paper aims to minimize the total in-transit time of both coal and non-coal cargo, while ensuring that coal demand at unloading stations is satisfied.

$$\min Z_1 = T_1 + T_2 + T_3 \tag{1}$$

T_1: Represents the total in-transit time for coal cargo dispatched from each origin station, including the loading time at the station, combination and decomposition operation

time, and section operation time:

$$T_1 = \sum_{i\in I}\sum_{t\in T}\sum_{g\in G}\sum_{n\in N}\sum_{m\in M}\sum_{x\in X}\sum_{j\in J}\sum_{a\in A}\sum_{p\in P}\partial_1 \times y_{(i,a,t,j)}^{(g,n,m,x,p)} \times (t_{(i,n)} + t_g + t_{(i,j)}^+) \qquad (2)$$

T_2: Represents the total in-transit time for non-coal cargo:

$$T_2 = \sum_{i\in I}\sum_{t\in T}\sum_{g\in G}\sum_{n\in N}\sum_{m\in M}\sum_{x\in X}\sum_{j\in J}\sum_{b\in B}\partial_2 \times y_{(i,b,t,j)}^{(g,n,m,x)} \times (t_{(i,n)} + t_g + t_{(i,j)}^+) \qquad (3)$$

(2) Minimize the in-transit time for reverse transportation cargo:

$$\min Z_2 = \sum_{j\in J}\sum_{t\in T}\sum_{g\in G}\sum_{n\in N}\sum_{m\in M}\sum_{x\in X}\sum_{i\in I}\left(\sum_{c\in C}\sum_{d\in D}y_{(j,c,t,i,d)}^{(g,n,m,x)} \times (t_{(j,n)} + t_g + t_{(j,i)}^-)\right) \qquad (4)$$

(3) Minimize the delay in meeting the empty car demand at the loading stations.

For any loading station, empty car demand can be determined based on the heavy train operation plan. When formulating the empty car distribution plan, dynamic coordination is required to satisfy each loading station's requirements for empty car type, quantity, and arrival time. The successful execution of the heavy train operation plan depends on the rationality of the empty car distribution plan. To ensure the smooth completion of subsequent loading plans, the empty car distribution plan for the current stage should aim to minimize deviations in empty car arrival times at loading stations, which include detention time costs and delay penalties for empty cars.

$$\min Z_3 = M_1 \times L_{(i,t,x)} + M_2 \times O_{(i,t,x)} \qquad (5)$$

3.2.2. *Restraint conditions.* The restraint conditions are as follows.

(1) Constraints on Matching Trains Dispatched from Origin Stations with Cargo Transportation Demand/Empty car Units

To avoid empty freight trains at loading stations, taking a 10,000-ton ($n = 2$) train as an example, each dispatch of a 10,000-ton ($n = 2$) train requires the simultaneous matching of two coal (or non-coal) cargo transportation demands, and each dispatch of a 10,000-ton ($n = 2$) empty train from any empty car supply station requires the provision of two unit empty cars. The corresponding mathematical expressions are given as:

$$\sum_{m\in M}\sum_{j\in J}\sum_{a\in A}y_{(i,a,t,j)}^{(g,n,m,x,p)} = n \times \varphi_{(i,g,t,n,x,p)}^+ \qquad (6)$$

$$\forall i \in I, \forall g \in G, \forall t \in T^+, \forall n \in N, \forall x \in X, \forall p \in P$$

$$\sum_{m\in M}\sum_{j\in J}\sum_{b\in B}y_{(i,b,t,j)}^{(g,n,m,x)} = n \times \varphi_{(i,g,t,n,x)}^+ \qquad (7)$$

$$\forall i \in I, \forall g \in G, \forall t \in T^+, \forall n \in N, \forall x \in X$$

$$\sum_{m\in M}\sum_{i\in I}\sum_{c\in C}\sum_{d\in D}y_{(j,c,t,i,d)}^{(g,n,m,x)} = n \times \varphi_{(j,g,t,n,x)}^{re} \qquad (8)$$

$$\forall j \in J, \forall g \in G, \forall t \in T^-, \forall n \in N, \forall x \in X$$

$$\sum_{m\in M}\sum_{i\in I}\sum_{c\in C}y_{(j,c,t,i)}^{(g,n,m,x)} = n \times \varphi_{(j,g,t,n,x)}^- \qquad (9)$$

$$\forall j \in J, \forall g \in G, \forall t \in T^-, \forall n \in N, \forall x \in X$$

$\varphi^{+}_{(i,g,t,n,x,p)}$, $\varphi^{+}_{(i,g,t,n,x)}$, $\varphi^{re}_{(j,g,t,n,x)}$, $\varphi^{-}_{(j,g,t,n,x)}$ in the above constraints are intermediate variables, representing the number of trains of different vehicle types x using different combination and decomposition plans g, dispatched by the origin station i/j and empty car supply station j in time period t.

(2) Constraints on Matching Trains Received at Destination Stations with Cargo Transportation Demand/Empty car Units.

Taking a 10,000-ton train as an example, for any destination station, each receipt of a 10,000-ton ($m = 2$) heavy-haul train requires the simultaneous arrival of two coal (or non-coal) cargo transportation demands, and each receipt of a 10,000-ton ($m = 2$) empty train at any empty car demand station requires the combination of two unit empty cars to form the arriving empty train. The corresponding mathematical expressions are given as:

$$\sum_{n \in N} \sum_{i \in I} \sum_{a \in A} y^{(g,n,m,x,p)}_{(i,a,t,j)} = m \times \varphi^{+}_{(j,g,t,m,x,p)} \qquad (10)$$

$$\forall j \in J, \forall g \in G, \forall t \in T^{+}, \forall m \in M, \forall x \in X, \forall p \in P$$

$$\sum_{n \in N} \sum_{i \in I} \sum_{b \in B} y^{(g,n,m,x)}_{(i,b,t,j)} = m \times \varphi^{+}_{(j,g,t,m,x)} \qquad (11)$$

$$\forall j \in J, \forall g \in G, \forall t \in T^{+}, \forall m \in M, \forall x \in X$$

$$\sum_{n \in N} \sum_{j \in J} \sum_{c \in C} \sum_{d \in D} y^{(g,n,m,x)}_{(j,c,t,i,d)} = m \times \varphi^{re}_{(i,g,t,m,x)} \qquad (12)$$

$$\forall i \in I, \forall g \in G, \forall t \in T^{-}, \forall m \in M, \forall x \in X$$

$$\sum_{n \in N} \sum_{j \in J} \sum_{c \in C} y^{(g,n,m,x)}_{(j,c,t,i)} = m \times \varphi^{-}_{(i,g,t,m,x)} \qquad (13)$$

$$\forall i \in I, \forall g \in G, \forall t \in T^{-}, \forall m \in M, \forall x \in X$$

$\varphi^{+}_{(j,g,t,m,x,p)}$, $\varphi^{+}_{(j,g,t,m,x)}$, $\varphi^{re}_{(i,g,t,m,x)}$, $\varphi^{-}_{(i,g,t,m,x)}$ in the above constraints are intermediate variables, representing the number of trains of different vehicle types x using different combination and decomposition plans g, received by the destination station i/j and empty car demand station i in time period t.

(3) Loading Station Loading Capacity Constraints.

For each loading station, the number of trains of each type that can be organized is limited by the station's maximum loading capacity and the availability of empty cars. The corresponding mathematical expression is:

$$\sum_{j \in J} \sum_{g \in G} \sum_{n \in N} \sum_{m \in M} \sum_{x \in X} \left(\sum_{b \in B} y^{(g,n,m,x)}_{(i,b,t,j)} + \sum_{a \in A} \sum_{p \in P} y^{(g,n,m,x,p)}_{(i,a,t,j)} \right) \leq C^{t}_{i}, \forall i \in I, \forall t \in T^{+} \qquad (14)$$

$$\sum_{j \in J} \sum_{g \in G} \sum_{n \in N} \sum_{m \in M} \left(\sum_{b \in B} y^{(g,n,m,x)}_{(i,b,t,j)} + \sum_{a \in A} \sum_{p \in P} y^{(g,n,m,x,p)}_{(i,a,t,j)} \right) \leq Q_{(i,t,x)} \qquad (15)$$

$$\forall i \in I, \forall t \in T^{+}, \forall x \in X$$

$$\sum_{i \in I} \sum_{g \in G} \sum_{n \in N} \sum_{m \in M} \sum_{x \in X} \sum_{c \in C} \sum_{d \in D} y^{(g,n,m,x)}_{(j,c,t,i,d)} \leq C^{t}_{j}, \forall j \in J, \forall t \in T^{-} \qquad (16)$$

Additionally, in the forward heavy-haul freight train operation plan, the type of train x supplied by loading station in time period t is determined by the available empty cars, which consist of empty cars delivered from the unloading end in the current period and

those remaining from the previous period. The corresponding recursive expression is given as follows:

$$Q_{(i,t,x)} = S_{(i,t,x)} + Q_{(i,t-1,x)} - \sum_{j\in J}\sum_{g\in G}\sum_{n\in N}\sum_{m\in M}\left(\sum_{b\in B}y_{(i,b,t-1,j)}^{(g,n,m,x)} + \sum_{a\in A}\sum_{p\in P}y_{(i,a,t-1,j)}^{(g,n,m,x,p)}\right) \quad (17)$$

$$\forall i \in I, \forall t \in T^+ \text{ and } t \geq 2, \forall x \in X$$

(4) Uniqueness Constraints of Cargo Transportation Demand and Unit Empty Car Operations.

The corresponding mathematical expressions are given as:

$$\sum_{i\in I}\sum_{t\in Tk}\sum_{j\in J}\sum_{g\in G}\sum_{n\in N}\sum_{m\in M}\sum_{x\in X}\sum_{p\in P}y_{(i,a,t,j)}^{(g,n,m,x,p)} = 1, \forall a \in A \quad (18)$$

$$\sum_{i\in I}\sum_{t\in T^+}\sum_{j\in J}\sum_{g\in G}\sum_{n\in N}\sum_{m\in M}\sum_{x\in X}y_{(i,b,t,j)}^{(g,n,m,x)} = 1, \forall b \in B \quad (19)$$

$$\sum_{i\in I}\sum_{t\in T^-}\sum_{j\in J}\sum_{g\in G}\sum_{n\in N}\sum_{m\in M}\sum_{x\in X}\sum_{c\in C}y_{(j,c,t,i,d)}^{(g,n,m,x)} = 1, \forall d \in D \quad (20)$$

$$\sum_{i\in I}\sum_{t\in T^-}\sum_{j\in J}\sum_{g\in G}\sum_{n\in N}\sum_{m\in M}\sum_{x\in X}\left(y_{(j,c,t,i)}^{(g,n,m,x)} + \sum_{d\in D}y_{(j,c,t,i,d)}^{(g,n,m,x)}\right) = 1, \forall c \in C \quad (21)$$

(6) Capacity Constraints on the Forward and Reverse Direction Tracks.

If section is located before, between, or after the combination station and decomposition station specified in the combination and decomposition plan, its track capacity utilization depends on the number of trains associated with this plan. Specifically, a train occupies the capacity of a section only if its origin or destination lies beyond the section, while for sections between the combination and decomposition stations, capacity utilization is determined by the number of combined heavy-haul trains. The corresponding mathematical expressions are given as:

Before the Combination Station:

$$L_{(l,t)}^1 = \sum_{g\in G}\left(\psi_{g,l}^1 \times \sum_{g\in G}\sum_{n\in N}\sum_{x\in X}\sum_{i\in I}v_{i,l}^+ \times \left(\varphi_{(i,g,t,n,x)}^+ + \sum_{p\in P}\varphi_{(i,g,t,n,x,p)}^+\right)\right) \quad (22)$$

$$\forall t \in T^+, \forall l \in L, \forall g \in G^+$$

$$L_{(l,t)}^2 = \sum_{g\in G}\left(\psi_{g,l}^1 \times \sum_{g\in G}\sum_{n\in N}\sum_{x\in X}\sum_{j\in J}v_{j,l}^- \times \left(\varphi_{(j,g,t,n,x)}^- + \sum_{p\in P}\varphi_{(j,g,t,n,x)}^{re}\right)\right) \quad (23)$$

$$\forall t \in T^-, \forall l \in L, \forall g \in G^-$$

Between the Combination and Decomposition Station:

$$L_{(l,t)}^3 = \sum_{g\in G}\left(\psi_{g,l}^2 \times K_g \times \sum_{i\in I}\sum_{g\in G}\sum_{n\in N}\sum_{m\in M}\sum_{x\in X}\sum_{j\in J}\left(\sum_{b\in B}y_{(i,b,t,j)}^{(g,n,m,x)} + \sum_{a\in A}\sum_{p\in P}y_{(i,a,t,j)}^{(g,n,m,x,p)}\right)\right)$$
$$(24)$$

$$\forall t \in T^+, \forall l \in L, \forall g \in G^+$$

$$L_{(l,t)}^4 = \sum_{g\in G}\left(\psi_{g,l}^2 \times K_g \times \sum_{j\in J}\sum_{g\in G}\sum_{n\in N}\sum_{m\in M}\sum_{x\in X}\sum_{c\in C}\sum_{i\in I}\left(y_{(j,c,t,i)}^{(g,n,m,x)} + \sum_{d\in D}y_{(j,c,t,i,d)}^{(g,n,m,x)}\right)\right) \quad (25)$$

$$\forall t \in T^-, \forall l \in L, \forall g \in G^-$$

After the Decomposition Station:

$$L^5_{(l,t)} = \sum_{g \in G} \left(\psi^3_{g,l} \times \sum_{g \in G} \sum_{m \in M} \sum_{x \in X} \sum_{j \in J} v^+_{j,l} \times \left(\varphi^+_{(j,g,t,m,x)} + \sum_{p \in P} \varphi^+_{(j,g,t,m,x,p)} \right) \right) \tag{26}$$

$$\forall t \in T^+, \forall l \in L, \forall g \in G^+$$

$$L^6_{(l,t)} = \sum_{g \in G} \left(\psi^3_{g,l} \times \sum_{g \in G} \sum_{m \in M} \sum_{x \in X} \sum_{i \in I} v^-_{i,l} \times \left(\varphi^-_{(i,g,t,m,x)} + \varphi^{re}_{(i,g,t,m,x)} \right) \right) \tag{27}$$

$$\forall t \in T^-, \forall l \in L, \forall g \in G^-$$

Accordingly, the unified expressions for track capacity utilization in both forward and reverse directions are:

$$L^1_{(l,t)} + L^3_{(l,t)} + L^5_{(l,t)} \leq L^+_{(l,t)} \tag{28}$$

$$L^2_{(l,t)} + L^4_{(l,t)} + L^6_{(l,t)} \leq L^-_{(l,t)} \tag{29}$$

(7) Train Number Balance Constraint Before and After Combination and Decomposition Operations.

Forward Combination:

$$\sum_{i \in I} \varphi^+_{(i,g,t,n,x,p)} = \psi_{(g,n)} \times f^+_{(g,t,n,x,p)}, \forall g \in G^+, \forall t \in T^+, \forall x \in X, \forall n \in N \tag{30}$$

$$\sum_{i \in I} \varphi^+_{(i,g,t,n,x)} = \psi_{(g,n)} \times f^+_{(g,t,n,x)}, \forall g \in G^+, \forall t \in T^+, \forall x \in X, \forall n \in N \tag{31}$$

$$f^+_{(g,t,n_1,x,p)} = f^+_{(g,t,n_2,x,p)} \tag{32}$$

$$\forall g \in G^+, \forall t \in T^+, \forall x \in X, \forall p \in P, \forall n_1 \neq n_2 \in N \text{ and } \theta_{(g,n_1,n_2)} = 1$$

$$f^+_{(g,t,n_1,x)} = f^+_{(g,t,n_2,x)} \tag{33}$$

$$\forall g \in G^+, \forall t \in T^+, \forall x \in X, \forall n_1 \neq n_2 \in N \text{ and } \theta_{(g,n_1,n_2)} = 1$$

Forward Decomposition:

$$\sum_{j \in J} \varphi^+_{(j,g,t,m,x,p)} = \psi_{(g,m)} \times f^+_{(g,t,m,x,p)}, \forall g \in G^+, \forall t \in T^+, \forall x \in X, \forall m \in M \tag{34}$$

$$\sum_{j \in J} \varphi^+_{(j,g,t,m,x)} = \psi_{(g,m)} \times f^+_{(g,t,m,x)}, \forall g \in G^+, \forall t \in T^+, \forall x \in X, \forall m \in M \tag{35}$$

$$f^+_{(g,t,m_1,x,p)} = f^+_{(g,t,m_2,x,p)} \tag{36}$$

$$\forall g \in G^+, \forall t \in T^+, \forall x \in X, \forall m_1 \neq m_2 \in M \text{ and } \theta_{(g,m_1,m_2)} = 1$$

$$f^+_{(g,t,m_1,x)} = f^+_{(g,t,m_2,x)} \tag{37}$$

$$\forall g \in G^+, \forall t \in T^+, \forall x \in X, \forall m_1 \neq m_2 \in M \text{ and } \theta_{(g,m_1,m_2)} = 1$$

Reverse Combination:

$$\sum_{j \in J} \varphi^-_{(j,g,t,n,x)} = \psi_{(g,n)} \times f^-_{(g,t,n,x)}, \forall g \in G^-, \forall t \in T^-, \forall x \in X, \forall n \in N \tag{38}$$

$$\sum_{j \in J} \varphi^{re}_{(j,g,t,n,x)} = \psi_{(g,n)} \times f^{re}_{(g,t,n,x)}, \forall g \in G^-, \forall t \in T^-, \forall x \in X, \forall n \in N \tag{39}$$

$$f^-_{(g,t,n_1,x)} = f^-_{(g,t,n_2,x)} \tag{40}$$

$$\forall g \in G^-, \forall t \in T^-, \forall x \in X, \forall n_1 \neq n_2 \in N \text{ and } \theta_{(g,n_1,n_2)} = 1$$

$$f^{re}_{(g,t,n_1,x)} = f^{re}_{(g,t,n_2,x)} \tag{41}$$

$$\forall g \in G^-, \forall t \in T^-, \forall x \in X, \forall n_1 \neq n_2 \in N \text{ and } \theta_{(g,n_1,n_2)} = 1$$

Reverse Decomposition:

$$\sum_{i \in I} \varphi_{(i,g,t,m,x)}^{-} = \psi_{(g,m)} \times f_{(g,t,m,x)}^{-}, \forall g \in G^{-}, \forall t \in T^{-}, \forall x \in X, \forall m \in M \tag{42}$$

$$\sum_{i \in I} \varphi_{(i,g,t,m,x)}^{re} = \psi_{(g,m)} \times f_{(g,t,m,x)}^{re}, \forall g \in G^{-}, \forall t \in T^{-}, \forall x \in X, \forall m \in M \tag{43}$$

$$f_{(g,t,m_1,x)}^{-} = f_{(g,t,m_2,x)}^{-} \tag{44}$$
$$\forall g \in G^{-}, \forall t \in T^{-}, \forall x \in X, \forall m_1 \neq m_2 \in M \text{ and } \theta_{(g,m_1,m_2)} = 1$$

$$f_{(g,t,m_1,x)}^{re} = f_{(g,t,m_2,x)}^{re} \tag{45}$$
$$\forall g \in G^{-}, \forall t \in T^{-}, \forall x \in X, \forall m_1 \neq m_2 \in M \text{ and } \theta_{(g,m_1,m_2)} = 1$$

The $f_{(g,t,n,x,p)}^{+}$, $f_{(g,t,n,x)}^{+}$, $f_{(g,t,n,x)}^{-}$, $f_{(g,t,n,x)}^{re}$ in the above constraints are intermediate variables, representing that in time period t, for each combined heavy-haul train of vehicle type x using combination and decomposition plan g, $\psi_{(g,n)}$ trains of vehicle type x and load capacity n are needed to form the combined heavy-haul train. Similarly, the $f_{(g,t,m,x,p)}^{+}$, $f_{(g,t,m,x)}^{+}$, $f_{(g,t,m,x)}^{-}$, $f_{(g,t,m,x)}^{re}$ in the above constraint are intermediate variables, representing that in time period t, for each combined heavy-haul train of vehicle type x using combination and decomposition plan g, $\psi_{(g,m)}$ trains of vehicle type x and load capacity m are produced after decomposition. Taking a 15,000-ton combined heavy-haul train as an example, this combination and decomposition plan consists of one 10,000-ton train and one 5,000-ton train, i.e., $\theta_{(g,n_1,n_2)} = 1$. The constraints to be satisfied are constraints (32), (33), (36), (37), (40), (41), (44), and (45).

(8) Combination and Decomposition Capacity Constraint.

The number of heavy-haul trains combined or decomposed at each station is limited by the corresponding combination or decomposition capacity of that station.

$$\sum_{i \in I} \sum_{j \in J} \sum_{g \in G} \sum_{n \in N} \sum_{m \in M} \sum_{x \in X} \left(\omega_{(g,z)} \times \left(\sum_{b \in B} y_{(i,b,t,j)}^{(g,n,m,x)} + \sum_{a \in A} \sum_{p \in P} y_{(i,a,t,j)}^{(g,n,m,x,p)} \right) \right) \leq C_t^z, \forall t \in T^{+}, \forall z \in Z^{+} \tag{46}$$

$$\sum_{i \in I} \sum_{j \in J} \sum_{g \in G} \sum_{n \in N} \sum_{m \in M} \sum_{x \in X} \left(\omega_{(g,f)} \times \left(\sum_{b \in B} y_{(i,b,t,j)}^{(g,n,m,x)} + \sum_{a \in A} \sum_{p \in P} y_{(i,a,t,j)}^{(g,n,m,x,p)} \right) \right) \leq C_t^f, \forall t \in T^{+}, \forall f \in F^{+} \tag{47}$$

$$\sum_{i \in I} \sum_{j \in J} \sum_{g \in G} \sum_{n \in N} \sum_{m \in M} \sum_{x \in X} \left(\omega_{(g,z)} \times \left(y_{(j,c,t,i)}^{(g,n,m,x)} + \sum_{d \in D} y_{(j,c,t,i,d)}^{(g,n,m,x)} \right) \right) \leq C_t^z, \forall t \in T^{-}, \forall z \in Z^{-} \tag{48}$$

$$\sum_{i \in I} \sum_{j \in J} \sum_{g \in G} \sum_{n \in N} \sum_{m \in M} \sum_{x \in X} \left(\omega_{(g,f)} \times \left(y_{(j,c,t,i)}^{(g,n,m,x)} + \sum_{d \in D} y_{(j,c,t,i,d)}^{(g,n,m,x)} \right) \right) \leq C_t^f, \forall t \in T^{-}, \forall f \in F^{-} \tag{49}$$

(9) Coal Demand Satisfaction Constraint.

The heavy train operation plan must ensure that coal demand at all stations is satisfied while optimizing transportation routes.

$$X_{(j,p)} = \sum_{x \in X} \sum_{t \in T} \sum_{g \in G} \sum_{m \in M} m \times \varphi_{(j,g,t,m,x,p)}^{+}, \forall j \in J, \forall p \in P \tag{50}$$

$$X_{(j,p)} \geq X_{(j,p)}^{need}, \forall j \in J, \forall p \in P \tag{51}$$

(10) Loading Station Empty Car Demand Constraint.

When performing empty car distribution, the empty car demand at each loading station during the decision-making period of the heavy train operation plan must be met (which is input as a parameter for the empty car distribution as the heavy train operation plan is determined). The constraint expression is as follows:

$$S_{(i,t_2,x)} = \sum_{j \in J} \sum_{c \in C} \sum_{t_1 \in T^-} \sum_{i \in I} \sum_{g \in G} \sum_{n \in N} \sum_{m \in M} \sum_{x \in X} \left(y_{(j,c,t_1,i)}^{(g,n,m,x)} \times \gamma_{(j,n,g,i,t_1,t_2)}^- + \sum_{d \in D} y_{(j,c,t_1,i,d)}^{(g,n,m,x)} \times \gamma_{(j,n,g,i,t_1,t_2)}^{re} \right)$$

$$(52)$$

$$\forall i \in I, \forall t_2 \in T^+, t_1 \in T^-, \forall x \in X$$

$$S_{(i,t,x)} - O_{(i,t,x)} + L_{(i,t,x)} = N_{(i,t,x)}, \forall i \in I, \forall x \in X, t = 1 \tag{53}$$

$$S_{(i,t,x)} - O_{(i,t,x)} + L_{(i,t,x)} + O_{(i,t-1,x)} = N_{(i,t,x)}, \forall i \in I, \forall t \geq 2 \in T^+, \forall x \in X \tag{54}$$

$$O_{(i,t,x)} \leq M \times P_{(i,t,x)} \tag{55}$$

$$L_{(i,t,x)} \leq M \times (1 - P_{(i,t,x)}) \tag{56}$$

$$O_{(i,t,x)} \geq 0 \tag{57}$$

$$L_{(i,t,x)} \geq 0 \tag{58}$$

If the number of empty cars supplied by the empty car distribution plan at time t for loading station i is insufficient to satisfy the heavy train operation plan ($L_{(i,t,x)} \geq 0$), the empty car distribution plan cannot guarantee the feasibility of the heavy train operation plan, and the heavy train operation plan must be adjusted and reintroduced as a constraint. Conversely, if the supplied empty cars exceed the required number ($O_{(i,t,x)} \geq 0$), the surplus empty cars are carried over to the next operation period and must satisfy constraint (54). The variable $P_{(i,t,x)}$ is an intermediate binary variable used to enforce mutual exclusivity between variables $O_{(i,t,x)}$ and $L_{(i,t,x)}$, such that at most one can be positive.

4. Case Study.

4.1. Road Network and its Parameter Setting.
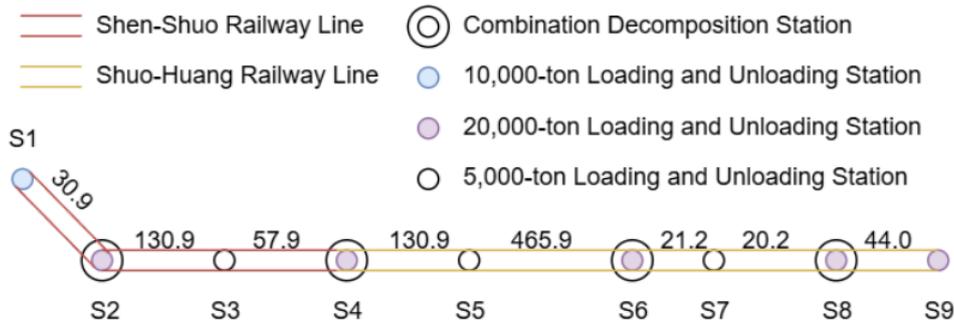The selected route consists of 9 stations and 8 sections, as shown in Figure 1.

Figure 1. Schematic diagram of the line

4.1.1. *Station Capacity and Section Operation Time Data.* To simplify the model, line capacity is set to 8 heavy-haul trains per time period. Loading times at forward loading stations are 40, 80, and 160 minutes for 5,000-, 10,000-, and 20,000-ton trains, respectively, with a maximum loading capacity of 25,000 tons per period, while loading times at reverse loading stations are 60, 120, and 240 minutes, with a maximum capacity of 20,000 tons per period. Combination times are 40, 60, and 80 minutes for 10,000-, 15,000-, and 20,000-ton trains, respectively, and decomposition time is fixed at 15 minutes. Detailed section operation times and loading station characteristics are provided in Table 1 and Table 2.

Table 1. Nature of loading and unloading station

Number	Station Name	Station Type
S1	Daliuta South Station	10,000-ton loading and unloading station
S2	Shenmu North Station	20,000-ton loading and unloading station (technical station)
S3	Hanjialou Station	5,000-ton loading and unloading station
S4	Shenchi South Station	20,000-ton loading and unloading station (technical station)
S5	Dongye Station	Intermediate Station
S6	Cangzhou West (Power Plant) Station	20,000-ton loading and unloading station (technical station)
S7	Litianmu (Power Plant) Station	5,000-ton loading and unloading station
S8	Huanghua South Station	20,000-ton loading and unloading station (technical station)
S9	Huanghua Port Station	20,000-ton loading and unloading station

Table 2. Train operation time within a section

Section	Train operation time	Section	Train operation time
S1-S2	23	S5-S6	372
S2-S3	98	S6-S7	17
S3-S4	43	S7-S8	16
S4-S5	105	S8-S9	36

4.1.2. *Freight Transportation Demand Data.* A day is divided into six 4-hour time periods. Based on forward coal and non-coal transportation demand, as well as reverse cargo demand, the corresponding transportation demand data are summarized in Table 3, Table 4, and Table 5.

4.1.3. *Empty Car Flow Data.* During the decision-making period for the empty car distribution plan, a total of 82 empty car units are available. The allocable dynamic empty car flows for each time period are shown in Table 6.

Table 3. Non-coal cargo transportation demand

No.	Load	Unload	Type	No.	Load	Unload	Type
1	S1	S6	C70	11	S3	S8	C80
2	S1	S7	C70	12	S3	S8	C80
3	S1	S8	C80	13	S3	S9	C80
4	S2	S6	C70	14	S4	S6	C70
5	S2	S8	C70	15	S4	S6	C70
6	S2	S7	C70	16	S4	S9	C70
7	S2	S7	C80	17	S4	S9	C70
8	S2	S9	C80	18	S4	S7	C80
9	S3	S6	C70	19	S4	S7	C80
10	S3	S7	C70	20	S4	S9	C80

Table 4. Reverse cargo transportation demand

No.	Load	Unload	Type	No.	Load	Unload	Type
1	S6	S1	C60	16	S8	S2	C60
2	S6	S1	C60	17	S8	S2	C60
3	S6	S2	C60	18	S8	S3	C60
4	S6	S2	C70	19	S9	S4	C60
5	S6	S3	C60	20	S9	S3	C60
6	S6	S3	C60	21	S9	S3	C60
7	S7	S1	C60	22	S9	S2	C80
8	S7	S1	C60	23	S9	S2	C60
9	S7	S2	C70	24	S9	S1	C60
10	S7	S2	C60	25	S8	S3	C60
11	S7	S3	C60	26	S8	S3	C60
12	S7	S3	C60	27	S8	S1	C60
13	S8	S1	C60	28	S9	S1	C80
14	S8	S1	C60	29	S9	S1	C70
15	S8	S1	C60	30	S9	S1	C70

4.2. **Result Analysis.**

4.2.1. *Collaborative Operation Plan for Forward and Reverse Cargo Transportation and Empty Car Distribution Plan.* The collaborative optimization of heavy and empty car flows is achieved by dynamically allocating empty car flows to meet the forward and reverse cargo transportation demands. The input for solving the reverse cargo train operation plan and empty car allocation plan is the demand for each type of empty car at each loading station in the initial forward cargo train operation plan. The empty car supply provided by the empty car allocation plan and the loaded cars waiting to be unloaded provided by the reverse cargo trains serve as feedback to re-optimize the forward cargo train operation plan. Using the Gurobi solver, the in-transit times for forward and reverse cargo transportation are 51,197 minutes and 22,654 minutes, respectively, and the empty car detention penalty is 3,200 minutes. The resulting forward and reverse cargo train

Table 5. Coal cargo transportation demand

No.	Station	Type	No.	Station	Type	No.	Station	Type
1	S1	Clean	22	S2	Clean	43	S3	Blended
2	S1	Clean	23	S2	Clean	44	S3	Blended
3	S1	Clean	24	S2	Clean	45	S3	Blended
4	S1	Clean	25	S2	Fine	46	S3	Clean
5	S1	Clean	26	S2	Fine	47	S3	Clean
6	S1	Clean	27	S2	Fine	48	S3	Clean
7	S1	Clean	28	S2	Fine	49	S3	Clean
8	S1	Clean	29	S2	Fine	50	S3	Clean
9	S1	Clean	30	S2	Fine	51	S4	Fine
10	S1	Fine	31	S3	Fine	52	S4	Fine
11	S1	Fine	32	S3	Fine	53	S4	Fine
12	S1	Fine	33	S3	Fine	54	S4	Fine
13	S1	Fine	34	S3	Fine	55	S4	Fine
14	S1	Blended	35	S3	Fine	56	S4	Fine
15	S1	Blended	36	S3	Fine	57	S4	Clean
16	S1	Blended	37	S3	Blended	58	S4	Clean
17	S1	Blended	38	S3	Blended	59	S4	Clean
18	S1	Blended	39	S3	Blended	60	S4	Clean
19	S1	Blended	40	S3	Blended	61	S4	Blended
20	S1	Blended	41	S3	Blended	62	S4	Blended
21	S2	Clean	42	S3	Blended			

operation plans and the empty car distribution plan are summarized in 4-hour periods, as shown in Figure 2.

Analysis of the forward cargo train operation plan shows that 15 out of 29 combined heavy-haul trains operate in a direct-combination mode, mainly because all unloading stations except Litianmu have a handling capacity of 20,000 tons. Consequently, direct combination is preferred under empty car shortages and tight line capacity to reduce decomposition-related time consumption.

Compared with the forward plan, the empty car distribution and reverse cargo train operation plans exhibit a higher proportion of 20,000-ton direct and combined trains, due to sufficient empty car accumulation and 20,000-ton handling capacity at most forward unloading stations, while only Shenchi South station supports 20,000-ton loading at the forward loading end, requiring additional decomposition for other stations.

For 5,000-ton unit direct trains, 10 out of 22 trains directly supply empty cars for forward transportation, while the remaining 12 first undertake reverse transportation and are reused for forward operations after unloading, ensuring full satisfaction of reverse demand and effective coordination between heavy and empty car flows. However, limited line capacity still results in 3,200 minutes of empty car detention, as forward loading stations must wait for reverse trains to unload before obtaining available empty cars, causing inter-period empty car backlog.

Table 6. Station storage unit empty car data

No.	Supply	Type	Time	No.	Supply	Type	Time
1	C6	C60	1	42	C8	C60	4
2	C6	C60	1	43	C8	C60	6
3	C6	C60	1	44	C8	C60	6
4	C6	C60	2	45	C8	C60	6
5	C6	C60	2	46	C9	C60	1
6	C6	C60	3	47	C9	C60	1
7	C6	C60	3	48	C9	C60	1
8	C6	C60	4	49	C9	C60	2
9	C6	C60	4	50	C9	C60	2
10	C6	C60	4	51	C9	C60	2
11	C6	C60	4	52	C9	C60	3
12	C6	C60	4	53	C9	C60	3
13	C6	C60	5	54	C9	C60	3
14	C6	C60	5	55	C9	C60	4
15	C6	C60	5	56	C9	C60	4
16	C6	C60	6	57	C9	C60	4
17	C6	C60	6	58	C9	C60	6
18	C6	C60	6	59	C9	C60	6
19	C6	C60	6	60	C9	C60	6
20	C7	C60	1	61	C9	C60	6
21	C7	C60	1	62	C9	C60	6
22	C7	C60	1	63	C6	C70	1
23	C7	C60	2	64	C6	C70	4
24	C7	C60	2	65	C6	C70	4
25	C7	C60	2	66	C6	C70	5
26	C7	C60	3	67	C6	C70	5
27	C7	C60	3	68	C7	C70	1
28	C7	C60	3	69	C7	C70	3
29	C7	C60	4	70	C7	C70	5
30	C7	C60	5	71	C7	C80	1
31	C7	C60	5	72	C7	C80	1
32	C8	C60	1	73	C7	C80	4
33	C8	C60	1	74	C8	C70	2
34	C8	C60	1	75	C8	C80	2
35	C8	C60	2	76	C8	C80	4
36	C8	C60	2	77	C8	C80	5
37	C8	C60	2	78	C9	C70	1
38	C8	C60	3	79	C9	C70	5
39	C8	C60	3	80	C9	C80	3
40	C8	C60	4	81	C9	C80	5
41	C8	C60	4	82	C9	C80	5

4.2.2. *Operation Plan for Forward and Reverse Cargo Transportation and Empty Car Distribution Plan with Forward Cargo Transportation Priority.* To further verify the necessity of collaborative optimization of forward and reverse transportation demands in heavy
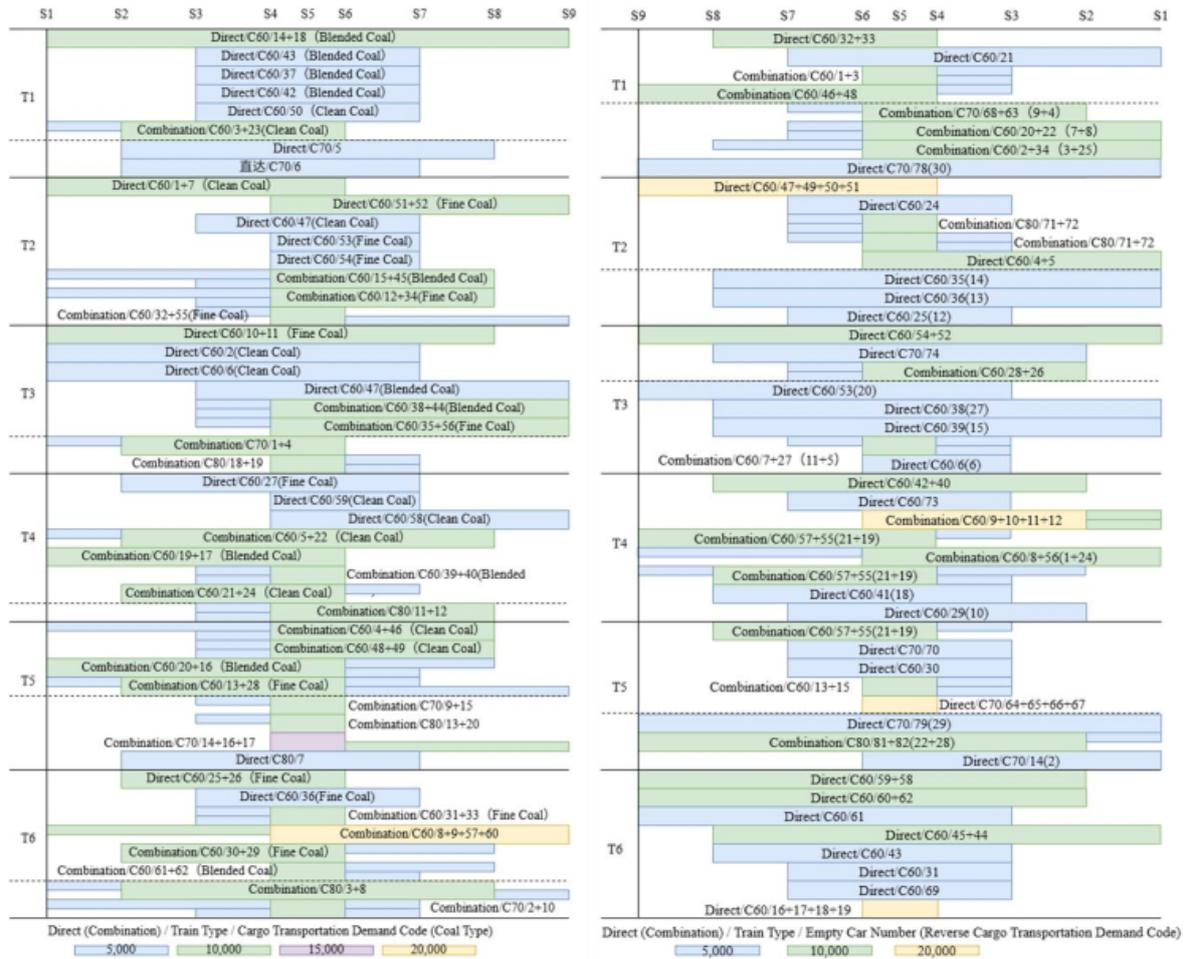
Figure 2. Forward Cargo Train Operation Plan, Empty Car Distribution Plan, and Reverse Cargo Train Operation for Collaborative Forward and Reverse Transportation

and empty car coordination, an optimization method prioritizing the forward heavy train operation plan was adopted to generate the forward and reverse heavy train operation plans and the empty car distribution plan. Using the Gurobi solver, the in-transit times for forward and reverse cargo transportation are 50,782 minutes and 23,483 minutes, respectively, with five reverse transportation demands unmet. The resulting heavy train operation plan and empty car distribution plan are shown in Figure 3.

Under the collaboration of the forward transportation priority empty car distribution plan and the heavy train operation plan of the forward and reverse transport, the loading demands at the loading stations in the forward heavy train operation plan were fully satisfied, and the detention time costs for empty cars at these stations were avoided. However, 5 reverse transportation demands remained unmet. Further analysis revealed that all of these unmet reverse transportation demands originated from Huanghua Port Station, with 2 demands having Shenmu North Station as the destination and 3 demands having Daliuta Station as the destination. The travel time between these three stations is the longest section on the entire line. If empty cars were to undertake these reverse transportation tasks, they would still need to complete unloading operations upon reaching the loading station, which would prevent them from being promptly allocated for forward

transportation, thus affecting the coordination of heavy and empty car flows. As a result, under the forward cargo transportation priority optimization method, these reverse transportation demands were excluded.
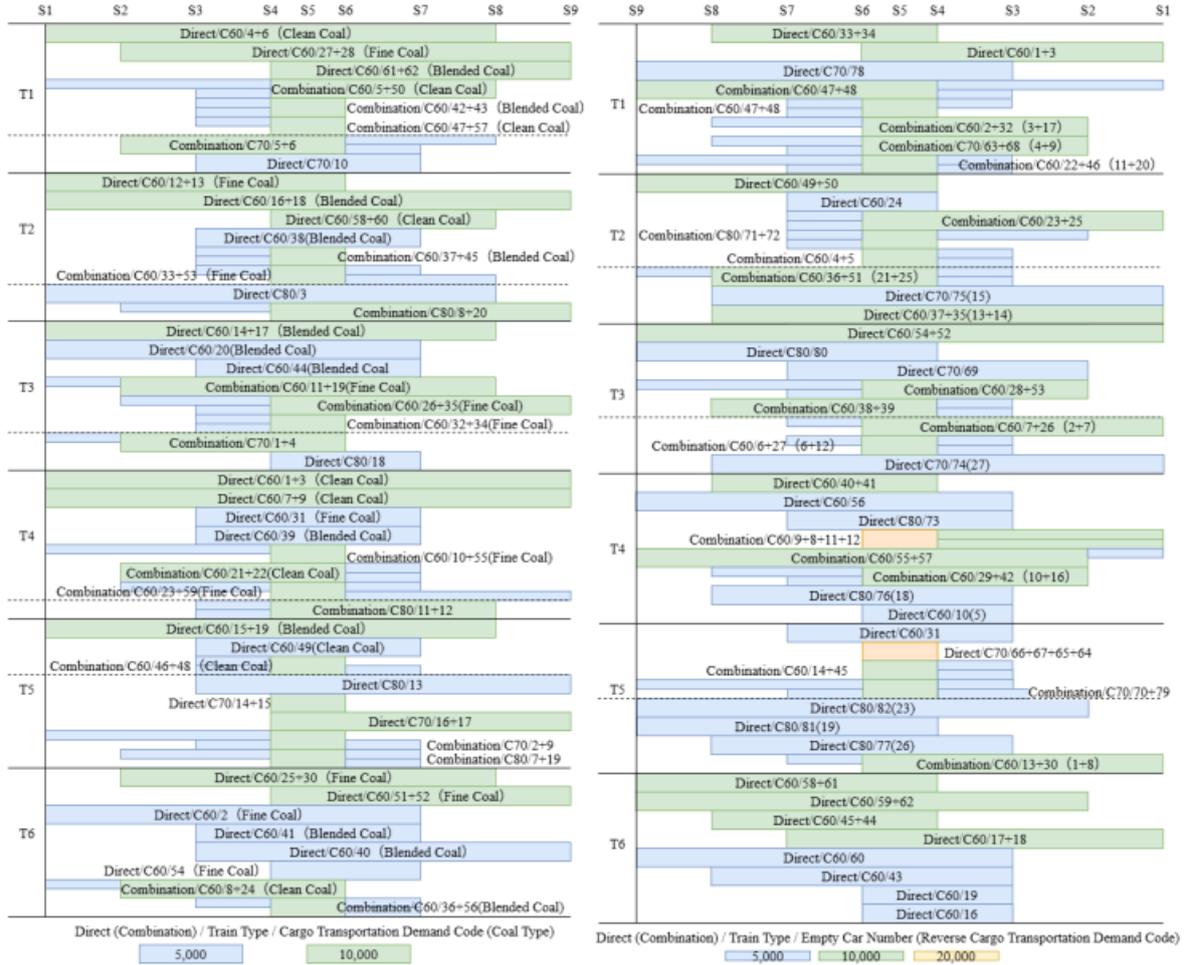

Figure 3. Forward Cargo Train Operation Plan, Empty Car Distribution Plan, and Reverse Cargo Train Operation with Forward Transportation Priority

4.2.3. *Comparison and Analysis of Optimization Results.* Based on a comparison between the forward transportation priority method and the collaborative forward–reverse transportation method, this section evaluates the effectiveness of heavy–empty car collaborative optimization considering reverse transportation, focusing on two aspects.

(1) The operation of forward heavy-haul trains after optimization

As shown in Table 7, considering collaborative forward–reverse optimization significantly increases the number of combined heavy-haul trains compared with the forward-priority plan, including the introduction of one 15,000-ton and one 20,000-ton combined train. This increase is mainly driven by imbalanced empty car arrivals caused by reverse transportation, particularly at Daliuta Station, which results in fluctuating loading intensity. When station-stored empty cars are insufficient, more direct trains are organized; when empty cars accumulate, combined trains are preferred to improve loading efficiency and relieve line congestion. As illustrated in Figure 2, direct trains dominate in the first time period, while the proportion of combined trains increases in later periods. The

Table 7. Types and Quantities of Forward Heavy-Haul Freight Trains

Method	Direct Train		Combined Train		
	5,000 tons	10,000 tons	Only Combined	Only Decomposed	Combined and Decomposed
Forward Transportation Priority	14	14	12	2	6
Collaborative Forward and Reverse Transportation	22	7	7	6	6

emergence of 15,000-ton combined trains is attributable to heterogeneous destinations of non-coal freight under line capacity and empty car constraints.

Overall, collaborative forward–reverse optimization effectively mitigates empty car supply imbalance, flexibly adjusts loading decisions, and enhances both station operation efficiency and line capacity utilization through adaptive combination and decomposition strategies.

(2) The completion of reverse cargo transportation demands, and the degree of matching between heavy and empty car demands. According to Table 8, the

Table 8. Completion Status of Reverse Cargo Transportation Demand

	Objective Function Value	Variation Range	Completion Status of Reverse Cargo Transportation Demand	Variation Range
Forward Transportation Priority	74265	+3.64%	83.3% (25/30)	+16.70%
Collaborative Forward and Reverse Transportation	76970		100% (30/30)	

total objective value under collaborative optimization is 76,970 minutes, comprising 51,197 minutes of forward in-transit time, 22,653 minutes of reverse in-transit time, and 3,120 minutes of empty car detention cost. Compared with the forward-priority plan, forward in-transit time increases by only 0.83%, while the total objective value increases by 3.64% after accounting for empty car detention and reverse demand penalties. However, the completion rate of reverse transportation demand improves by approximately 16.70%, generating additional revenue and significantly enhancing overall transportation efficiency.

In summary, by integrating dynamic demand characteristics, station empty car inventory adjustment, and flexible combination–decomposition strategies, the proposed optimization method ensures coordinated heavy–empty car operations, improves resource utilization, and enhances plan feasibility and economic performance, thereby validating the effectiveness and practicality of the proposed model and algorithm.

5. **Conclusion.** (1) This paper addresses the collaborative optimization of heavy and empty car flows in CHN ENERGY's heavy-haul railway transportation system. It establishes a two-stage optimization model with the objectives of minimizing the in-transit time for both forward and reverse cargo, as well as minimizing the delays in meeting empty car demand and the costs of empty car detention. The model's effectiveness was validated using the Gurobi solver in a complex scenario involving nine stations on the Shen–Shuo line and the Shuo–Huang line.

(2) Practical case validation indicates that, although the freight train operation plan incorporating reverse cargo transportation demand results in a slight 3.64% increase in overall transportation time, flexible combination and decomposition strategies, along with adjustments to the loading quantity decision, successfully resolved the issue of uneven empty car arrivals caused by reverse transportation and optimized the succession of heavy and empty car flows. On this basis, the completion rate of reverse cargo orders increased by 16.70 percentage points, fully unlocking the potential of reverse transportation resources. This optimization solution demonstrates the model's effectiveness in improving transportation efficiency, resource utilization, and economic benefits, providing a feasible solution to address the imbalance of bi-directional transportation in heavy-haul railways.

Future research could focus on integrating the operational profitability of coal and non-coal goods, developing a multi-objective planning model that also considers operational benefits, thus maximizing overall operational profitability while optimizing transportation efficiency. By balancing transportation timeliness, costs, and benefits, this would provide more accurate decision-making support for the practical operation of CHN ENERGY's bi-directional transportation and promote the intelligent and efficient development of its heavy-haul railway transportation system.

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